Minimum-Latency Scheduling for Group Communications

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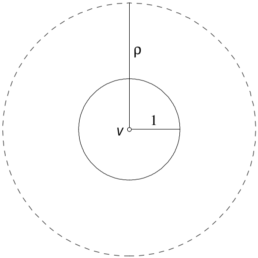
Outline

- Problem Description
- Dominating Tree
- Broadcast Scheduling
- Aggregation Scheduling
- Gathering Scheduling
- Gossiping Scheduling

Network Model

- V: a finite planar set of nodes
- Synchronous transmissions
- Protocol interference model:
  - unit communication radius: Communication topology G is the UDG on V
  - uniform interference radius $\rho \geq 1$



Group Communications

- Broadcast
- Data aggregation
- Data gathering
- Gossiping

Communication Schedule

- Routing
- Assignment of time-slot for each link
  - Link ordering should be followed
  - all links scheduled in the same time-slot are conflict-free

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Min-Latency Communication Scheduling: Compute a communication schedule of minimum latency for a specified communication task.

Summary on Scheduling Algorithms

Communication	Approx. Bound
broadcast	$2\beta_\rho$
aggregation with $\rho > 1$	$(\alpha_\rho + 12)\,\beta_\rho$
gathering	$2\beta_\rho$
gossiping	$4\beta_\rho$

$\alpha_\rho =$ max. # of points in a unit-disk whose mutual dist. $> \rho - 1$.
$\beta_\rho =$ max. # of points in a half $(\rho + 1)$-disk whose mutual dist. > 1

Bounds on The Two Packing Numbers

- For any $\rho > 1$,

$$\alpha_\rho \leq \left\lfloor \frac{2\pi/\sqrt{3}}{(\rho-1)^2} + \frac{\pi}{\rho-1} \right\rfloor + 1.$$

- For any $\rho \geq 1$,

$$\beta_\rho \leq \left\lfloor \frac{\pi}{\sqrt{3}} (\rho+1)^2 + \left(\frac{\pi}{2}+1\right)(\rho+1) \right\rfloor + 1.$$

Roadmap

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- **Dominating Tree**
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Root Selection

Root s:

- Broadcast: source

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- Broadcast: source
- Aggregation/Gathering: sink

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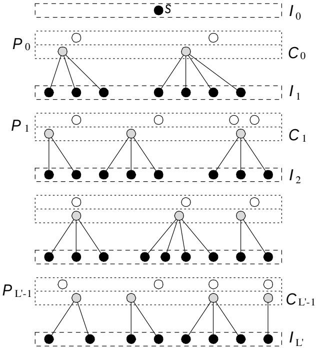
L: the graph radius of G w.r.t. s

A Sparse CDS

- I: first-fit MIS in BFS ordering w.r.t. s



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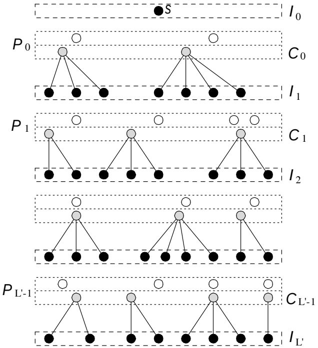

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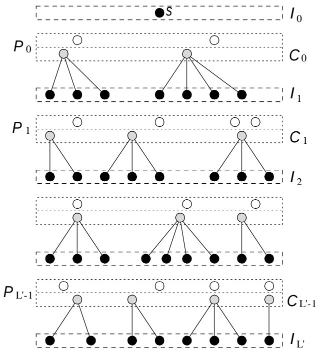

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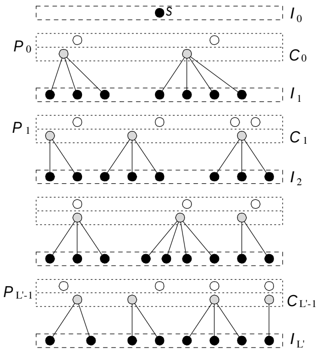

