Maximizing Network Capacity of MPR-Capable Wireless Networks

P.-J. Wan, F. Al-dhelaan, X. Jia, B. Wang, and G. Xing

wan@cs.iit.edu

Outline

- Introduction
- Exam Algorithms in Single Interference Domain
- Approximation Algorithms in General Networks
- PTAS
- Summary

Network Model

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  - Each $a \in A$ has rate $c(a)$

Independence (i.e. Feasibility) with MPR

Independence (i.e. feasibility) of $I \subseteq A$

1. **MPR Constraint**: Each node v is the receiver of at most $\tau(v)$ links in I.
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$\mathcal{I}$: collection of all independent subsets of A

Max-Weighted Independent Set (MWIS)

Given a non-negative weight function w *on* A*, find an independent subset* I *of* A *with maximum total weight* $\sum_{e \in I} w(e)$.

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Shortest Link Scheduling (SLS)

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$$\mathcal{S} = \{(I_j, x_j) \in \mathcal{I} \times \mathbb{R}^{+} : 1 \leq j \leq q\}$$

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- Capacity region of the network:

$$\left\{ d \in \mathbb{R}_+^A : \chi^*(d) \leq 1 \right\}$$

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  - $\sum_{j=1}^{k} f_j$: cumulative flow of f

Maximum Weighted Multiflow (MWMF)

Given that each request j has a weight $w_j > 0$ per unit of its flow, find a multiflow $f = \langle f_1, \cdots, f_k \rangle$ and a link schedule $\mathcal{S}$ of $\sum_{j=1}^{k} f_j$ such that the length of $\|\mathcal{S}\| \leq 1$ and the total weight of f given by

$$\sum_{j=1}^{k} val\left(f_j\right) w_j.$$

is maximized.

Maximum Concurrent Multiflow (MCMF)

Given that each request j has a demand $d_j > 0$, find a multiflow $f = \langle f_1, \cdots, f_k \rangle$ and a link schedule $\mathcal{S}$ of $\sum_{j=1}^{k} f_j$ such that $\|\mathcal{S}\| \leq 1$ and the concurrency of f given by

$$\min_{1 \leq j \leq k} val\left(f_j\right) / d_j.$$

is maximized.

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- Still the same approximality?

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Exact Algorithm for MWIS

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- Enumeration

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  - Length

$$\max\left\{\max_{a\in\delta_B^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a\in\delta_B^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)}\right\}.$$

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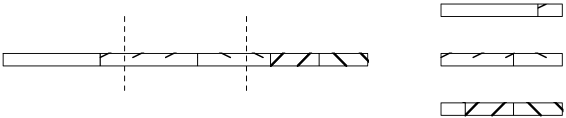
  - A wrap-around scheme


(a) (b)

Capacity Region

$$\Omega = \left\{ d \in \mathbb{R}_+^A : \sum_{v \in V} \max \left\{ \max_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\} \leq 1 \right\}.$$

Exact Algorithms for MWMF, MCMF

MWMF:

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\begin{array}{ll}
\max & \sum_{j=1}^{k} w_j \cdot val\left(f_j\right) \\
s.t. & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k; \\
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MCMF:

$$\begin{aligned} \max \quad & \phi \\ s.t. \quad & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k; \\ & val\left(f_j\right) \geq \phi d_j, \forall 1 \leq i \leq k; \\ & \sum_{j=1}^{k} f_j \in \Omega. \end{aligned}$$

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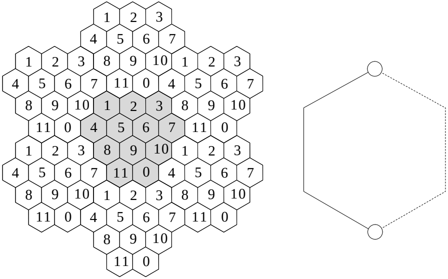
Spatial Division



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- hexagon cells of diameter $r - 1$

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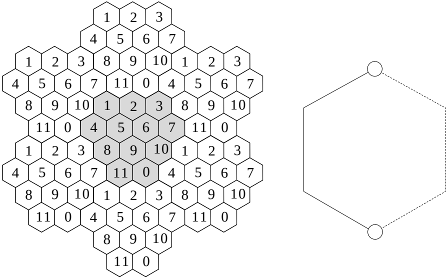

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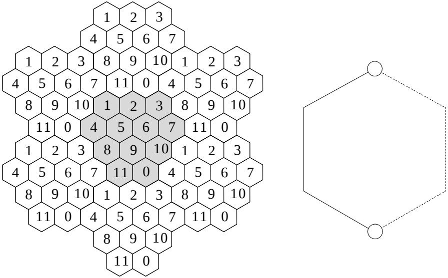

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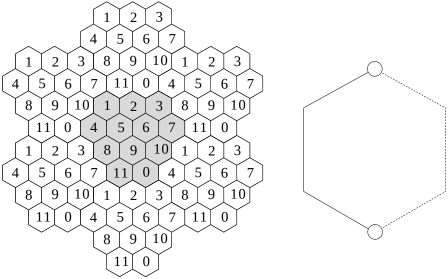

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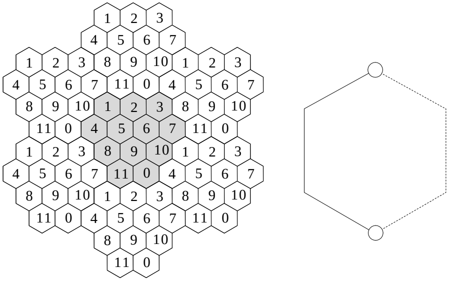

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- cell labelling: all co-label cells are apart $> r + 1$
  - # of labels $7 \leq \lambda \leq 12$ for $r \geq 3$
- $\forall 0 \leq i < \lambda$, $\mathcal{S}_i \leftarrow$ the set of non-empty cells with label i

Divide-And-Conquer Algorithm for MWIS

- **Conquer**: For each non-empty cell S, $I_S \leftarrow$ a MWIS of all links associated with S.

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- λ-approixmate

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- λ-approximate solution.

Approximate Capacity Region

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λ-approximate:

$$\Phi \subseteq \Omega \subseteq \lambda \Phi.$$

Exact Restricted Mutiflows

Φ-restricted MWMF:

$$\begin{array}{ll} \max & \sum_{j=1}^{k} w_j \cdot val\left(f_j\right) \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k; \\ & \sum_{j=1}^{k} f_j \in \Phi. \end{array}$$

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λ-approximations

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- PTAS for **SLS**, **MWMF**, and **MCMF**: approximation-preserving reductions from **MWIS**

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- Future work: arbitrary interference radii