Minimum Latency Link Scheduling under Protocol Interference Model

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Outline

- Problem Description
- Introduction to Graph Coloring
- First-Fit Link Scheduling
- Strip-wise Link Scheduling

Protocol Interference Model

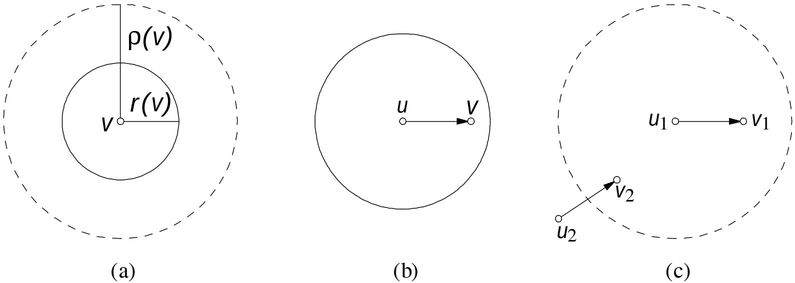

Figure: (a) Communication range and interference range of each node; (b) a communication link; (c) a conflicting pair of communication links.

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- Conflict graph H of A: a pair links in A are adjacent in H iff they conflict with each other
- A link schedule for A corresponds to a vertex coloring of H

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Summary on Scheduling Algorithms

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Independence Number and Clique number

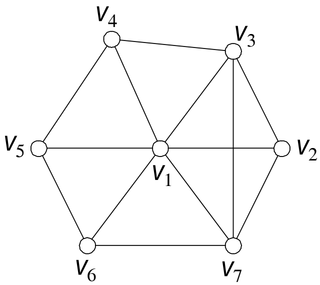
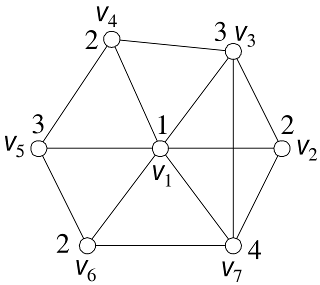


Figure: $\alpha(G) = 3$ and $\{v_2, v_4, v_6\}$ is a maximum IS. $\omega(G) = 4$ and $\{v_1, v_2, v_3, v_7\}$ is a maximum clique.

Vertex Coloring

- Vertex coloring: adj. nodes receive distinct colors
  - partition of vertices into IS's
- Minimum Vertex Coloring: NP-hard in general
  - chromatic number: $\chi(G)$

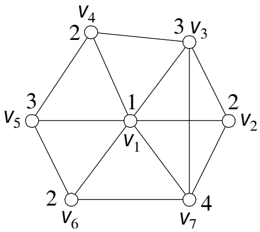
$$\chi(G) \geq \max\left\{\frac{|V|}{\alpha(G)}, \omega(G)\right\}.$$



First-Fit Coloring

- $\langle v_1, v_2, \cdots, v_n \rangle$: a vertex ordering of V

$$N_{\prec}(v_i) = \{v_j : 1 \le j < i, v_j \in N(v_i)\}.$$



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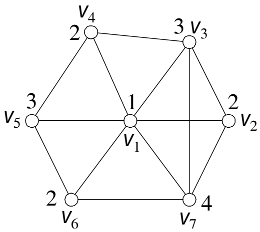

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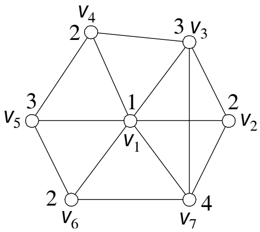

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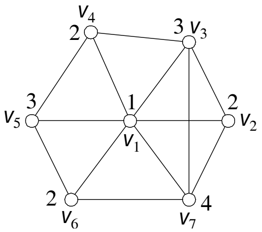

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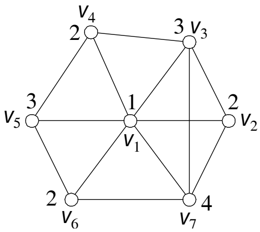

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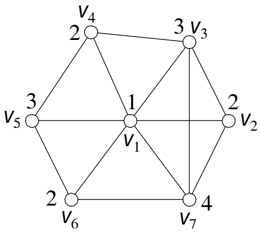
- number of colors $\leq 1 + \max_{1<i\leq n} |N_{\prec}(v_i)|$.
  - $\max_{1<i\leq n} |N_{\prec}(v_i)|$: inductivity of $\langle v_1, v_2, \cdots, v_n \rangle$



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Smallest-Degree-Last Ordering

