Broadcast (Radio, TV) Networks

1 Broadcasting Systems

In its most common form, radio is used for the transmission of sounds (voice and music) and pictures (television). The sounds and images are converted into electrical signals by a microphone (sounds) or video camera (images), amplified, and used to modulate a carrier wave that has been generated by an oscillator circuit in a transmitter. The modulated carrier is also amplified, then applied to an antenna that converts the electrical signals to electromagnetic waves for radiation into space. Such waves radiate at the speed of light and are transmitted not only by line of sight but also by deflection from the ionosphere. Receiving antennas intercept part of this radiation, change it back to the form of electrical signals, and feed it to a receiver. The most efficient and most common circuit for radio-frequency selection and amplification used in radio receivers is the superheterodyne. In that system, incoming signals are mixed with a signal from a local oscillator to produce intermediate frequencies (IF) that are equal to the arithmetical sum and difference of the incoming and local frequencies. One of those frequencies is applied to an amplifier. Because the IF amplifier operates at a single frequency, namely the intermediate frequency, it can be built for optimum selectivity and gain. The tuning control on a radio receiver adjusts the local oscillator frequency. If the incoming signals are above the threshold of sensitivity of the receiver and if the receiver is tuned to the frequency of the signal, it will amplify the signal and feed it to circuits that demodulate it, i.e., separate the signal wave itself from the carrier wave.

There are certain differences between AM and FM receivers. In an AM transmission the carrier wave is constant in frequency and varies in amplitude (strength) according to the sounds present at the microphone; in FM the carrier is constant in amplitude and varies in frequency. Because the noise that affects radio signals is partly, but not completely, manifested in amplitude variations, wideband FM receivers are inherently less sensitive to noise. In an FM receiver, the limiter and discriminator stages are circuits that respond solely to changes in frequency. The other stages of the FM receiver are similar to those of the AM receiver but require more care in design and assembly to make full use of FMs advantages. FM is also used in television sound systems. In both radio and television receivers, once the basic signals have been separated from the carrier wave they are fed to a loudspeaker or a display device (usually a cathode-ray tube), where they are converted into sound and visual images, respectively.

1.1 Radio Broadcasting

1.1.1 AM Radio Broadcasting

Standard broadcasting refers to the transmission of voice and music received by the general public in the 535- to 1705-kHz frequency band. A total of 117 carrier frequencies are allocated from 540 to 1700 kHz in 10-kHz intervals. Each carrier frequency is required by the FCC rules to deviate no more than ±20 Hz from the allocated frequency, to minimize heterodyning from two or more interfering stations. Typical modulation frequencies for voice and music range from 50 Hz to 10 kHz. Each channel is generally thought of as 10 kHz in width, and thus the frequency band is designated from 535 to 1705 kHz. In addition stations may also use multiplexing to transmit stereophonic programming. The FCC adopted Motorola's C-QUAM (compatible quadrature amplitude modulation) in 1994. Approximately 700 AM stations transmit in stereo.

In standard broadcast (AM), stations are classified according to their operating power, protection from interference, and hours of operation. A Class A station operates with 10 to 50 kW of power servicing a large area with primary, secondary, and intermittent coverage and is protected from interference both day and night. These stations are called clear channel stations because the channel is cleared of nighttime interference over a major portion of the country. Class B stations operate full time with transmitter powers of 0.25 to 50 kW and are designed to render primary service only over a principal center of population and the rural area contiguous thereto. While nearly all Class A stations operate with 50 kW, most Class B stations must restrict their power to 5 kW or less to avoid interfering with other stations. Class B stations operating in the 1605 to 1705 kHz band are restricted to a power level of 10 kW daytime and 1 kW nighttime. Class C stations operate on six designated channels (1230, 1240, 1340, 1400, 1450, and 1490) with a maximum power of 1 kW or less full time and render primarily local service to smaller communities. Class D stations operate on Class A or B frequencies with Class B transmitter powers during daytime, but nighttime operation, if permitted at all, must be at low power (less than 0.25 kW) with no protection from interference.

