Fractional Wireless Link Scheduling and Polynomial Approximate Capacity Regions of Wireless Networks

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Outline

- Introduction
- A New Paradigm
- Profitable Feasible Set
- Applications
- Summary

Problem Description

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- Polynomial (α, β)-approximate capacity region Φ:

$$\frac{1}{\alpha}\Omega \subseteq \Phi \subseteq \beta\Omega.$$

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  - β depends only on d

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  - $O\left(\varepsilon^{-2} |A| \ln |A|\right)$ calls of $\mathcal{A}$

Design Overview

// **initialization**
$\Gamma \leftarrow \emptyset$, //schedule
$P \leftarrow \mathbf{0}$, // vector of proportion of demands served by Γ
$\phi \leftarrow \frac{\ln\|A\|+\varepsilon}{\varepsilon(1+\varepsilon)+\ln(1-\varepsilon)}$; //inactive threshold
$S \leftarrow A$, // active links
// **schedule augmentations**
$\vdots$
// **scaling**
return $\frac{1}{\phi}\Gamma$.

Schedule Augmentations

while $S \neq \emptyset$ do

// **augmentation**

$I \leftarrow$ IS of S output by $\mathcal{A}$ w.r.t. *exponential* weight

$$w(a) = (1-\varepsilon)^{P(a)}, \forall a \in S$$

$t \leftarrow \min_{a \in I} d(a)$;

$\Gamma \leftarrow \Gamma \cup \{(I, t)\}$;

// **updates**

for each $a \in I$ do

$P(a) \leftarrow P(a) + \frac{t}{d(a)}$; // **update the profit**

if $P(a) \geq \phi$ then $S \leftarrow S \setminus \{a\}$; // **inactive**

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- Algorithm **PFS**: For any $w \in \mathbb{R}_{++}^{A}$ and $S \subseteq A$, output a $4\Delta(d)$-profitable ρ-feasible set $I \subseteq S$

Extraction Procedure

$$\overline{\rho}\left(b,a\right):=\frac{\overline{w}\left(b\right)}{\overline{w}\left(a\right)}\rho\left(a,b\right)+\rho\left(b,a\right),\forall a,b\in S$$

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- **Initialization**: $I \leftarrow \emptyset$
- **Growing Phase:** While $S \neq \emptyset$, remove any a from S, and add it to I if

$$\overline{\rho}(I,a) + \frac{1}{2\Delta(d)} \sum_{b \in S} \overline{\rho}(b,a)\, d(b) < 1.$$

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$$\overline{\rho}(I,a) + \frac{1}{2\Delta(d)} \sum_{b \in S} \overline{\rho}(b,a)\, d(b) < 1.$$
- **Pruning Phase:** While I is not ρ-feasible, remove from I any a with $\rho(I \setminus \{a\}, a) \geq 1$

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- Polynomial $(\mu, 4)$-approximate capacity region:

$$\Phi = \left\{ d \in \mathbb{R}_+^A : \Delta(d) \leq 1 \right\}.$$

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  - **(Sender Constraint)**

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  - **Bipartition of Streams:** $(U' \to U'') \cup (U' \leftarrow U'')$; return the one with largest return rate

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- Explicit polynomial approximate capacity region