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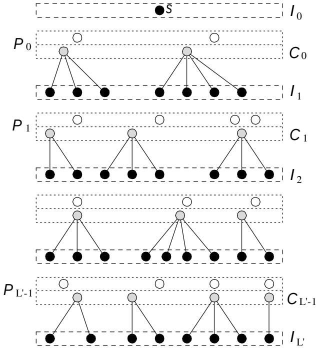

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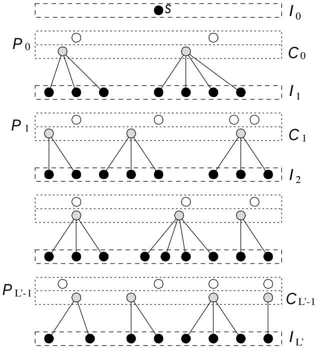
  - I_l: dominators of depth l in G'



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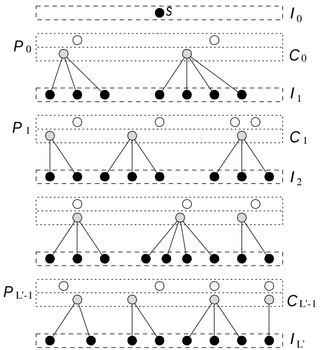

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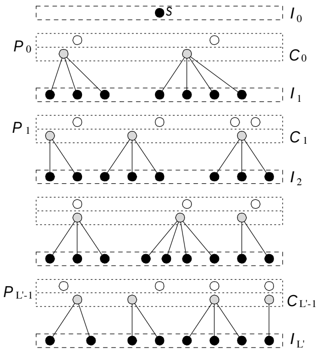

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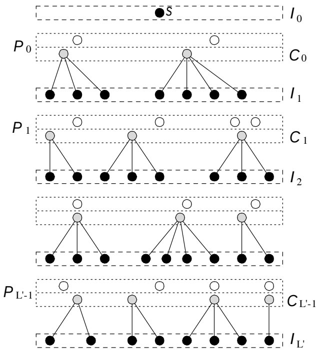
- $W = V \setminus (I \cup C)$: dominatees



Size, Radius And Sparsity

$$|C| \leq |I| - 1,$$
$$Rad\left(G\left[I \cup C\right], s\right) \leq 2\left(L - 1\right).$$

Lemma

$|C_0| \leq 12$ and each dominator in I_l with $2 \leq l \leq L' - 1$, is adjacent to at most 11 connectors in C_l.

An Equilateral Triangle Property

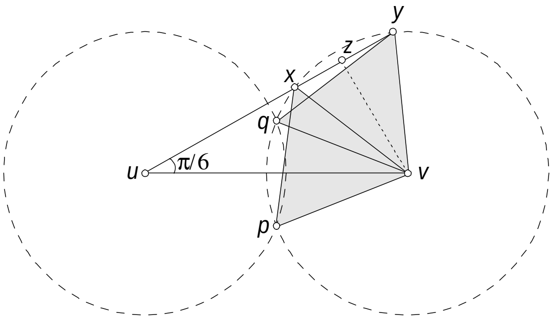


Figure: The two circles have unit radius, and $1 \leq \|uv\| \leq 2$. Then, both $\triangle pvx$ and $\triangle qvy$ are equilateral.

A Geometric Lemma on Angle Separation

Lemma

Consider three nodes u, v and w satisfying that $1 < \|uw\| \leq \|uv\| \leq 2$ and $\|vw\| > 1$. If $\widehat{vuw} \leq 2\arcsin\frac{1}{4} \approx 28.955^\circ$, then $B(u) \cap B(v) \subseteq B(w)$.

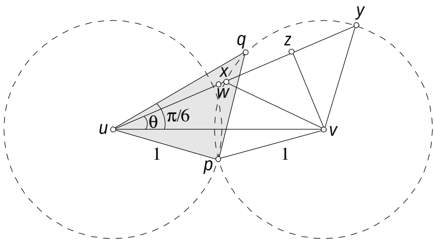


Figure: If $\theta \leq 2\arcsin\frac{1}{4}$, then $\|uy\| \geq \|uv\|$, and hence $w \in ux \subset \triangle upq$.

