Minimum Latency Edge Scheduling under 802.11 Interference Model

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Outline

- Problem Description
- First-Fit Edge Scheduling
- Strip-wise Edge Scheduling

802.11 Interference Model

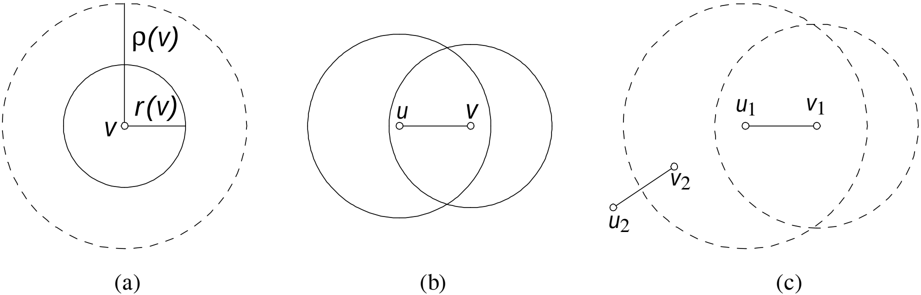

Figure: (a) Communication range and interference range of each node; (b) a communication edge; (c) a conflicting pair of communication edges.

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Given a set A of communication edges

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- An edge schedule for A corresponds to a vertex coloring of H

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  - the positions of all nodes are available

Summary on Scheduling Algorithms

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  - uniform communication/interference radii: better approx.

Roadmap

- Problem Description
- **First-Fit Edge Scheduling**
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First-fit edge scheduling: first-fit coloring of the conflict graph in the smallest-degree-last ordering

Theorem

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Proof overview: find an ordering with inductive independence number at most 7 or 23.

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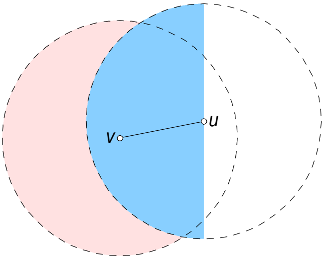
Lemma

$\alpha^* \leq 7$.



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$\alpha^* \leq 23$.

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Fix an edge $e = uv$ with $\rho(u) \geq \rho(v)$ and an $I \in \mathcal{I}$ in $N_{\prec}(e)$.

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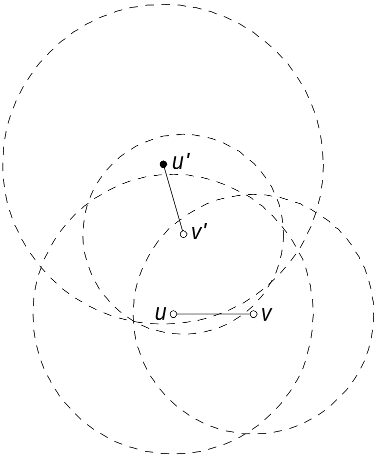
- Classification of edges in I into four types
  - I_j with $1 \leq j \leq 4$: the edges in I of the j-th type
- Prove $|I_1 \cup I_2| \leq 12$ and $|I_3 \cup I_4| \leq 11$, which together imply that $|I| \leq 23$.

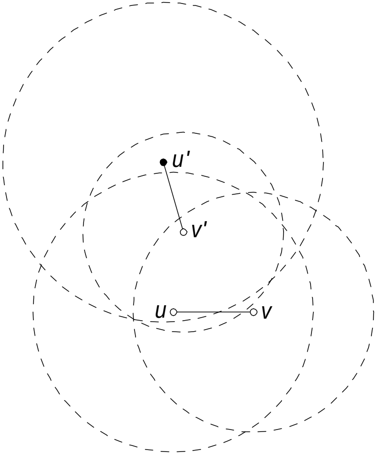
The First Type

- At least one end s.t. (1) it has larger or equal interference radius than u, and (2) its interference range contains u.



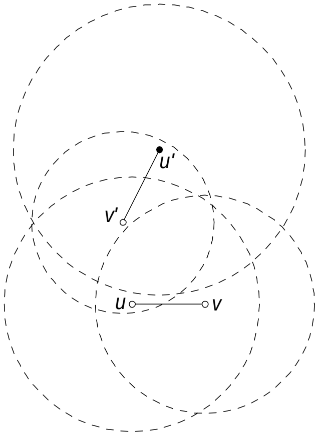
The First Type

- At least one end s.t. (1) it has larger or equal interference radius than u, and (2) its interference range contains u.
- Any such end is chosen as its representative.



