Joint Selection And Transmission Scheduling of Point-to-Point Communication Requests in Multi-Channel Wireless Networks

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Outline

- Overview
- Light Requests
- Heavy Requests
- Summary

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- **MWFS**: Given $w \in \mathbb{R}_+^A$, find an $F \in \mathcal{F}$ maximizing $w(F) = \sum_{a \in F} w(a)$.

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- Feasibility test is NP-hard alone!

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$$\Delta^{\prec}(B) := \max_{a \in B} \left[d(a) + \sum_{b \in N(a) \cap B_{\prec a}} \varrho(a,b)\, d(b) \right].$$

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  - *inductivity* of B in an ordering

Selection: Divide & Conquer

- **Division**:

$$A_1 \leftarrow \left\{ a \in A : d\left(a\right) \leq \frac{1}{2} \right\} \text{ // light requests}$$

$$A_2 \leftarrow \left\{ a \in A : d\left(a\right) > \frac{1}{2} \right\} \text{ // heavy requests}$$

- **Conquer**: Apply a μ_i-approx. alg. to select a feasible subset F_i of A_i for $i = 1, 2$.
- **Combination**: Return the better one between F_1 and F_2.
  - a $(\mu_1 + \mu_2)$-approximate solution.

Strategies for Conquer

- Local-ratio (primal-dual) scheme: ordering based
- Fractional local-ratio (primal-dual) scheme: orientation based

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for each $a \in S$ in $\prec$ do
    if $d(a) + \sum_{b \in N(a) \cap F} \varrho(a, b)\, d(b) \leq 1$, $F \leftarrow F \cup \{a\}$;
return F. //*maximal inductively feasible subset* of S in $\prec$

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- (Fractional) Local-ratio (equivalently, primal-dual) scheme

Local-Ratio Scheme

- Applicability: Admitting an ordering $\prec$ with small BLIN

$$\mu := \max_{a \in A} \max_{I \in \mathcal{I}_G} \left| I_{\preceq a} \cap N\left[a\right] \right|$$

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Interference Radii	Ordering	BLIN
arbitrary	[Wan, Mobihoc' 09]	23
symmetric	[Wan et, al, Infocom' 11]	8
uniform	[Joo, et, al,, IEEE TAC'09]	6

Local-Ratio Scheme

- **Phase 1**: Selection of the candidate set S:

$$S \leftarrow \emptyset;$$

for each $a \in A$ in the *reverse* order of $\prec$ do

$$\overline{w}(a) \leftarrow w(a) - d(a) \sum_{b \in N(a) \cap S} \varrho(a, b) \frac{\overline{w}(b)}{1-d(b)};$$

if $\overline{w}(a) > 0$, $S \leftarrow S \cup \{a\}$;

- **Phase 2**: Compute the maximal inductively feasible subset F of S in $\prec$.

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Mode	Orientation	ILIN
Unidirectional	[Wan, Mobihoc' 09]	$\left\lceil \pi / \arcsin \frac{1-c}{2} \right\rceil - 1$
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Fractional Local-Ratio Scheme

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\begin{array}{ll}
\max & \sum_{a\in A} \frac{w(a)}{d(a)} x(a) \\
s.t. & x(a) + \sum_{b\in N_D^{in}(a)} \varrho(a,b)\, x(b) \leq 1/2, \forall a \in A \\
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\end{array}
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- **Phase 0**: Compute an optimal *partial demand* x and a smallest-last ordering $\prec$ of A w.r.t. x.
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Approximation bound: $2\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right)$, where μ is the BLIN of $\prec$

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  - An orientation with ILIN $\mu \Longrightarrow 4\left(\mu + 2\left(1 - \frac{1}{\lambda}\right)\right)$-approx.

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- Restriction: inductive feasibility, inductive compatibility,