A Geometric Lemma on Angle Separation

$$\|uy\| \geq \|uv\| \Leftrightarrow \widehat{uvy} \geq \widehat{uyv} = \widehat{uxy} \Leftrightarrow \widehat{xvy} \geq \theta.$$

$$\|vz\| = \|uv\| \sin\theta \leq 2\sin\theta = 4\sin\frac{\theta}{2}\cos\frac{\theta}{2} \leq \cos\frac{\theta}{2}$$
$$\Rightarrow \widehat{xvy} = 2\arccos\|vz\| \geq 2\arccos\left(\cos\frac{\theta}{2}\right) = \theta.$$

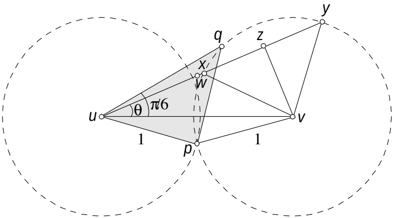


Figure: If $\theta \leq 2\arcsin\frac{1}{4}$, then $\|uy\| \geq \|uv\|$.

Proof of Sparsity

$|C_0| \leq 12$:

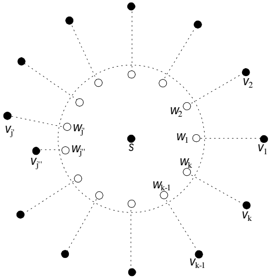


Figure: $w_1, w_2, \cdots, w_k$ are the connectors in C_0. Each v_j is a private dominator neighbor of w_j in I_1 with respect to C_0.

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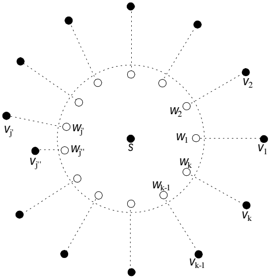


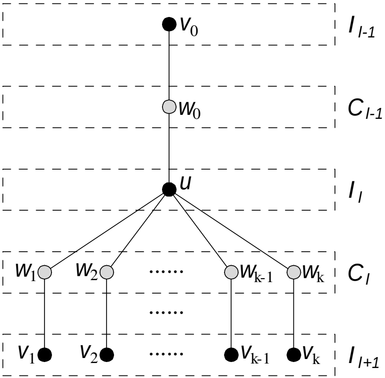
Figure: $w_1, w_2, \cdots, w_k$ are the connectors in C_0. Each v_j is a private dominator neighbor of w_j in I_1 with respect to C_0.

If $k \geq 13$, then there exist two dominators $v_{j'}$ and $v_{j''}$ s.t. $\angle v_{j'} u v_{j''} \leq \frac{2\pi}{13}$. Assume by symmetry $v_{j''}$ is closer to u then $v_{j'}$. Then,

$$w_{j'} \in B(u) \cap B(v_{j'}) \subseteq B(v_{j''}).$$

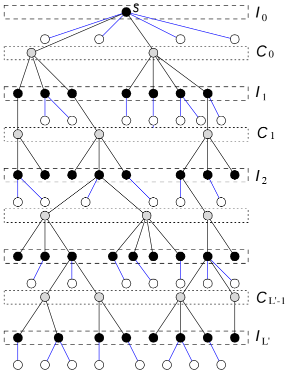
Proof of Sparsity

Each dominator $u \in I_l$ is adj. to at most 11 connectors in C_l:



Dominating Tree

- Parent of each node $v \neq s$



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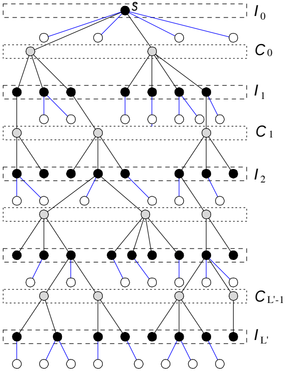
- Parent of each node $v \neq s$
  - $v \in I_l$: adj. connector of least ID in C_{l-1}