The Second Type

- Not of the first type



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- Not of the first type
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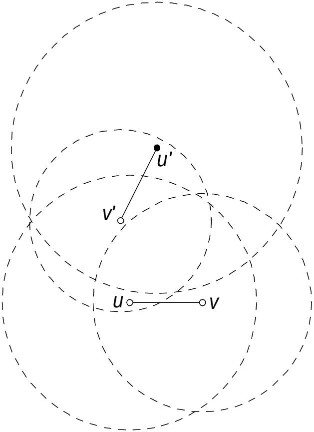
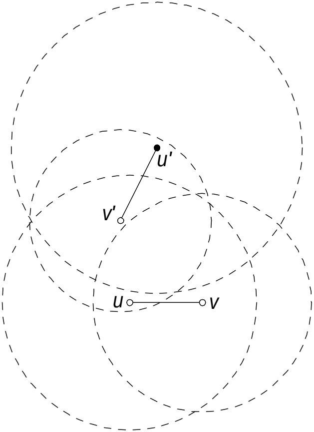


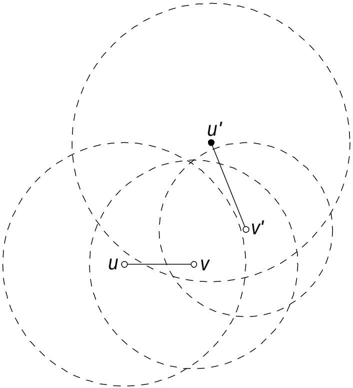
The Second Type

- Not of the first type
- At least one end lies in the interference range of u.
- Its end with larger interference radius is chosen as its representative.



The Third Type

- Not of the first and second types



The Third Type

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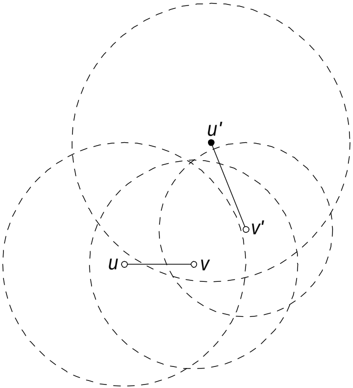
- At least one end s.t. (1) it has larger or equal interference radius than v, and (2) its interference range contains v.



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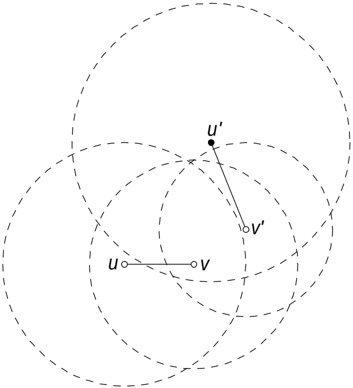
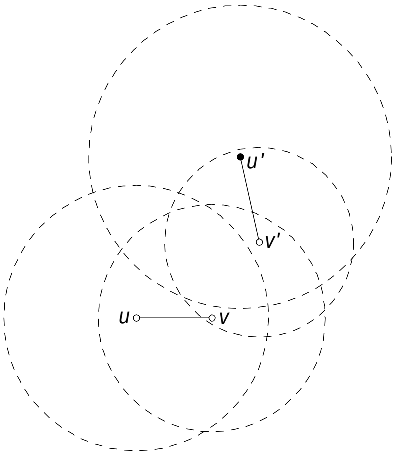


The Fourth Type

- Any remaining edge



The Fourth Type

- Any remaining edge
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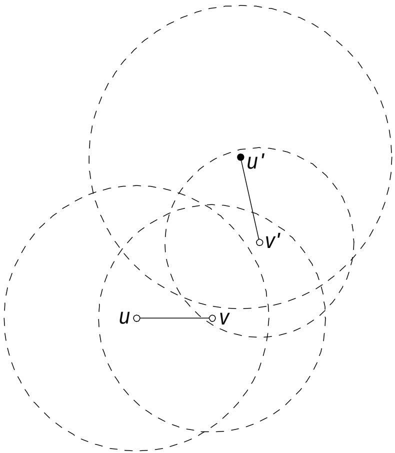