1.1.2 FM Radio Broadcasting

Frequency-modulation (FM) broadcasting refers to the transmission of voice and music received by the general public in the 88- to 108-MHz frequency band. FM is used to provide higher-fidelity reception than is available with standard (AM) broadcast. FM broadcast is typically limited to line-of-sight ranges. As a result, FM coverage is localized to a range of 75 mi (120 km) depending on the antenna height and ERP.

The 100 carrier frequencies for FM broadcast range from 88.1 to 107.9 MHz and are equally spaced every 200 kHz. The channels from 88.1 to 91.9 MHz are reserved for educational and noncommercial broadcasting and those from 92.1 to 107.9 MHz for commercial broadcasting. Each channel has a 200-kHz bandwidth. The maximum frequency swing under normal conditions is ±75 kHz. The carrier frequency is required to be maintained within ±2000 Hz. The frequencies used for FM broadcasting generally limit the coverage to the line-of-sight or a slightly greater distance. The actual coverage area

Station Class	Maximum ERP (kW)	HAAT (m)	Distance (km)
A	6	100	28
B1	25	100	39
B	50	150	52
C3	25	100	39
C2	50	150	52
C1	100	299	72
C	100	600	92

Table 1: FM Station Classifications, Powers, and Tower Heights. Source: FCC Rules and Regulations, Revised 1991; vol. III, Part 73.211(b)(1).

is determined by the ERP of the station and the height of the transmitting antenna above the average terrain in the area. Either increasing the power or raising the antenna will increase the coverage area.

In FM broadcast, stations are classified according to their maximum allowable ERP and the transmitting antenna height above average terrain in their service area. Class A stations provide primary service to a radius of about 28 km with 6000 W of ERP at a maximum height of 100 m. The most powerful class, Class C, operates with maximums of 100,000 W of ERP and heights up to 600 m with a primary coverage radius of over 92 km. The powers and heights above average terrain (HAAT) for all of the classes are shown in Table ??. All classes may operate at antenna heights above those specified but must reduce the ERP accordingly. Stations may not exceed the maximum power specified, even if antenna height is reduced. The classification of the station determines the allowable distance to other co-channel and adjacent channel stations.

1.1.3 Digital Audio Broadcasting

In the years since the early 1980s, the consumer marketplace has undergone a great shift toward digital electronic technology. The explosion of personal computer use has led to greater demands for information, including multimedia integration. Over the same time period, compact disc (CD) digital audio technology has overtaken long-playing records (and has nearly overtaken analog tape cassettes) as the consumer audio playback media of choice. Similar digital transcription methods and effects also have been incorporated into commonly available audio and video equipment. Additionally, it is virtually certain that the upcoming transition to a high-definition television broadcast system will incorporate full digital methods for video and audio transmission. Because of these market pressures, the radio broadcast industry has determined that the existing analog methods of broadcasting must be updated to keep pace with the advancing audio marketplace.

In addition to providing significantly enhanced audio quality, DAB systems are being developed to overcome the technical deficiencies of existing AM and FM analog broadcast systems. The foremost problem of current broadcast technology, as perceived by the industry, is its susceptibility to interference. AM medium-wave broadcasts, operating in the 530- to 1700-kHz frequency range, are prone to disruption by fluorescent lighting and by power system distribution networks, as well as by numerous other manufactured unintentional radiators, including computer and telephone systems. Additionally, natural effects, such as nighttime skywave propagation interference between stations and lightning, cause irritating service disruption to AM reception. FM broadcast transmissions in the 88- to 108-MHz band are much more resistant to these types of interference. However, multipath propagation and abrupt signal fading, especially found in urban and mountainous areas containing a large number of signal reflectors and shadowers (e.g., buildings and terrain), can seriously degrade FM reception, particularly in automobiles.

1.2 TV Broadcasting

1.2.1 Analog Television

There are three incompatible primary color transmission standards in use today:

- NTSC (National Television Systems Committee): Used in the United States, Canada, Central America, most of South America, and Japan. In addition, NTSC is used in various countries or possessions heavily influenced by the United States. NTSC standard specifies 525 scanning lines per picture, a field rate of 59.94 per second (nominally 60 Hz), and 2:1 interlaced scanning (although there are about 60 fields per second, there are only 30 new frames per second). The aspect ratio (ratio of width to height) is 4:3. The bandwidth of the television signal is 6 MHz, including the sound signal.
- PAL (Phase Alternation each Line): Used in England, most countries and possessions influenced by the British Commonwealth, many western European countries and China. Variation exists in PAL systems. This specifies 625 scanning lines per picture and a field rate of 50 per second. The bandwidth of this type of television signal is 8 MHz.
- SECAM (Sequential Color with [Avec] Memory): Used in France, countries and possessions influenced by France, the USSR (generally the former Soviet Bloc nations), and other areas influenced by Russia.

