Multiflows in Multihop Wireless Networks

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Outline

- Problem Description
- Practical Approximation Algorithms
- Polynomial-Time Approximation Scheme
- Summary

A General Specification of Multihop Wireless Networks

$(V, A, \mathcal{I})$:

- V: network nodes

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- V: network nodes
- A: communication links
  - (V, A): communication topology
- $\mathcal{I}$: collection of independent (i.e. conflict-free) links in A
  - implicitly given by an interference model (IM)

Protocol Interference Model (IM)

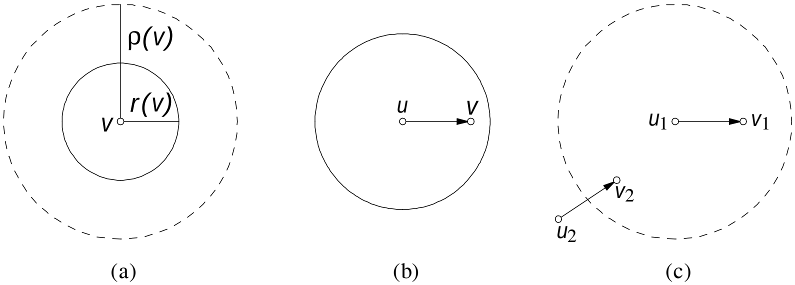

Figure: (a) Communication range and interference range of each node; (b) a communication link; (c) a conflicting pair of communication links.

802.11 Interference Model (IM)

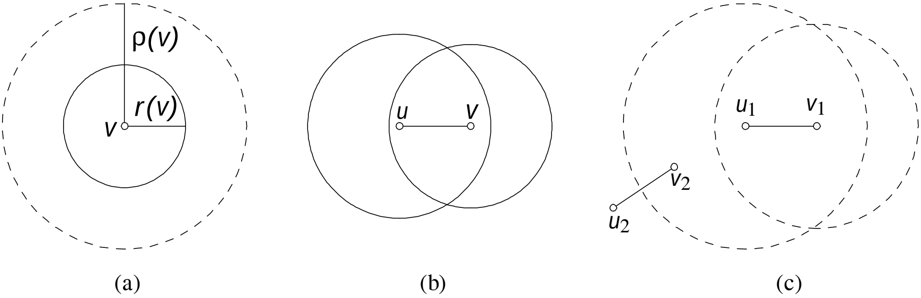

Figure: (a) Communication range and interference range of each node; (b) a communication edge; (c) a conflicting pair of communication edges.

(Fractional) Link Schedule

$$S = \{(I_j, \lambda_j) \in \mathcal{I} \times \mathbb{R}_+ : 1 \leq j \leq k\}$$

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If the length of $S \le 1$, it determines a link capacity function

$$c_S = \sum_{1 \le j \le k} \lambda_j \mathbf{1}^{I_j}.$$

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Capacity Region $P \subset \mathbb{R}_+^A$: convex hull of $\left\{\mathbf{1}^I : I \in \mathcal{I}\right\}$, or equivalently

$$P = \{c_S : S \text{ is a link schedule of length} \leq 1\}.$$

(Fractional) Multiflow

1. **Maximum Multiflow** (**MMF**): Given a set of commodities, find a link schedule S of length at most one such that the maximum multiflow subject to c_S is maximized.

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2. **Maximum Concurrent Multiflow** (**MCMF**): Given a set of commodities with demands, find a link schedule S of length at most one such that the maximum concurrent multiflow subject to c_S is maximized.

Related Problems

1. **Maximum Weighted Independent Set of Links (MWISL)**: Given $d \in \mathbb{R}_{+}^{A}$, find an $I \in \mathcal{I}$ such that $d(I)$ is maximized

Related Problems

1. **Maximum Weighted Independent Set of Links** (**MWISL**): Given $d \in \mathbb{R}_+^A$, find an $I \in I$ such that $d(I)$ is maximized
2. **Shortest Weighted Link Schedule** (**SWLS**) : Given $d \in \mathbb{R}_+^A$, find a shortest link schedule

$$S = \{(I_j, \lambda_j) \in \mathcal{I} \times \mathbb{R}_+ : 1 \leq j \leq k\}$$

such that $d = \sum_{1 \leq j \leq k} \lambda_j \mathbf{1}^{I_j}$.