Objective: find a least-inductivity ordering $\langle v_1, v_2, \cdots, v_n \rangle$

- $H \leftarrow G$.
- For $i = n$ down to 1,
  - $v_i \leftarrow$ a vertex of H with least degree
  - $H \leftarrow H - \{v_i\}$

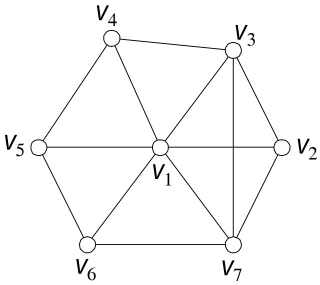

Figure: $\langle v_1, v_7, v_6, v_5, v_4, v_3, v_2 \rangle$ is a smallest-last ordering.

Smallest-Degree-Last Ordering

Inductivity of G:

$$\delta^*(G) = \max_{U \subseteq V} \delta(G[U])$$

Theorem

The smallest-degree-last ordering $\langle v_1, v_2, \cdots, v_n \rangle$ achieves the smallest inductivity $\delta^*(G)$ among all vertex orderings.

Smallest-Degree-Last Ordering

For each $1 < i \leq n$,

$$\left|N_{\prec}\left(v_i\right)\right| = \delta\left(G\left[\left\{v_1, v_2, \cdots, v_i\right\}\right]\right) \leq \delta^*\left(G\right).$$

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Let U be such that $\delta^*\left(G\right) = \delta\left(G\left[U\right]\right)$ and

$$j = \max\left\{1 \leq i \leq n : v_i \in U\right\}.$$

Then,

$$\delta^*\left(G\right) = \delta\left(G\left[U\right]\right) \leq \deg_{G[U]}\left(v_j\right) \leq \left| N_{\prec}\left(v_j\right)\right|.$$

Inductive Local Independence Number (LIN)

Inductive LIN of $\langle v_1, v_2, \cdots, v_n \rangle$:

$$\alpha^* = \max_{1 < i \leq n} \left\{ |I| : I \in \mathcal{I}, I \subseteq N_{\prec}(v_i) \right\}.$$

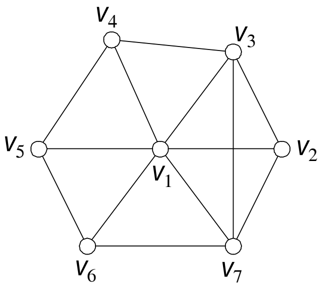

Figure: Inductive indepence number of $\langle v_1, v_2, v_3, v_4, v_5, v_6, v_7 \rangle$ is 2, while the Inductive indepence number of $\langle v_2, v_3, v_4, v_5, v_6, v_7, v_1 \rangle$ is 3.

Application of Inductive LIN

Lemma

Any vertex ordering with inductive LIN α^* *has inductivity at most* $\alpha^*\left(\chi\left(G\right)-1\right)$.

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For any $1 < i \leq n$,

$$\chi \left(G \right) \geq \left| N_{\prec} \left(v_i \right) \right| / \alpha^* + 1 \Rightarrow \left| N_{\prec} \left(v_i \right) \right| \leq \alpha^* \left(\chi \left(G \right) - 1 \right)$$

Inward Local Independence Number (LIN)

- $D = (V, A)$: an orientation of G

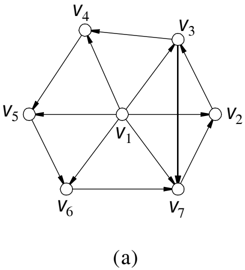
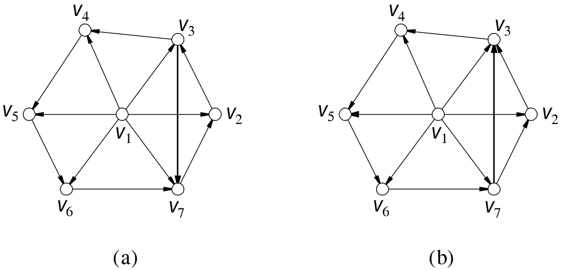


Figure: Two graph orientations: (a) the local independence number is two, (b) the local independence number is one.

