Fractional Wireless Link Scheduling and Polynomial Approximate Capacity Regions of Wireless Networks

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Outline

- Introduction
- A New Paradigm
- Profitable Feasible Set
- Applications
- Summary
Wireless network \((V, A; \mathcal{I})\)
Problem Description

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- Link demand \(d \in \mathbb{R}_+^A\)
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- Capacity region \(\Omega := \{d \in \mathbb{R}^A_+ : \chi^*(d) \leq 1\}\)
- Polynomial \((\alpha, \beta)\)-approximate capacity region \(\Phi: \frac{1}{\alpha} \Omega \subseteq \Phi \subseteq \beta \Omega\).
Prior Art

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Roadmap

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Profitable Subset

- Link weight $w \in \mathbb{R}_{++}^A$
Profitable Subset

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- \textit{Return rate of} \( a \in A \): \( \overline{w}(a) := \frac{w(a)}{d(a)} \)
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- Return rate of $I \subseteq A$: $\overline{w}(I)$
Profitable Subset

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- Return rate of $a \in A$: $\overline{w}(a) := \frac{w(a)}{d(a)}$
- Return rate of $I \subseteq A$: $\overline{w}(I)$
- $\beta$-profitable subset of $S \subseteq A$: $I \subseteq S$ s.t. $\overline{w}(I) \geq \frac{w(S)}{\beta}$
Profitable Subset

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- *Return rate* of $I \subseteq A$: $\overline{w}(I)$
- $\beta$-profitable subset of $S \subseteq A$: $I \subseteq S$ s.t. $\overline{w}(I) \geq \frac{w(S)}{\beta}$
  - $\beta$ depends only on $d$
A New Paradigm for Link Scheduling

- $\beta$-Profitable oracle $A$: for any $S \subseteq A$, compute a $\beta$-profitable IS $I$ of $S$
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- $\epsilon \in (0, 1/2]$: accuracy-efficiency trade-off parameter
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- $LS(\varepsilon)$: computes a link schedule of $d$
• \( \beta \)-Profitable oracle \( \mathcal{A} \): for any \( S \subseteq A \), compute a \( \beta \)-profitable IS \( I \) of \( S \)

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  • length \( \leq (1 + \varepsilon) \beta \)
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- $\beta$-Profitable oracle $A$: for any $S \subseteq A$, compute a $\beta$-profitable IS $I$ of $S$
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- $LS(\epsilon)$: computes a link schedule of $d$
  - length $\leq (1 + \epsilon) \beta$
  - $O(\epsilon^{-2} \vert A \vert \ln \vert A \vert)$ calls of $A$
```
// initialization
Γ ← ∅, //schedule
P ← 0, // vector of proportion of demands served by Γ
φ ← \frac{\ln|A|+\varepsilon}{\varepsilon(1+\varepsilon)+\ln(1-\varepsilon)}; //inactive threshold
S ← A, // active links

// schedule augmentations
...

// scaling
return \frac{1}{φ} Γ.
```
Schedule Augmentations

```latex
while \( S \neq \emptyset \) do

    \begin{align*}
    &\text{// augmentation}\nonumber \\
    &I \leftarrow \text{IS of } S \text{ output by } \mathcal{A} \text{ w.r.t. exponential weight} 
onumber \\
    &w(a) = (1 - \varepsilon)^{P(a)}, \forall a \in S 
    
    &t \leftarrow \min_{a \in I} d(a); 
    
    &\Gamma \leftarrow \Gamma \cup \{(l, t)\}; 
    \end{align*}

    \begin{align*}
    &\text{// updates} 
    
    &\text{for each } a \in I \text{ do} 
    
    &P(a) \leftarrow P(a) + \frac{t}{d(a)}; \text{// update the profit} 
    
    &\text{if } P(a) \geq \phi \text{ then } S \leftarrow S \setminus \{a\}; \text{// inactive} 
    \end{align*}
```
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Conflict factor $\rho : A \times A \rightarrow [0, 1]$
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$\rho$-feasible set $I$: $\rho (I \setminus \{a\}, a) < 1, \forall a \in I$
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$\rho$-feasible set $I$: $\rho(I \setminus \{a\}, a) < 1, \forall a \in I$

Maximum local load

$$\Delta (d) = \max_{a \in A} \left[ d(a) + \sum_{b \in A \setminus \{a\}} \rho(b, a) d(b) \right]$$
Profitable Feasible Set

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- $\rho$-feasible set $I$: $\rho (I \setminus \{a\}, a) < 1, \forall a \in I$
- Maximum local load

$$\Delta (d) = \max_{a \in A} \left[ d(a) + \sum_{b \in A \setminus \{a\}} \rho(b, a) d(b) \right]$$

- Algorithm **PFS**: For any $w \in \mathbb{R}_+^A$ and $S \subseteq A$, output a $4\Delta (d)$-profitable $\rho$-feasible set $I \subseteq S$
Extraction Procedure

\[ \bar{\rho}(b, a) := \frac{\overline{w}(b)}{\overline{w}(a)} \rho(a, b) + \rho(b, a), \forall a, b \in S \]
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\]