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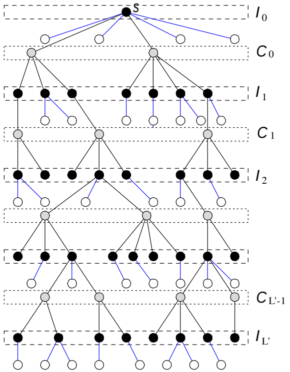

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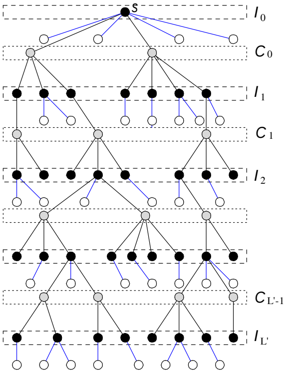
  - $v \in W$: adj. dominator of least (depth,ID)



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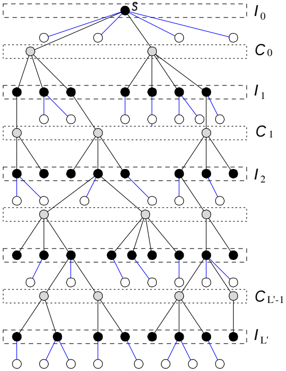

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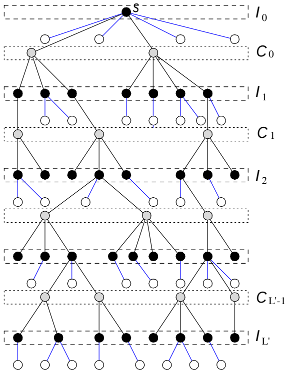

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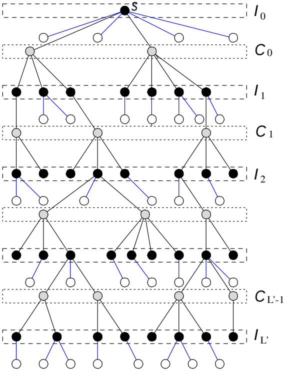
- Depth: $\leq 2L' + 1 \leq 2L - 1$
- s: ≤ 12 connector children
- other dominators: ≤ 11 connector children



Distance-$(\rho + 1)$ Coloring of Dominators

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- Problem Description
- Dominating Tree
- **Broadcast Scheduling**
- Aggregation Scheduling
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Broadcast Scheduling

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- Scheduling: $2L' + 1$ rounds sequentially dedicated to the transmissions by

$$I_0, C_0, I_1, C_1, \cdots, I_{L'-1}, C_{L'-1}, I_{L'}$$

resp.

Individual Rounds

For each $1 \leq l \leq L'$, compute a first-fit distance-$(\rho + 1)$ coloring of I_l in the lexicographic order.

- Round for I_0: s transmits in one time-slot.
- Round for C_0: all nodes in C_0 transmit one by one in ≤ 12 time-slots.
- Round for I_l with $1 \leq l \leq L'$: a dominator of i-th color transmits in i-th time slot. Latency $\leq \beta_\rho$
- Round for C_l with $1 \leq l \leq L' - 1$: a connector with a child dominator of i-th color transmits in i-th time slot. Latency $\leq \beta_\rho$

Approximation Bound

- Total latency at most

$$1 + 12 + (2L' - 1)\,\beta_\rho \le 13 + \beta_\rho\,(2L - 3)$$
$$= 2\beta_\rho L - (3\beta_\rho - 13)\,.$$

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- Min. broadcast latency $\geq L$

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- Total latency at most

$$1 + 12 + \left(2L' - 1\right)\beta_\rho \leq 13 + \beta_\rho \left(2L - 3\right)$$
$$= 2\beta_\rho L - \left(3\beta_\rho - 13\right).$$

- Min. broadcast latency $\geq L$
- Approx ratio $\leq 2\beta_\rho$

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B: a set of links whose receiving ends are all dominators and whose transmitting ends are distinct

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        - a link whose dominator end has the i-th color is scheduled in the i-th time-slot
    - Latency: $\leq \phi\beta_\rho$.