Television transmitters in the United States operate in three frequency bands:

- Low-band VHF (very high frequency), channels 2 through 6: $[54, 60] + 6(c - 2)$ MHz
- High-band VHF, channels 7 through 13: $[174, 180] + 6(c - 7)$ MHz
- UHF (ultra-high frequency), channels 14 through 83: $[470, 476] + 6(c - 14)$ MHz (Channel 37, 608-614 MHz is reserved exclusively for the radio astronomy service; UHF channels 70 through 83 (806–890MHz) currently are assigned to mobile radio services).

Because of the wide variety of operating parameters for television stations outside the United States, this section will focus primarily on TV transmission as it relates to the Unites States.

Maximum power output limits are specified by the FCC for each type of service. The maximum effective radiated power (ERP) for low-band VHF is 100 kW; for high-band VHF it is 316 kW; and for UHF it is 5 MW. The ERP of a station is a function of transmitter power output (TPO) and antenna gain. ERP is determined by multiplying these two quantities together and subtracting transmission line loss.

The second major factor that affects the coverage area of a TV station is antenna height, known in the broadcast industry as height above average terrain (HAAT). HAAT takes into consideration the effects of the geography in the vicinity of the transmitting tower. The maximum HAAT permitted by the FCC for a low- or high-band VHF station is 1000 ft (305 m) east of the Mississippi River and 2000 ft (610 m) west of the Mississippi. UHF stations are permitted to operate with a maximum HAAT of 2000 ft (610 m) anywhere in the United States (including Alaska and Hawaii).

The ratio of visual output power to aural output power can vary from one installation to another; however, the aural is typically operated at between 10 and 20% of the visual power. This difference is the result of the reception characteristics of the two signals. Much greater signal strength is required at the consumer's receiver to recover the visual portion of the transmission than the aural portion. The aural power output is intended to be sufficient for good reception at the fringe of the station's coverage area but not beyond. It is of no use for a consumer to be able to receive a TV station's audio signal but not the video.

In addition to high power stations, two classifications of low-power TV stations have been established by the FCC to meet certain community needs. They are:

- Translator: A low-power system that rebroadcasts the signal of another station on a different channel. Translators are designed to provide "fill-in" coverage for a station that cannot reach a particular community because of the local terrain. Translators operating in the VHF band are limited to 100 W power output (ERP), and UHF translators are limited to 1 kW.
- Low-Power Television (LPTV):A service established by the FCC designed to meet the special needs of particular communities. LPTV stations operating on VHF frequencies are limited to 100 W ERP, and UHF stations are limited to 1 kW. LPTV stations originate their own programming and can be assigned by the FCC to any channel, as long as sufficient protection against interference to a full-power station is afforded.

1.2.2 Digital TV

HDTV systems nominally double the number of scan lines in a frame and change the aspect ratio to 16:9. Of course, if we were willing to start from scratch and abandon all existing television systems, we could set the bandwidth of each channel to a number greater than 6 (or 8) MHz, thereby achieving higher resolution. The Japan Broadcasting Corporation (NHK) has done just this in their HDTV system. This system permits 1125 lines per frame with 30 frames per second and 60 fields per second (2:1 interlaced scanning). The aspect ratio is 16:9. The system is designed for a bandwidth of 10

MHz per channel. With the 1990 launching of the BS-3 satellite, two channels were devoted to this form of HDTV. To fit the channel within a 10-MHz bandwidth (instead of the approximately 50 MHz that would be needed to transmit using traditional techniques), bandwidth compression was required. It should be noted that the Japanese system is primarily analog frequency modulation (FM) (the sound is digital). The approach to decreasing bandwidth is multiple sub-Nyquist encoding (MUSE). The sampling below Nyquist lowers the bandwidth requirement, but moving images suffer from less resolution.