Angle Separations

1. w_1, w_2: representatives of two edges in $I_1 \cup I_2$. Then, $\widehat{w_1 u w_2} > 2 \arcsin \frac{1}{4}$.
2. w_1, w_2: representatives of two edges in $I_3 \cup I_4$. Then, $\widehat{w_1 v w_2} > 2 \arcsin \frac{1}{4}$.
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$$
\begin{aligned}
|I_1 \cup I_2| &\leq \left\lceil \frac{2\pi}{2 \arcsin \frac{1}{4}} \right\rceil - 1 = 12, \\
|I_3 \cup I_4| &\leq \left\lceil \frac{2\pi - \frac{\pi}{3}}{2 \arcsin \frac{1}{4}} \right\rceil = 11.
\end{aligned}
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- Better scheduling algorithm

A Height Function

$$h(\rho) = \sqrt{\rho^2 - \frac{1}{4}} \cos\left(\frac{\pi}{6} + \arcsin\frac{1}{2\rho}\right), \rho \in [1, \infty)$$

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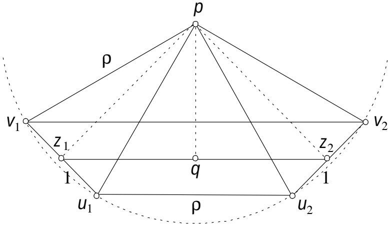

Figure: The distance between p and $z_1 z_2$ is exactly $h(\rho)$.

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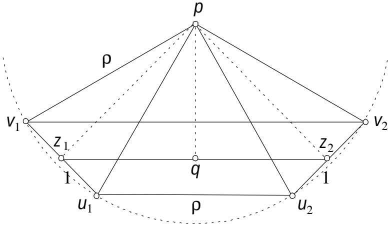

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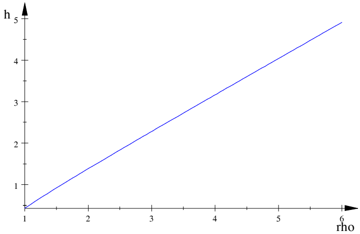


Figure: Plot of $h(\rho)$.

A Nature of the 802.11 Interference

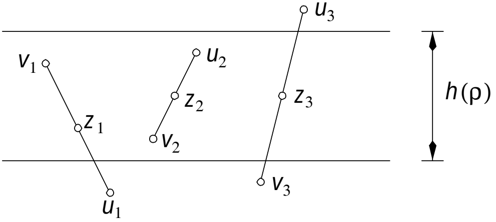


Figure: Under the 802.11 interference model, if (u_1, v_1) and (u_3, v_3) conflict with each other, then at least one of them conflicts with (u_2, v_2).

Strip-wise Edge Scheduling

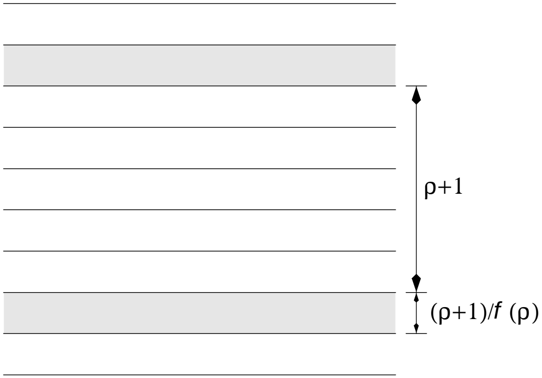
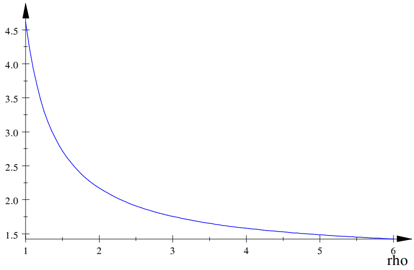


Figure: Partition of the plane into strips of height $(\rho+1)/f(\rho)$ where $f(\rho)=\lceil(\rho+1)/h(\rho)\rceil$.

Approximation Bound

approx. ratio $\leq 1 + f(\rho)$,

$$f(\rho) = \begin{cases} 5 & \text{if } \rho \in [1, 1.0891); \\ 4 & \text{if } \rho \in [1.0891, 1.3609); \\ 3 & \text{if } \rho \in [1.3609, 2.2907); \\ 2 & \text{if } \rho \in [2.2907, \infty). \end{cases}$$



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