Individual Rounds

- Round for W: previous link schedule. Latency $\leq (\Delta - 1)\,\beta_\rho$
- Round for C_l with $1 \leq l \leq L' - 1$: previous link schedule. Latency $\leq 11\beta_\rho$
- Round for C_0: all nodes in C_0 transmit one by one. Latency ≤ 12.
- Round for I_l with $1 \leq l \leq L'$:
  - compute a first-fit distance-$(\rho + 1)$ coloring of I_l in the lexicographic order
  - each dominator of the i-th color transmit in the i-th time slot
  - latency $\leq \beta_\rho$

Approximation Bound

- Total latency: at most

$$\begin{aligned}&(\Delta-1)\,\beta_\rho+11\beta_\rho\left(L'-1\right)+12+L'\beta_\rho\\&=\Delta\beta_\rho+12\beta_\rho\left(L'-1\right)+12\\&\leq\Delta\beta_\rho+12\beta_\rho\left(L-2\right)+12\end{aligned}$$

Approximation Bound

- Total latency: at most

$$\begin{aligned}
&(\Delta - 1)\,\beta_\rho + 11\beta_\rho\left(L' - 1\right) + 12 + L'\beta_\rho \\
&= \Delta\beta_\rho + 12\beta_\rho\left(L' - 1\right) + 12 \\
&\leq \Delta\beta_\rho + 12\beta_\rho\,(L - 2) + 12
\end{aligned}$$

- Minimum aggregation latency $\geq \max\left\{L, \Delta/\alpha_\rho\right\}$

Approximation Bound

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- Minimum aggregation latency $\geq \max\left\{L, \Delta/\alpha_\rho\right\}$
- Approx ratio $\leq \left(\alpha_\rho+12\right)\beta_\rho$

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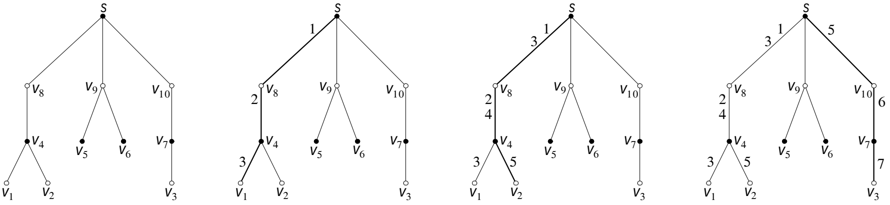
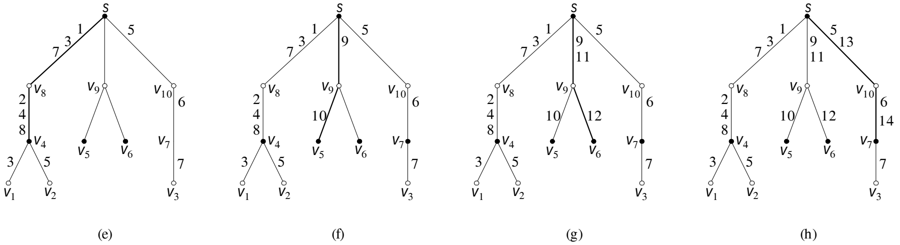
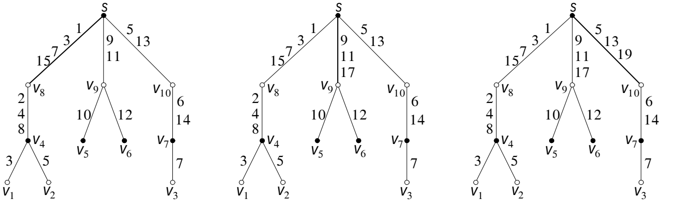
Multi-labelling of Dominating Tree



(a) (b) (c) (d)


s
1
3
7
5
9
11
13
v8
v9
v10
2
4
8
10
12
6
14
v4
v5
v6
v7
3
5
7
v1
v2
v3
(e)
(f)
(g)
(h)

