Minimum-Latency Beaconing Schedule

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Outline

- Problem Description
- First-Fit Beaconing Scheduling
- Strip-wise Beaconing Scheduling

Network Model

- V: a finite planar set of nodes
- Synchronous transmissions
- Beaconing radius: one
- Interference radius

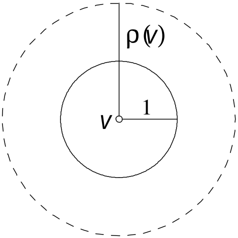

Figure: The beaconing range and interfrence range of each node.

Primary And Secondary Conflicts

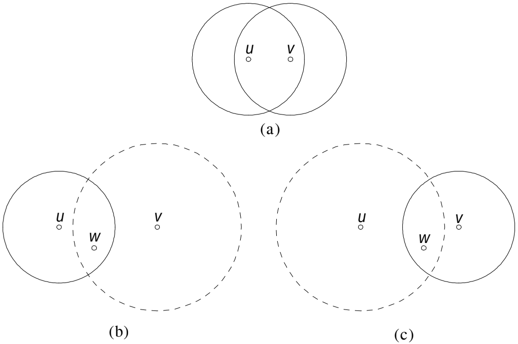

Figure: (a): Primary conflict; (b) and (c): secondary conflict.

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- Conflict graph H: a pair of nodes are adjacent in H iff they conflict with each other
- A beaconing schedule corresponds to a vertex coloring of H

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  2. Strip-wise beaconing scheduling: uniform interference radii

Roadmap

- Problem Description
- **First-Fit Beaconing Scheduling**
- Strip-wise Beaconing Scheduling

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First-fit coloring of H in smallest-degree-last ordering

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$$\beta_\rho = \left\lfloor \frac{\pi}{\sqrt{3}} \left(\frac{\rho+1}{\max\{1, \rho-1\}} \right)^2 + \left(\frac{\pi}{2} + 1 \right) \frac{\rho+1}{\max\{1, \rho-1\}} \right\rfloor + 1.$$

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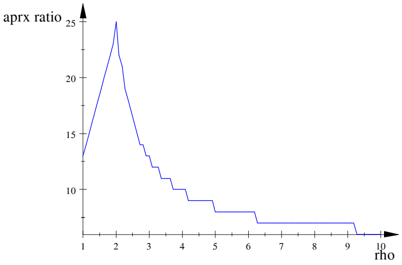

Figure: The plot of β_ρ.

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$$\left|U \cap S\right| \leq \frac{2\pi}{\sqrt{3}} r^2 + \pi r + 1,$$
$$\left|U \cap S'\right| \leq \frac{\pi}{\sqrt{3}} r^2 + \left(\frac{\pi}{2} + 1\right) r + 1.$$

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B_u, B_v and B_w: unit-disks centered at u, v and w resp. s.t.$1 < \|uv\| \leq 2$ and $\|vw\| > 1$.

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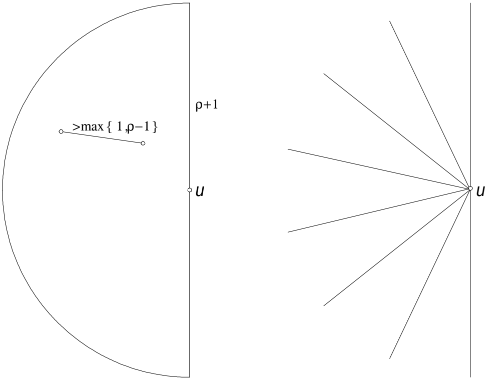
Then, $B_u \cap B_v \subseteq B_w$ if one of the following two conditions holds

1. $\|uw\| \leq 1$ and $\widehat{vuw} \leq \frac{\pi}{6}$.
2. $1 < \|uw\| \leq \|uv\|$ and $\widehat{vuw} \leq 2 \arcsin \frac{1}{4}$.

Uniform interference Radii

α^*: inductive independence number of lexicographic ordering

Lemma $\alpha^* \leq \beta_\rho$ in general and $\alpha^* \leq 7$ when $\rho = 1$.



(a) (b)

Arbitrary Interference Radii

α^*: inductive independence number of interference radius decreasing ordering

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Lemma $\alpha^* \leq 61$.

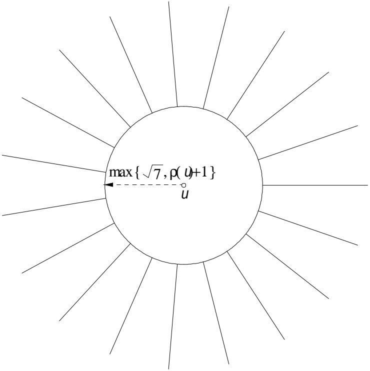
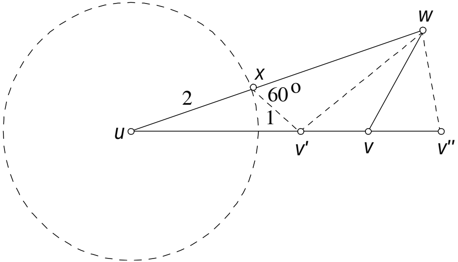


Figure: At most 45 inside the disk and 18 inside.