Europe began its HDTV project in mid-1986 with a joint initiative involving West Germany (Robert Bosch GmbH), the Netherlands (NV Phillips), France (Thomson SA), and the United Kingdom (Thorn/EMI Plc.). The system, termed Eureka 95 or D2-MAC, has 1152 lines per frame, 50 fields per second, 2:1 interlaced scanning, and a 16:9 aspect ratio. A more recent European proposed standard is for 1250 scanning lines at 50 fields per second. This is known as the Eureka EU95. It is significant to note that the number of lines specified by Eureka EU95 is exactly twice that of the PAL and SECAM standard currently in use. The field rate is the same, so it is possible to devise compatible systems that would permit reception of HDTV by current receivers (of course, with adapters and without enhanced definition). The HDTV signal requires nominally 30 MHz of bandwidth.

In the United States, the FCC has ruled (in March 1990) that any new HDTV system must permit continuation of service to contemporary NTSC receivers. This significant constraint applies to terrestrial broadcasting (as opposed to videodisk, videotape, and cable television). The HDTV signals will be sent on taboo channels, those that are not used in metropolitan areas to provide adequate separation. Thus, these currently unused channels would be used for simulcast signals. Since the proposed HDTV system for the United States uses digital transmission, transmitter power can be less than that used for conventional television this reduces interference with adjacent channels. Indeed, in heavily populated urban areas (where many stations are licensed for broadcast), the HDTV signals will have to be severely limited in power. When a color television signal is converted from analog to digital (A/D), the luminance, hue, and saturation signals must each be digitized using 8 bits of A/D per sample. Digital transmission of conventional television therefore requires a nominal bit rate of about 216 megabits/s, while uncompressed HDTV nominally requires about 1200 megabits/s. If we were to use a digital modulation system that transmits 1 bit per hertz of bandwidth, we see that the HDTV signal requires over 1 GHz of bandwidth, yet only 6 MHz is allocated. Clearly significant data compression is required!

2 Channel Assignments

2.1 Interference Graph

The region in which the signal transmitted by a particular broadcast station can be received is called a *cell*. In an uneven area, e.g. in the mountains, cells can have a very irregular shape. However, in order to grasp the basic structure of the interference graphs, we must make some simplifying assumptions on the shape of the cells. If the signals that are transmitted by the broadcast stations can equally spread in all directions then the cells have the shape of disks. This could be the case in rural, plain areas.

If two cells overlap then the corresponding broadcast stations should use different radio channels in order to prevent interferences. For the purpose of this assignment, the network is modeled as a so-called *interference graph*. Each vertex of the interference graph represents a broadcast station and is associated with a disk representing the station's coverage area. There is an edge between two vertices if and only if their disks intersect. Thus, the interference graph is also called disk graph. When all stations have the same transmission power, then all disks are of the same radius and the disk graph is called as a unit-disk graph (UDG). Now, the channel assignment problem can then be formulated as the vertex coloring of the interference graph.

The determination of the chromatic number is NP-hard even for UDGs. NP-completeness of the 3-coloring has been proven in [1], and this result has been generalized in [4] to k-coloring of UDGs for an arbitrary k. Therefore, only polynomial-time approximation algorithms can be expected. The performance of a polynomial-time approximation algorithm is measured by its approximation ratio, which is the supreme, over all instances, of the ratio of the number of channels output by this algorithm to the minimum number of channels. It has been noted in [5] that no polynomial time algorithm for the coloring problem of UDGs can have performance ratio better than $\frac{4}{3}$, unless P = NP. In spite of these hardness results, the chromatic number of disk graphs can be efficiently approximated. It turns out that the chromatic number of disk graphs exceeds their clique number only by a small constant. The clique number of a unit disk graph can be obtained in $O\left(|V|^3\right)$ time [1].