Roadmap

- Problem Description
- **Practical Approximation Algorithms**
- Polynomial-Time Approximation Scheme
- Summary

A Polyhedral Approach

Theorem If there is a capacity sub-region $Q \subseteq P$ s.t.

1. Q is a μ-approximation of P, i.e., $P \subseteq \mu Q$,
2. Q has an explicit polynomial representation, and
3. $\forall d \in Q$, a fractional link schedule of length at most one for d can be computed in poly. time.

Then, both **MMF** and **MCMF** have a polynomial μ-approximation.

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Any $Q \subseteq P$ meeting the three above conditions is called a poly. μ-approx. capacity subregion.

Link-Flow LP

$\mathcal{F}_i$: set of s_i–t_i flows

max. multiflow

$$\begin{array}{ll} \max & \sum_{j=1}^{k} val\left(f_j\right) \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k \\ & \sum_{j=1}^{k} f_j \in P \end{array}$$

max. concurrent multiflow

$$\begin{array}{ll} \max & \phi \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k \\ & val\left(f_j\right) \geq \phi d\left(j\right), \forall 1 \leq j \leq k \\ & \sum_{j=1}^{k} f_j \in P \end{array}$$

Restricted Multiflow

max. Q-restricted multiflow

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Step 1: solve the Q-restricted LP to obtain a k-flow $\langle f_1, f_2, \cdots, f_k \rangle$,
Step 2: compute a fractional link schedule of length at most one for $\sum_{j=1}^{k} f_j$.

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  - **Episode: Independence Polytope And Fractional Coloring**
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- Independence polytope $P \subset \mathbb{R}_+^V$: convex hull of $\{\mathbf{1}^I : I \in \mathcal{I}\}$.

Fractional (Weighted) Coloring

- For any $d \in \mathbb{R}_{+}^{V}$, a *fractional coloring* of (G, d) is a set of k pairs $(I_j, \lambda_j) \in \mathcal{I} \times \mathbb{R}_{+}$ s.t.

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$$\chi_f(G, d) \geq \frac{d(V)}{\alpha(G)}.$$

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Fractional (Weighted) Coloring: Example

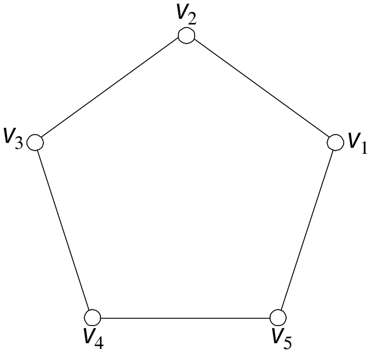

Figure: For the all-one demand vector d, $\chi_f(G, d) = 2.5$. On the other hand, $\chi(G) = 3$.

Poly. Approx. of Independence Polytope

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- Coloring weight of $S \leq \max_{1 \leq i \leq n} d\left(N_{\preceq}\left(v_i\right)\right)$, where

$$N_{\preceq}\left(v_i\right) = \left\{v_j : 1 \leq j \leq i, v_j \in N\left(v_i\right)\right\}$$
$$N_{\prec}\left(v_i\right) = \left\{v_j : 1 \leq j < i, v_j \in N\left(v_i\right)\right\}.$$

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$$

  - $\max_{1 \leq i \leq n} d\left(N_{\preceq}\left(v_i\right)\right)$: (closed) d-inductivity of $\langle v_1, v_2, \cdots, v_n \rangle$

First-Fit Fractional Weighted Coloring

- k: # of iterations/colors
- $\forall 1 \leq j \leq k$,
  - U_j: the subset of nodes with residue demands at the beginning of the j-th iteration
  - (I_j, λ_j): the pair selected in the j-th iteration.