- **Initialization:** \( I \leftarrow \emptyset \)
Extraction Procedure

\[ \bar{\rho}(b, a) := \frac{w(b)}{w(a)}\rho(a, b) + \rho(b, a), \forall a, b \in S \]

- **Initialization:** \( I \leftarrow \emptyset \)

- **Growing Phase:** While \( S \neq \emptyset \), remove any \( a \) from \( S \), and add it to \( I \) if
  \[ \bar{\rho}(I, a) + \frac{1}{2\Delta(d)} \sum_{b \in S} \bar{\rho}(b, a) d(b) < 1. \]
Extraction Procedure

$$\bar{\rho}(b, a) := \frac{w(b)}{w(a)} \rho(a, b) + \rho(b, a), \forall a, b \in S$$

- **Initialization:** $I \leftarrow \emptyset$

- **Growing Phase:** While $S \neq \emptyset$, remove any $a$ from $S$, and add it to $I$ if

$$\bar{\rho}(I, a) + \frac{1}{2\Delta(d)} \sum_{b \in S} \bar{\rho}(b, a) d(b) < 1.$$ 

- **Pruning Phase:** While $I$ is not $\rho$-feasible, remove from $I$ any $a$ with $\rho(I \setminus \{a\}, a) \geq 1$
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∃ conflict factor $\rho$ s.t.,
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\( \rho \)-feasibility \iff \) independence
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- ρ-feasibility ⇔ independence
- Δ(d) ≤ μχ∗(d) for constant μ (under linear transmission power)
∃ conflict factor $\rho$ s.t.,

- $\rho$-feasibility $\iff$ independence
- $\Delta(d) \leq \mu \chi^*(d)$ for constant $\mu$ (under linear transmission power)

- $4\Delta(d)$-Profitable oracle $A$: PFS
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- $\rho$-feasibility $\iff$ independence
- $\Delta(d) \leq \mu \chi^*(d)$ for constant $\mu$ (under linear transmission power)

4$\Delta(d)$-Profitable oracle $\mathcal{A}$: PFS

Schedule length: $\leq 4(1 + \varepsilon)\Delta(d) \leq 4(1 + \varepsilon)\mu \chi^*(d)$
∃ conflict factor ρ s.t.,

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4Δ(d)-Profitable oracle \( \mathcal{A} \): PFS

Schedule length: \( \leq 4(1 + \varepsilon)\Delta(d) \leq 4(1 + \varepsilon)\mu\chi^*(d) \)

Polynomial (\( \mu, 4 \))-approximate capacity region:

\[
\Phi = \left\{ d \in \mathbb{R}_+^A : \Delta(d) \leq 1 \right\}.
\]
MIMO Wireless Networks under Protocol IM

- DoF $\tau$

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- Links $\leftrightarrow$ streams
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- DoF $\tau$
- Links $\leftrightarrow$ streams
- Independence under receiver-side interference suppression
DoF $\tau$

Links $\leftrightarrow$ streams

Independence under receiver-side interference suppression

Half-Duplex Constraint
MIMO Wireless Networks under Protocol IM

- DoF $\tau$
- Links $\leftrightarrow$ streams
- Independence under receiver-side interference suppression
  - **Half-Duplex Constraint**
  - **Receiver Constraint**: Each receiving node $v$ is interfered by $< \tau$ streams
MIMO Wireless Networks under Protocol IM

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- Links $\leftrightarrow$ streams
- Independence under receiver-side interference suppression
  - **Half-Duplex Constraint**
  - **Receiver Constraint**: Each receiving node $v$ is interfered by $< \tau$ streams
  - **(Sender Constraint)**
Conflict factor

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- $\rho$-feasibility $\iff$ meeting Receiver Constraint
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- $\Delta(d) \leq \mu \chi^*(d)$ for constant $\mu$
Phase 1: **PFS** to produce a $4\Delta(d)$-profitable $J \subseteq S$ meeting the Receiver Constraint
Oracle for Profitable IS

- **Phase 1**: **PFS** to produce a $4\Delta (d)$-profitable $J \subseteq S$ meeting the Receiver Constraint
- **Phase 1**: extracting a $16\Delta (d)$-profitable $I \subseteq J$ meeting the Half-duplex Constraint
Oracle for Profitable IS

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  - **Bipartition of Nodes**: $U = U' \cup U''$ s.t. $J[U', U'']$ has $1/2^+$ return rate of $J$
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  - **Bipartition of Nodes:** $U = U' \cup U''$ s.t. $J[U', U'']$ has $1/2^+$ return rate of $J$
    - greedy partition $\sim$ Maximum Cut
  
  - **Bipartition of Streams:** $(U' \rightarrow U'') \cup (U' \leftarrow U'')$; return the one with largest return rate
Schedule length: \[ \leq 16 (1 + \varepsilon) \Delta (d) \leq 16 (1 + \varepsilon) \mu \chi^* (d) \]
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• Polynomial $(\mu, 16)$-approximate capacity region:

$$\Phi = \left\{ d \in \mathbb{R}_+^A : \Delta (d) \leq 1 \right\}. $$
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Profitable IS $\leftrightarrow$ Link Schedule $\leftrightarrow$ Poly. Approx. Capacity Region
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- Explicit polynomial approximate capacity region