For any graph, a vertex coloring can be induced by a vertex ordering in the following first-fit manner: Coloring the vertices sequentially according to the given vertex ordering by assigning each vertex the least possible color. There always exists a vertex coloring which induces an optimal vertex coloring. The number of colors used by the vertex coloring induced by a vertex ordering can be bound in terms of the *inductivity* of the vertex ordering, which is the least integer q such that each vertex is adjacent to at most q prior vertices. Obviously, the vertex coloring induced by a vertex ordering of inductivity q uses at most $q + 1$ colors. This suggests the use of a vertex ordering of small inductivity to induce a vertex coloring. The following vertex orderings of the interference graph will be used in this paper:

1. Smallest-last ordering: This vertex ordering has the least inductivity and can be found in polynomial time using a greedy algorithm given by Matula and Beck [6].

2. Radius-decreasing ordering: In this ordering, the vertices are sorted in the decreasing order of their transmission radii.

3. Distance-decreasing ordering: In this ordering, the vertices are sorted in the decreasing order of their Euclidean distances from an arbitrary fixed point.

4. Lexicographic ordering: In this ordering, the vertices are sorted in the lexicographic order of their coordinates.

For the sake of brevity, the algorithm which outputs the vertex coloring induced by a specified vertex ordering is referred to as *FIRST-FIT* in that specified vertex ordering. Corresponding to the above four vertex orderings, there are four FIRST-FIT heuristics: FIRST-FIT in smallest-last ordering, FIRST-FIT in radius-decreasing ordering, FIRST-FIT in distance-decreasing ordering, and FIRST-FIT in lexicographic ordering.

We introduce some symbols and notations that will be used throughout this paper. The set of given radio nodes is denoted by V. For each node $v \in V$, its transmission radius is denoted by r_v. We use B_v to denote the disk centered at v with radius r_v, and C_v to denote the boundary circle of B_v. The nodes in V are said to *uniform* transmission radii is all r_v's are equal. The interference graph of V is denoted by G. We use $\chi(G), \omega(G), \Delta(G)$ to denote its chromatic number, clique number, and maximum degree respectively. The inductivity of the smallest-last ordering is also called the *inductivity* of G and is denoted by $ind(G)$.

The remaining of this section is arranged as follows. In Section 2.2, we explore topological properties of the neighborhood of the node with the smallest transmission radius in the interference graph. Base on these topological properties, in Section 2.3 we derive upper bounds on the approximation ratios of all FIRST-FIT greedy algorithms. In Section 2.4, we present the algorithm STRIP coloring for nodes with uniform transmission radii and analyze its approximation ratio. Finally we conclude the paper in Section 3.

2.2 Neighborhood of The Node with The Smallest Transmission Radius

In this section, we fix u to be the node with minimum transmission radius. By proper scaling, we assume that $r_u = 1$. We study the sufficient conditions for two neighbors of u to be adjacent. We partition the plane into six 60^o-sectors as shown in Figure 1(a). The next lemma shows that all neighbors of u in each sector form a clique.

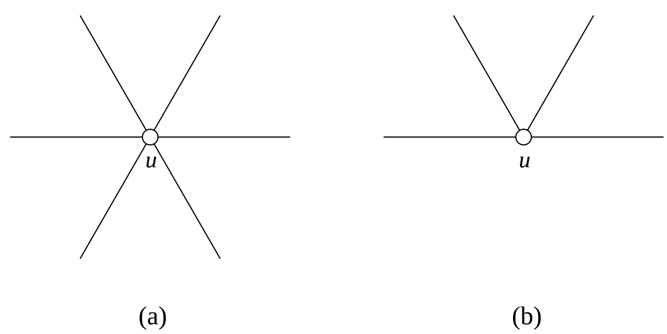

Figure 1: Partitions of the neighborhood of u.

Lemma 1 *All neighbors of u lying in each of the six sectors in the partition shown in Figure 1 (a) form a clique. Consequently, u has at most five independent (i.e., pairwise nonadjacent) neighbors.*

Proof. It is sufficient to show that any two neighbors, v and w, of u lying in the same sector are adjacent. If v and w lie in the unit disk, then $\widehat{vuw} \leq 60^o$. Since $|uv| \leq \max\{|uw|, 1\}$, then in $\triangle uvw$,

$$|vw| \leq \max\{|uv|, |uw|\} \leq \max\{|uw|, 1\} \leq r_w.$$

Thus, $v \in B_w$. ■

Finally, if all neighbors of u lie in a half-plane , we adopt the partition shown in Figure 1(b) of the half-plane containing all neighbors of u. Following the similar argument in the proof of Lemma 1, we can prove the following lemma.