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Consider an arbitrary node $v_i \in U_k$. $\forall\ 1 \leq j \leq k$, let $V_{i,j} = N_{\preceq}(v_i) \cap U_j$. Then, $\forall 1 \leq j \leq k$, $I_j \cap V_{i,j} \neq \emptyset$. Hence,

$$d\left(N_{\preceq}(v_i)\right) = \sum_{j=1}^{k} \lambda_j \left|I_j \cap V_{i,j}\right| \geq \sum_{j=1}^{k} \lambda_j.$$

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Question: how to compute a vertex ordering with least closed d-inductivity?

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- $H \leftarrow G$.
- For $i = n$ down to 1,
    - $v_i \leftarrow$ a vertex of smallest closed weighted degree in H
    - $H \leftarrow H - \{v_i\}$

Smallest-Last Ordering

closed d-inductivity of G:

$$\overline{\delta}^{*}(G,d)=\max_{U\subseteq V}\min_{u\in U} d\left(N_{G[U]}[u]\right)$$

Theorem

The smallest-last ordering achieves the smallest d-inductivity $\overline{\delta}^{*}(G,d)$ among all vertex orderings.

Proof of The Theorem

For any ordering $\langle v_1, v_2, \cdots, v_n \rangle$,

$$\max_{1 \leq i \leq n} d\left(N_{\preceq}\left(v_i\right)\right) \geq \overline{\delta}^*\left(G, d\right).$$

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$$\max_{1 \leq i \leq n} d\left(N_{\preceq}(v_i)\right) \geq \overline{\delta}^*(G, d).$$

Let $U \subseteq V$ be s.t.

$$\overline{\delta}^*(G, d) = \min_{u \in U} d\left(N_{G[U]}[u]\right)$$

and j be the last index such that $v_j \in U$. Then,

$$\max_{1 \leq i \leq n} d\left(N_{\preceq}(v_i)\right) \geq d\left(N_{\preceq}(v_j)\right) \geq d\left(N_{G[U]}[v_j]\right) = \overline{\delta}^*(G, d).$$

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$$\max_{1 \leq i \leq n} d\left(N_{\preceq}\left(v_i\right)\right) \leq \overline{\delta}^*\left(G, d\right).$$

For any $1 \leq i \leq n$, let $V_i = \{v_1, v_2, \cdots, v_i\}$. Then,

$$d\left(N_{\preceq}\left(v_i\right)\right) = \min_{u \in V_i} d\left(N_{G[V_i]}\left[u\right]\right) \leq \overline{\delta}^*\left(G, d\right).$$

Hence,

$$\max_{1 \leq i \leq n} d\left(N_{\preceq}\left(v_i\right)\right) \leq \overline{\delta}^*\left(G, d\right).$$

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- $\forall d \in \mathbb{R}_+^V$,

$$\overline{\delta}^* (G, d) \leq \max_{1 \leq i \leq n} d (N_{\preceq}(v_i)) \leq \alpha^* \chi_f (G, d) .$$

Inductive LIN: The Second Inequality

$\forall 1 \leq i \leq n$,

$$\begin{aligned} \chi_f(G,d) &\geq d(N_{\prec}(v_i))/\alpha^* + d(v_i) \\ \Rightarrow d(N_{\preceq}(v_i)) &\leq \alpha^*(\chi_f(G,d) - d(v_i)) + d(v_i) \\ &= \alpha^*\chi_f(G,d) - (\alpha^* - 1)\, d(v_i) \\ &\leq \alpha^*\chi_f(G,d) - (\alpha^* - 1) \min_{v \in V} d(v). \end{aligned}$$

Inductive Independence Polytope

The polytope

$$Q = \left\{ d \in \mathbb{R}_+^V : \max_{1 \leq i \leq n} d\left(N_{\preceq}\left(v_i\right)\right) \leq 1 \right\}.$$

is a polynomial α^*-approx. of P.