Inward Local Independence Number (LIN)

- $D = (V, A)$: an orientation of G
- Inward LIN of D:

$$\beta^* = \max_{u \in V} \left\{ |I| : I \in \mathcal{I}, I \subseteq N_D^{in}(u) \right\}.$$



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Consider a node $v \in V$ with largest in-degree in D. Then,

$$\chi(G) \geq \lceil \Delta^{in}(D) / \beta^* \rceil + 1 \Rightarrow \Delta^{in}(D) \leq \beta^*(\chi(G) - 1).$$

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Let U be such that $\delta^*(G) = \delta(G[U])$. $D[U]$ contains at least one node u with in-degree $\geq$ out-degree. Thus,

$$\begin{aligned}\delta^*(G) &\leq \deg_{G[U]}(u) = \deg^{in}_{D[U]}(u) + \deg^{out}_{D[U]}(u) \\ &\leq 2\deg^{in}_{D[U]}(u) \leq 2\deg^{in}_{D}(u) \leq 2\Delta^{in}(D).\end{aligned}$$

A Master Theorem on First-Fit Coloring

Theorem

Let $G = (V, E)$ be an undirected graph.

1. *If G has an vertex ordering with inductive LIN α^*, then the first-fit coloring of G in the smallest-degree-last ordering uses at most $\alpha^* \chi(G) - (\alpha^* - 1)$ colors and hence is a α^*-approximation.*
2. *If G has an orientation D with inward LIN β^*, then the first-fit coloring of G in the smallest-degree-last ordering uses at most $2\beta^* \chi(G) - (2\beta^* - 1)$ colors and hence is a $2\beta^*$-approximation.*

Perfect Graphs

A graph $G = (V, E)$ is *perfect* if it satisfies any of the following three equivalent conditions:

1. For any $U \subseteq V$, $\chi(G[U]) = \omega(G[U])$.
2. For any $U \subseteq V$, $\alpha(G[U]) \cdot \omega(G[U]) \geq |U|$.
3. Neither G nor its complement contains an induced odd cycle of size larger than three.

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Coloring of perfect graphs are solvable in poly. time.

Cocomparability Graphs

Cocomparability ordering $\langle v_1, v_2, \cdots, v_n \rangle$: $i < j < k$ and $v_i v_k \in E \Rightarrow$ either $v_i v_j \in E$ or $v_j v_k \in E$.

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- Perfect
- Minimum vertex coloring: reduced to maximum bipartite matching

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Theorem

The approximation bound of the first-fit link scheduling is at most $2\left(\left\lceil \pi / \arcsin \frac{c-1}{2c} \right\rceil - 1\right)$.

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Lemma

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Angle Separation

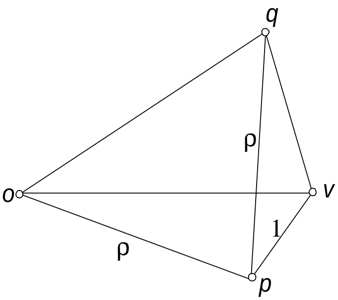
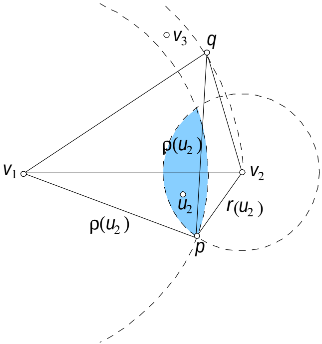


Figure: In a convex quadruple *upvq* with $\|pu\| = \|pq\| = \rho \geq 1 = \|pv\|$ and $\|uq\| = \|uv\|$, we have $\widehat{quv} \geq 2 \arcsin \frac{\rho-1}{2\rho}$.

Consider three links $a_i = (u_i, v_i)$ for $1 \leq i \leq 3$ s.t. (1) v_1 is in the interference ranges of u_2 and u_3 (2) a_2 and a_3 are conflict-free.



Local Independence Number

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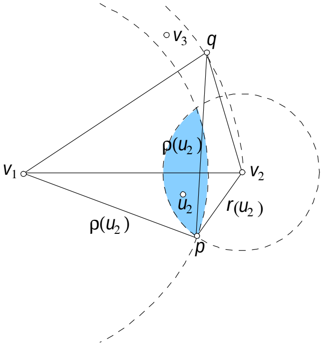

$$\widehat{v_2 v_1 v_3} > \widehat{v_2 v_1 q}$$
$$\geq 2 \arcsin \frac{\rho(u_2) - r(u_2)}{2\rho(u_2)}$$
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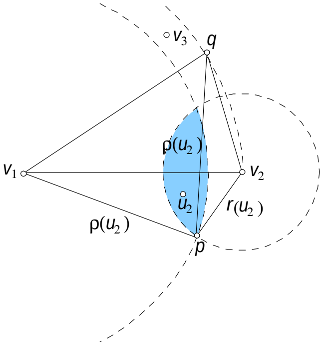

$$\begin{aligned}\widehat{v_2 v_1 v_3} &> \widehat{v_2 v_1 q} \\ &\geq 2 \arcsin \frac{\rho(u_2) - r(u_2)}{2\rho(u_2)} \\ &\geq 2 \arcsin \frac{c-1}{2c}.\end{aligned}$$