Lemma 2 *Suppose that all neighbors of u lie in a half-plane. Then all neighbors of u lying in each of the three sectors in the partition shown in Figure 1(b) of the half-plane containing all neighbors of u form a clique. Consequently, u has at most three independent neighbors.*

2.3 FIRST-FIT Coloring

Since FIRST-FIT in a vertex ordering of inductivity q uses at most $q+1$ colors, our analyses of the FIRST-FIT colorings will be based on the upper bounds on the inductivities of the corresponding vertex orderings. Since the smallest-last ordering has the smallest inductivity, an upper bound on the inductivity of any other vertex ordering applies to the smallest-last ordering as well.

We begin with upper-bounding the inductivity of the radius-decreasing ordering.

Lemma 3 *The inductivity of any radius-decreasing ordering is at most* $\min\{6\omega(G)-7, 5\chi(G)-4\}$.

Proof. Consider an arbitrary radius-decreasing ordering and let q be its inductivity. Let u be a node with q prior neighbors in this ordering. Note that the transmission radii of these q prior neighbors of u are no less than that of u. By proper scaling, we can assume that the transmission radii of u is one. Then the transmission radii of these q prior neighbors of u are at least one.

We first show that $q \leq 6\omega(G)-7$. We partition the plane as shown in Figure 1(a) such that at least one of these q prior neighbors lie in the boundary of one of those six sectors. Note u has at most $\omega(G)-1$ neighbors in each sector, and at least one neighbor is counted more than once. Therefore,

$$q \leq 6(\omega(G)-1)-1 = 6\omega(G)-7.$$

Now we show that $q \leq 5\chi(G)-4$. By Lemma 1, at most five prior neighbors of u are independent can get the same color in any proper coloring. Therefore, the prior neighbors of u requires at least $\lceil \frac{q}{5} \rceil$ colors. Since u must be colored differently than its neighbors, u and all its q prior neighbors requires at least $1+\lceil \frac{q}{5} \rceil$ colors. This implies that

$$\chi(G) \geq 1+\left\lceil \frac{q}{5} \right\rceil.$$

Therefore,

$$q \leq 5\chi(G)-5.$$

■

From the above lemma, we have

$$ind\left(G\right) \leq \min\left\{6\omega\left(G\right) - 7, 5\chi\left(G\right) - 5\right\}.$$

Thus, we have the following theorem.

Theorem 4 *Both FIRST-FIT in smallest-last ordering and FIRST-FIT in radius-decreasing ordering use at most* $\min\left\{6\omega\left(G\right) - 6, 5\chi\left(G\right) - 4\right\}$ *colors, and hence have approximation ratios of at most 5.*

As a corollary of the above theorem,

$$\omega\left(G\right) \leq \chi\left(G\right) \leq 6\omega\left(G\right) - 6.$$

We also remark that when all nodes have the uniform transmission radii, then every vertex ordering is a radius-decreasing ordering. In this case, FIRST-FIT in an *arbitrary* vertex ordering still has an approximation ratio of at most 5. This also follows from the fact if all nodes have the uniform transmission radii, then

$$\Delta\left(G\right) \leq \min\left\{6\omega\left(G\right) - 7, 5\chi\left(G\right) - 4\right\}.$$

The proof of this fact is similar to that of Lemma 3 and is omitted here.

Next, we upper-bound the inductivities of distance-decreasing ordering and lexicographic ordering when all nodes have uniform transmission radii.

Lemma 5 *Suppose that all nodes have uniform transmission radii. Then the inductivity of any distance-decreasing ordering or any lexicographic ordering is at most* $3\omega\left(G\right) - 3$*.*

Proof. A key property of both distance-decreasing ordering and any lexicographic ordering is that for each node u, all its prior neighbors all lie in a half-plane with u at the boundary. Because of this property, Lemma 2 can be applied. The theorem can then be proven by following the similar but simpler argument to the proof of Lemma 3. The detail is omitted. ■

From the above lemma, if all nodes have uniform transmission radii, then

$$ind\left(G\right) \leq 3\omega\left(G\right) - 3.$$

Thus, we have the following theorem.