Yet Angle Separation

Lemma Suppose that $\sqrt{7} \leq \|uv\| \leq \|uw\| \leq 1 + r$ and $\widehat{vuw} \leq \arccos \frac{5}{2\sqrt{7}} \approx 19.107^\circ$. Then $\|vw\| \leq \max\{1, r-1\}$.



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- **Strip-wise Beaconing Scheduling**

The Height Function

$$h(\rho) = \begin{cases} \cos \frac{\arccos \frac{1}{2\rho} + \arccos \frac{\rho}{2}}{2} & \text{if } 1 \leq \rho \leq \sqrt{\frac{3+\sqrt{13}}{2}}; \\ \sin \left(\arccos \frac{1}{2\rho} - \arcsin \frac{1}{\rho} \right) & \text{if } \sqrt{\frac{3+\sqrt{13}}{2}} \leq \rho \leq 2; \\ (\rho - 1) \sin \left(\arccos \frac{\rho - 1}{2\rho} - \arcsin \frac{1}{\rho} \right) & \text{if } \rho > 2. \end{cases}$$

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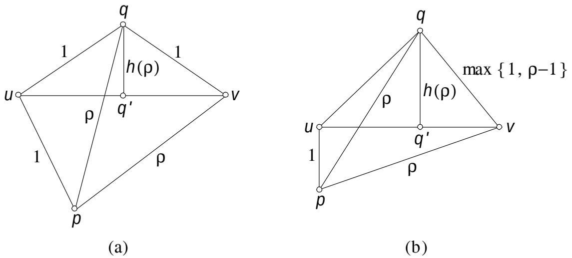
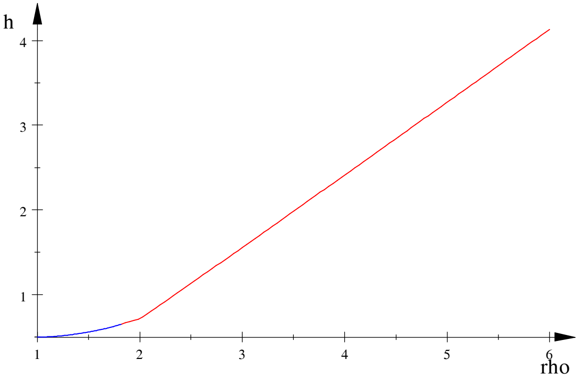


Figure: (a) $1 \le \rho \le \sqrt{\frac{3+\sqrt{13}}{2}}$, (b) $\rho > \sqrt{\frac{3+\sqrt{13}}{2}}$.

Plot of The Height Function



A Transitivity of Independence

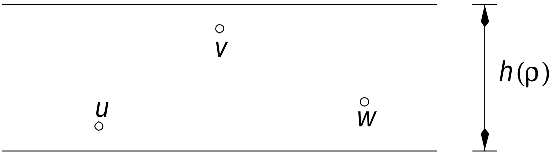


Figure: Suppose u and v are indepdent, v and w are independent. Then, u and w are independent.

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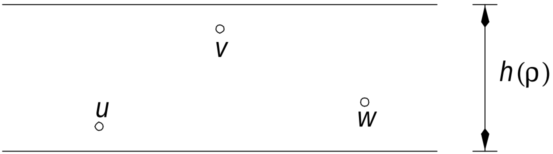

Figure: Suppose u and v are indepdent, v and w are independent. Then, u and w are independent.

Implication: The conflict graph of all nodes in a horizontal strip is a cocomparability graph, and its minimum coloring can be computed in polynomial time.

Strip-wise Scheduling: Spatial Divide & Conquer & Reuse

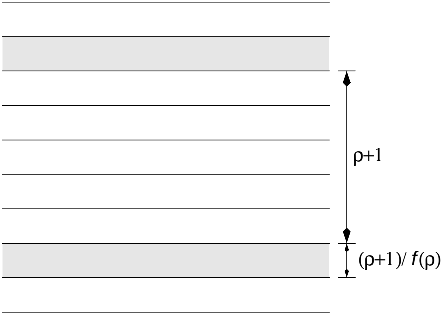

Figure: Partition of the plane into strips of height $(\rho+1)/f(\rho)$ where $f(\rho) = \lceil(\rho+1)/h(\rho)\rceil$.

Approximation Bound

$$\text{approx. ratio} \leq 1 + f(\rho)$$

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$$f(\rho) = \begin{cases} 4 & \text{if } \rho = 1; \\ 5 & \text{if } 1 < \rho < 2.0632; \\ 4 & \text{if } 2.0632 \leq \rho < 2.5689; \\ 3 & \text{if } 2.5689 \leq \rho < 4.2462; \\ 2 & \text{if } \rho \geq 4.2462. \end{cases}.$$