Capacity Maximization in Wireless MIMO Networks with Receiver-Side Interference Suppression

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Outline

- Introduction
- Computational Hardness
- Practical Constant-Approximation Algorithms
- Summary

Network Model

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- (V, A): stream-level communication topology

Receiver-Side Interference Suppression under PrIM

When $I \subseteq A$ transmits at the same time, the transmission by a stream $a \in I$ from a sender u to a receiver v succeeds if the following constraints are satisfied:

1. **Half-Duplex Constraint**: u (resp. v) is not the receiver (resp. sender) of any other stream in I.
2. **Sender Constraint**: u is the sender is at most $\tau(u)$ streams in I.
3. **Receiver Constraint**: v lies in the interference range of at most $\tau(v)$ streams in I.

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- **MIS**: $\{0, 1\}$-weighted variant of **MWIS**

Algorithmic Issues of MWIS

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- Do there exist a PTAS?
- Do there exist poly. time approx. algorithms with constant approximation bound and practical running time?

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- NP-hard even when the input streams with positive weight are *node-disjoint*: due to the **receiver-side interference suppression**
- APX-hard when the nodes have *arbitrary* number of antennas: due to the **half-duplex constraint**.
- PTAS when the maximum number of antennas at all nodes is bounded by a *constant*.
- Practical constant-approx. algorithms when all streams have *uniform* interference radii or all nodes have *uniform* number of antennas

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NP-Hardness

Theorem

The problem **MIS** *is NP-hard even when restricted to node-disjoint streams with uniform interference radii and when all nodes have uniform and fixed number of antennas.*

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Reduction from maximum k-restricted independent set (**MAX** k-**RIS**) in UDGs

APX-Hardness

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With uniform but arbitrarily many antennas at each node, the problem **MIS** *is NP-hard and APX-hard even when restricted to pairwise conflicting streams.*

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Reduction from the maximum directed cut (**MAX Di-Cut**),

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When $\max_{v \in V} \tau(v)$ *is bounded by a constant, the problem* **MWIS** *admits a PTAS.*

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- shifting strategy + dynamic programming
- Sparsity of IS: If a point o lies in the interference ranges of an independent set I of streams, then $|I| = O(1)$.

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Relaxation

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- Relaxation of independence on **Half-Duplex Constraint**

Extraction

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- A^+: the set of streams in A with positive weight.

Spatial Division

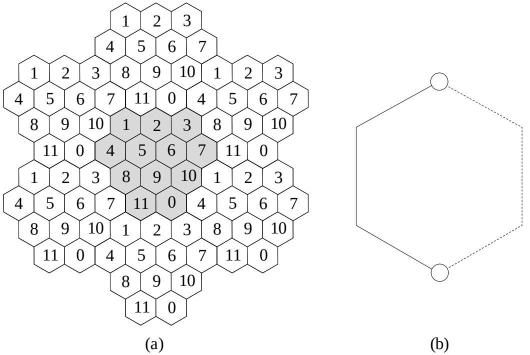

Figure: Tiling of the plane into half-open half closed hexagons of diameter diameter $r - 1$. A stream is said to be associated with a cell if its sender lies in this cell

Conquer

Computes a heaviest weakly independent subset S of the streams B in A^+ associated with a non-empty cell.

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- $S \leftarrow$ the heaviest one among $S_1, S_2, \cdots, S_k$

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Theorem

The output I is a 4λ-approximate solution.

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LP-Relaxation And Rounding

- Uniform number τ of antennas but arbitrary interference radii
  - **Receiver Constraint** $\Rightarrow$ **Sender Constraint**.
- LP based constant-approximation approximation algorithm

Interference Factor

For any $a, b \in A^+$,

- $\rho(a, b) = 1/\tau$ if the receiver of b lies within the interference range of a,
- $\rho(a, b) = 0$ otherwise.

LP Relaxation

Compute an optimal solution x to the LP:

$$\begin{array}{ll} \max & \sum_{a \in B} w(a)\, x(a) \\ s.t. & \sum_{b \in A^+ \setminus \{a\}} \rho(a,b)\, x(a) \leq \frac{1}{2}, \forall b \in A^+ \\ & 0 \leq x(a) \leq \frac{1}{2}, \forall a \in A^+. \end{array}$$

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$$\sum_{b \in A^+ \setminus \{a\}} \left(\frac{w(b)}{w(a)} \rho(a,b) + \rho(b,a) \right) x(b) < 1,$$

set $x(a) \leftarrow 1$; otherwise set $x(a) \leftarrow 0$.

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    - remove a from J

Extraction of IS

- $I \leftarrow$ the IS I extracted from J by **ExtractIS**

Theorem

Suppose that the interference radius of each stream is at least η times its length. Then, I is a $16\left(\left\lceil \pi/\arcsin\frac{1-1/\eta}{2}\right\rceil - 1\right)$-approximate solution.

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- PTAS in case of constant-bounded antennas
- Practical constant-approx. algorithms in case of uniform interference radii or uniform number of antennas
- **Open Problem**: whether there exists a practical constant-approximation algorithm in the most general setting of arbitrary interference radii and arbitrary number of antennas