Theorem 6 *Suppose that all nodes have uniform transmission radii. Then FIRST-FIT in smallest-last ordering, FIRST-FIT in distance-decreasing ordering, and FIRST-FIT in lexicographic ordering all use at most* $3\omega\left(G\right) - 2$ *colors, and hence have approximation ratios of at most 3.*

As a corollary of the above theorem, if all nodes have uniform transmission radii, then

$$\omega(G) \leq \chi(G) \leq 3\omega(G) - 2.$$

The bound on the inductivity $ind(G) \leq 3\omega - 3$ is tight.

Theorem 7 *For each given $\omega > 0$, there exists a UDG G with $\omega(G) = \omega$ and such that every vertex has degree $3\omega - 3$.*

Proof. For $\omega = 1$, the graph containing a single vertex has the required property.
For $\omega > 1$, consider two concentric circles C_1 and C_2. The outer circle C_1 has radius

$$r_1 = \frac{1}{2\sin\frac{\omega-1}{n}\pi}.$$

Now n nodes are placed at equidistance positions on the circle C_1. Then, the distance between any vertex and a corresponding ωth consecutive vertex is exactly one. Therefore, each ω consecutive vertices on C_1 form a clique (but not $\omega + 1$ vertices). The same construction is repeated for the inner circle C_2, which has a radius

$$r_2 = r_1 - \alpha$$

for some constant $0 < \alpha \leq 1$. If ω is even, then the n vertices on the circle C_2 are placed exactly opposite the vertices on the circle C_1. Otherwise, the n vertices on the circle C_2 are shifted by $\frac{\pi}{2n}$. Then, the distance between any vertex and a corresponding ωth consecutive vertex is less than one. On the other hand, the distance between any vertex and a corresponding $(\omega + 1)$th consecutive vertex is greater than one. Furthermore, each vertex on one circle is adjacent to exactly $\omega - 1$ nodes on the other circle. Consequently, each vertex has degree $3\omega - 3$. ■

Theorem 8 *For each given $\omega > 0$, there exists a UDG G with $\omega(G) = \omega$ and $\chi(G) \geq \left\lfloor \frac{3\omega}{2} \right\rfloor$.*

Proof. Consider a UDG over $3\omega - 1$ points uniformly distributed on a circle with radius $\frac{1}{\sqrt{3}}$. Each vertex has $\omega - 1$ neighbors to the left and $\omega - 1$ neighbors to the right. Hence we have $\omega(G) = \omega$. On the other hand, $\alpha(G) = 2$ and therefore $\chi(G) \geq \left\lceil \frac{3\omega-1}{2} \right\rceil = \left\lfloor \frac{3\omega}{2} \right\rfloor$. ■

2.4 STRIP Coloring

Throughout of this section, we assume that all nodes have uniform transmission radii equal to one. Under this assumption, the interference graph is the square of the unit-disk graph over all the nodes. Such property allows us to develop an approximation algorithm, called STRIP coloring, with better approximation ratio of at most 4. Before we present this algorithm, we first briefly introduce cocomparability graphs and discuss their properties.

A graph H is said to be a *cocomparability graph* if there is a vertex ordering $[v_1, v_2, \cdots, v_n]$ of $V(H)$ such that if $i < j < k$ and $v_i v_k \in E(H)$ then either $v_i v_j \in E(H)$ or $v_j v_k \in E(H)$. Such ordering is referred to as its *cocomparability ordering.* Every cocomparability graph H is perfect [3], i.e., $\chi(H) = \omega(H)$. Furthermore, the minimum vertex coloring of a cocomparability graph can be found in polynomial time, either by solving a maximum bipartite matching [2] or by solving a minimum-flow problem [3].

A unit-disk graph is in general not a cocomparability graph. However, if all nodes lie in a horizontal strip of width $\sqrt{3}/2$, then their unit-disk graph is a cocomparability graph and the lexicographic ordering is a cocomparability ordering [4]. Therefore, when all nodes lie in a horizontal strip of height $\sqrt{3}/2$, an optimal vertex coloring can be obtained in polynomial time.

