A New Paradigm for Multiflow in Wireless Networks: Theory And Applications

Peng-Jun Wan

wan@cs.iit.edu

Outline

- Introduction
- An Adaptive Coupled Game
- Maximum Concurrent Multiflow
- Summary

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  - A k-flow is a sequence $f = \langle f_1, f_2, \cdots, f_k \rangle$ with $f_j \in \mathcal{F}_j$ $\forall j \in [k]$

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Given k requests $\{(s_j, t_j; d_j) : j \in [k]\}$, find a link schedule $\mathcal{S}$ of unit length and a k-flow $(f_1, \cdots, f_k)$ with $\sum_{j\in[k]} f_j \leq c_S$ such that

$$\min_{j\in[k]} val\,(f_j)\,/\,d_j$$

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- NP-hard

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- Computationally inefficient

A New Combinatorial Paradigm

- $\mathcal{A}$: μ-approximation algorithm for **MWIS**

  Given a non-negative weight function w on A, find an independent subset I of A with maximum total weight $\sum_{a \in I} w(a)$.

- $\varepsilon \in (0, 1/2]$: trade-off between accuracy and efficiency
- $(1 + 2\varepsilon)\,\mu$-approx. alg. for **MCMF** by making
  - $O(\varepsilon^{-2})$ calls to $\mathcal{A}$
  - $O(\varepsilon^{-2})$ calls to Disjkstra's shortest-path algorithm

Applications

- MCMF in SC-SR wireless networks under physical interference model
- MCMF in MIMO wireless networks

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  - The agents update y according to the **MWU Rule**:

$$y(e) \leftarrow y(e)\left(1 - \varepsilon\left(p(e) - l(e)\right)\right), \forall e \in E$$

Analysis of The Game

- $P(e)$: cumulative profit of e
- $L(e)$: cumulative loss of e

Theorem

At the end of each round,

$$\frac{\ln(1+\varepsilon)}{\ln(1-\varepsilon)^{-1}} L(e) - P(e) \leq \frac{\ln m}{\ln(1-\varepsilon)^{-1}}, \forall e \in E$$

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- **Scaling Stage**: Down scale both $\frac{\ln(1+\varepsilon)}{\ln(1-\varepsilon)^{-1}} f$ and $\Gamma \cup \Gamma'$ by $\|\Gamma \cup \Gamma'\|$ and then return them.

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- Agent: **WMU Rule**
- Adversary: At the end of each round,

$$\begin{aligned} \text{cumulative profit of } a &= g(a) \\ \text{cumulative loss of } a &= \sum_{j \in [k]} f_j(a). \end{aligned}$$

Profits/Losses Generation In Each Round

$$\boxed{\begin{array}{l} I \leftarrow \text{a } y\text{-weighted IS in } \mathcal{I} \text{ output by } \mathcal{A}; \\ \forall j \in [k],\ P_j \leftarrow \text{a shortest } j\text{-path w.r.t. } y; \end{array}}$$

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$$\boxed{\begin{array}{l}\Gamma \leftarrow \Gamma \cup \{(I, \delta)\}; \\ \forall a \in I,\ g(a) \leftarrow g(a) + \delta; \\ \forall j \in [k] \text{ and } \forall a \in P_j,\ f_j(a) \leftarrow f_j(a) + \lambda \delta d_j;\end{array}}$$

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  - invariant property

$$d'(a) \leq \frac{\ln m}{\ln(1-\varepsilon)^{-1}}, \forall a \in A$$

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$$\|\Gamma'\| = \sum_{J \in \mathcal{J}} \max_{a \in J} d'(a) \leq \frac{l \ln m}{\ln (1-\varepsilon)^{-1}}.$$

Scaled Concurrency And Termination Criteria

- Concurrence of $\frac{1}{\|\Gamma\cup\Gamma'\|}\frac{\ln(1+\varepsilon)}{\ln(1-\varepsilon)^{-1}}f$: at least

$$\frac{\|\Gamma\|}{\|\Gamma\cup\Gamma'\|}\frac{\ln\left(1+\varepsilon\right)}{\ln\left(1-\varepsilon\right)^{-1}}\frac{opt}{\mu}\geq\frac{1}{1+\frac{\frac{l\ln m}{\ln(1-\varepsilon)^{-1}}}{\|\Gamma\|}}\frac{\ln\left(1+\varepsilon\right)}{\ln\left(1-\varepsilon\right)^{-1}}\frac{opt}{\mu}$$

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- Termination criteria: To ensure the scaled concurrency $\geq \frac{opt}{(1+2\varepsilon)\mu} \mu$, terminates when

$$\|\Gamma\| \geq \frac{l \ln m}{(1+2\varepsilon)\ln(1+\varepsilon) + \ln(1-\varepsilon)},$$

Upper Bound on # of Iterations

- h_j: the hop number of a min-hop path of the j-th request for each $j \in [k]$
- α: the size of a maximum-sized independent set of A
- In each iteration

$$\delta \geq \frac{1}{\max\left\{1, \frac{\alpha}{\min_{j\in[k]} h_j}\right\}}.$$

- # of iterations at most

$$\frac{\max\left\{1, \frac{\alpha}{\min_{j\in[k]} h_j}\right\} / \ln m}{(1+2\varepsilon)\ln(1+\varepsilon) + \ln(1-\varepsilon)} = O\left(\varepsilon^{-2} \max\left\{1, \frac{\alpha}{\min_{j\in[k]} h_j}\right\} / \ln m\right).$$

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- Combinatorial reduction from cross-layer MCMF to
  - link-layer **MWIS**
  - network-layer **Shortest Path**
- Faster and simpler
- Primary LS and complementary LS
- An adaptive coupled game