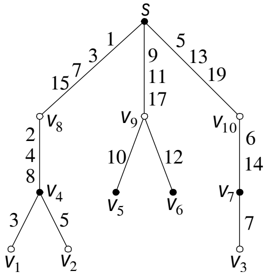
Multi-labelling of Dominating Tree



(i) (j) (k)

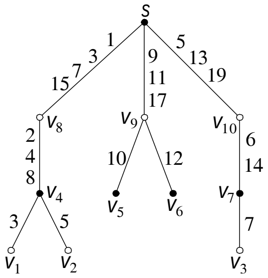
Properties of Multi-labelling

- For each edge $(v, p(v))$ between v and its parent $p(v)$:
  - $\#$ of labels $= \#$ of descendents (including v) of v;
  - all labels are even if $v \in I$, and odd otherwise.
- All edges across two consecutive layers receive distinct labels.
- The largest label is $2n - 3$.



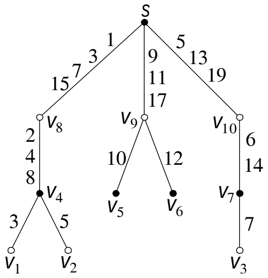
Grouping of Edges/Links by Labels

- For each $1 \leq k \leq 2n-3$,



Grouping of Edges/Links by Labels

- For each $1 \leq k \leq 2n-3$,
  - E_k: edges in T with label k,



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- For each $1 \leq k \leq 2n-3$,
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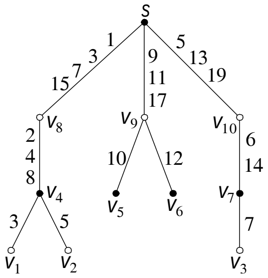

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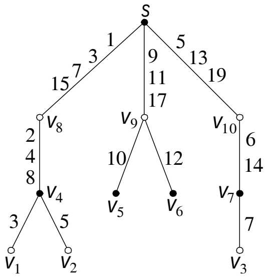

Grouping of Edges/Links by Labels

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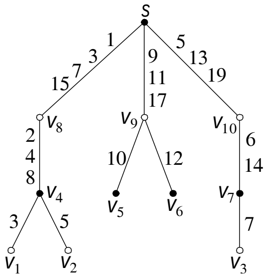
- All links in each A_k are disjoint
- For odd (resp., even) k, all the receiving (resp., transmitting) ends of links in A_k are dominators.



Link Scheduling

- $2n-3$ rounds sequentially dedicated to

$$A_{2n-3}, A_{2n-2}, \cdots, A_2, A_1$$

resp.

Link Scheduling

- $2n - 3$ rounds sequentially dedicated to

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- Round for A_k with $1 \leq k \leq 2n-3$:
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  - a link whose dominator end has the i-th color is scheduled in the i-th time-slot

Approximation Bound

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- Approx ratio $\leq 2\beta_\rho$

Roadmap

- Problem Description
- Dominating Tree
- Broadcast Scheduling
- Aggregation Scheduling
- Gathering Scheduling
- **Gossiping Scheduling**

Two-Phased Gossiping Scheduling

- s: graph center of G

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  - Scheduling: Pipelined broadcasting

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- Framing: frame = k-slot dominator subframe + k-slot connector subframe

dominator subframe	connector subframe	dominator subframe	connector subframe	dominator subframe	connector subframe

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- Upon receiving a packet in a connector subframe, each dominator of the i-th color transmits the received packet in the i-th time-slot of the dominator subframe in the subsequent frame.

Latency of The Second Phase

- After $n-1$ frames, s transmits the last packet.

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So, the latency of the second phase is at most

$$2k\left(n-1+L'\right)+k \leq 2k\left(n+L-2\right)+k$$
$$=2k\left(n+L-1.5\right) \leq 2\beta_{\rho}\left(n+L-1.5\right).$$

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- Approx ratio $\leq 4\beta_\rho$