Now we are ready to present the algorithm STRIP coloring. We first find a top-most node, a bottom-most node, a left-most node, and a right-most node in V. These nodes define an axis-parallel rectangle, which covers all nodes in V. We partition such rectangle into top-closed bottom-open horizontal strips, such that the top of the top-most strip aligns with the top of the rectangle, the heights of all strips except the bottom-most one are all equal to$\sqrt{3}/2$, and the height of the bottom-most strip is at most $\sqrt{3}/2$ (see Figure 2). Let the successive strips be numbered from top to bottom using integers $0, 1, \cdots$. For each strip i, let V_i denote the nodes in this strip and G_i denote the interference graph of V_i. For simplicity, each color is represented by a duple (c_1, c_2) where $0 \leq c_1 \leq 2$ and c_2 is a positive integer. By applying any optimal coloring algorithms for cocomparibility graphs, we color G_i using the colors

$$\{(i \bmod, c_2) : 1 \leq c_2 \leq \chi(G_i)\}.$$

The coloring is valid since for any i and j with $i < j$ and $i \bmod 3 = j \bmod 3$, any node in a strip i and any node in a strip j are separated by a distance of at least $\sqrt{3} > 1$ and thus are not adjacent.

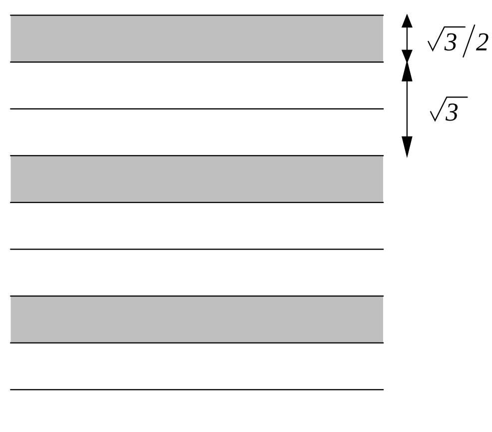

Figure 2: Partition into horizontal strips of height $\sqrt{3}/2$.

Next, we show that STRIP coloring uses at most $3\omega(G)$ colors. Since cocomparibility graphs are perfect, $\chi(G_i) = \omega(G_i)$ for all i. Since G_i is a subgraph of G, $\omega(G_i) \leq \omega(G)$. Therefore, all colors used by STRIP coloring belong to the set

$$\{(c_1, c_2) : 0 \leq c_1 \leq 2, 1 \leq c_2 \leq \omega(G)\}.$$

This implies that STRIP coloring uses at most $3\omega(G)$ colors.

Theorem 9 *STRIP coloring uses at most $3\omega(G)$ colors, and hence has an approximation ratio of at most 3.*

3 Summary

Channel assignment is a fundamental problem in broadcast networks. It is NP-hard even when all nodes are located in a plane and have equal transmission radii. In this paper, we provide an analysis of several greedy FIRST-FIT algorithms, FIRST-FIT in smallest-last ordering, FIRST-FIT in radius-decreasing ordering, FIRST-FIT in distance-decreasing ordering, and FIRST-FIT in lexicographic ordering. In addition, for nodes with equal transmission radii, we present a non-greedy algorithm, STRIP coloring and analyze its approximation ratio. Table 2 summarizes the upper bounds on the approximation ratios of these algorithms.

Table 2: Upper bounds on the approximation ratios. SL: smallest-last; RD: radius-decreasing; DD: distance-decreasing; LO: lexicographic.

radii	FIRST-FIT				STRIP
	SL	RD	DD	LG	
arbitrary	5	5	N/A	N/A	N/A
uniform	3	5	3	3	3

We also obtain relations among the inductivity, the clique number, and the chromatic number of the interference graph. Table 3 summarizes the upper bounds on the pairwise ratios of these parameters.

Table 3: Relations among inductivity, clique number, and chromatic number of the interference graph.

radii	$\frac{\omega(G)}{ind(G)}$	$\frac{\chi(G)}{ind(G)}$	$\frac{\omega(G)}{\chi(G)}$
arbitrary	6	5	5
uniform	3	3	3

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