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- $\forall d \in \mathbb{R}_+^V$,

$$\overline{\delta}^*(G, d) \leq \max_{v \in V} \left(d(v) + 2d\left(N_D^{in}(v) \right) \right) \leq 2\beta^* \chi_f(G, d).$$

Inward LIN: The First Inequality

For any $U \subseteq V$, $\exists u \in U$ s.t.

$$d\left(N_{D[U]}^{in}(u)\right) \geq d\left(N_{D[U]}^{out}(u)\right).$$

Thus,

$$\begin{aligned} d\left(N_{G[U]}[u]\right) &= d(u) + d\left(N_{D[U]}^{in}(u)\right) + d\left(N_{D[U]}^{out}(u)\right) \\ &\leq d(u) + 2d\left(N_{D[U]}^{in}(u)\right) \\ &\leq d(u) + 2d\left(N_{D}^{in}(u)\right) \\ &\leq \max_{v \in V}\left(d(v) + 2d\left(N_{D}^{in}(v)\right)\right). \end{aligned}$$

So,

$$\overline{\delta}^{*}(G, d) \leq \max_{v \in V}\left(d(v) + 2d\left(N_{D}^{in}(v)\right)\right).$$

Inward LIN: The Second Inequality

$\forall v \in V$,

$$\begin{aligned}
&\chi_f(G,d) \geq d\left(N_D^{in}(v)\right)/\beta^* + d(v) \\
&\Rightarrow d\left(N_D^{in}(v)\right) \leq \beta^*\left(\chi_f(G,d) - d(v)\right) \\
&\Rightarrow d(v) + 2d\left(N_D^{in}(v)\right) \leq 2\beta^*\chi_f(G,d) - (2\beta^* - 1)\,d(v)
\end{aligned}$$

Inward Independence Polytope

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$$Q = \left\{ d \in \mathbb{R}_{+}^{V} : \max_{v \in V} \left(d\left(v\right) + 2d\left(N_{D}^{in}\left(v\right)\right)\right) \leq 1 \right\}.$$

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Protocol IM

- Orientation of the conflict graph: for any a pair of conflicting links $a_1 = (u_1, v_1)$ and $a_2 = (u_2, v_2)$
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Lemma

$\beta^* \leq \left\lceil \pi / \arcsin \frac{c-1}{2c} \right\rceil - 1$, *and hence* Q *is a poly.* $2 \left(\left\lceil \pi / \arcsin \frac{c-1}{2c} \right\rceil - 1 \right)$*-approx capacity subregion.*

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$\alpha^* \leq 7$ and hence Q is a poly. 7-approx capacity subregion.

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Lemma

$\alpha^* \leq 23$ *hence* Q *is a poly. 23-approx capacity subregion.*

Roadmap

- Problem Description
- Practical Approximation Algorithms
- **Polynomial-Time Approximation Scheme**
- Summary

Approximation-Preserving Reduction to **MWISL**

Theorem Restricted to a network class $\mathcal{N}$, if there is a poly. (resp., a poly. μ-approx.) alg. for **MWISL**, then there is a poly. (resp., a poly. μ-approx.) alg. for (1) **SWLS**, (2) **MMF**, and (3) **MCMF**.

Ellipsoid Method with (Approx.) Separation Oracle

$\mathcal{P}_j$ for $1 \leq j \leq k$: the set of paths for commodity j
$\mathcal{P}$: union of $\mathcal{P}_1, \cdots, \mathcal{P}_k$
$\mathcal{P}_e$ for $e \in A$: the set of paths in $\mathcal{P}$ that use link e
$\mathcal{I}_e$ for $e \in A$: the set of I in $\mathcal{P}$ containing e

Primary (path-flow) LP for **MMF**	dual LP
$\max \quad x(\mathcal{P})$	$\min \quad \tau$
$s.t. \quad x(\mathcal{P}_e) \leq \lambda(\mathcal{I}_e), \forall e \in A$	$s.t. \quad y(p) \geq 1, \forall p \in \mathcal{P}$
$\lambda(\mathcal{I}) \leq 1$	$y(I) \leq \tau, \forall I \in \mathcal{I}$
$x \in \mathbb{R}_+^{\mathcal{P}}, \lambda \in \mathbb{R}_+^{\mathcal{I}}$	$y \in \mathbb{R}_+^A, \tau \in \mathbb{R}_+$

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Approaches:

- for the first two classes: shifting strategy + dynamic programming
- for the third class: polynomial growth

PTAS for All Four

Theorem Restricted to any of the following three network classes, all of **MWISL**, **SWLS**, **MMF**, and **MCMF** have a PTAS:

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- PTAS in broader ranges of networks