Hence,

$$\beta^* \leq \left\lceil \pi / \arcsin \frac{c-1}{2c} \right\rceil - 1.$$

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Uniform Communication/Interference Radii

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Uniform Communication/Interference Radii

- Communication radii: normalized to one
- Interference radii: $\rho > 1$.
- Better scheduling algorithm

A Height Function

$$h(\rho) = (\rho - 1)\sin\left(\arccos\frac{\rho - 1}{2\rho} - \arcsin\frac{1}{\rho}\right), \rho \in [1, \infty)$$

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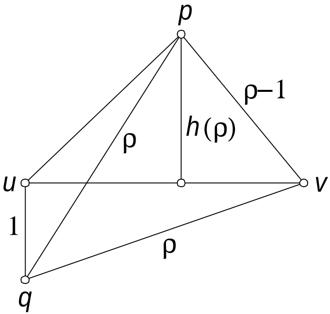

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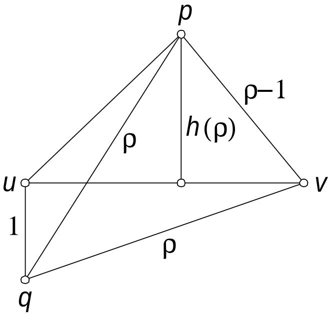

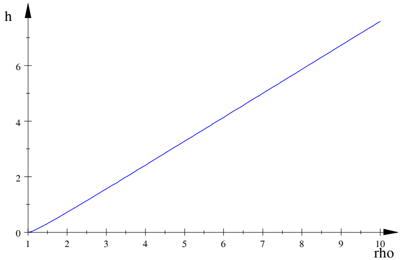

A Strip-wise Transitivity of Independence

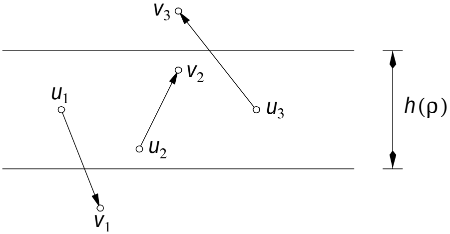

Figure: If both (u_1, v_1) and (u_3, v_3) are independent with (u_2, v_2), then they are independent with each other.

Strip-wise Link Scheduling with $\rho > 1$

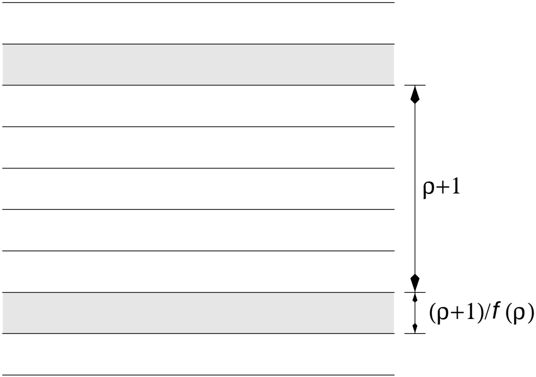

Figure: Partition of the plane into strips of height $(\rho + 1) / f(\rho)$ where $f(\rho) = \lceil (\rho + 1) / h(\rho) \rceil$.

Approximation Bound

$$\text{approx. ratio} \leq 1 + f(\rho) = 1 + k \text{ if } \rho \in [\rho_k, \rho_{k-1}),$$

where $\rho_1 = \infty$, and ρ_k with $k \geq 2$ is the root of $(\rho + 1) / h(\rho) = k$.

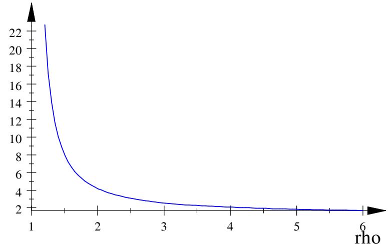

Figure: Plot of $(\rho + 1) / h(\rho)$.

k	ρ_k	k	ρ_k
2	4.2462	7	1.5715
3	2.5689	8	1.5009
4	2.0632	9	1.4476
5	1.8167	10	1.4058
6	1.6697	11	1.3721

Table: Numberic values of ρ_k for $2 \leq k \leq 11$.