Mechanism Design For Set Cover Games When Elements Are Agents

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Abstract. In this paper we study the set cover games when the elements are selfish agents. In this case, each element has a privately known valuation of receiving the service from the sets, *i.e.*, being covered by some set. Each set is assumed to have a fixed cost. We develop several approximately efficient truthful mechanisms, each of which decides, after soliciting the declared bids by all elements, which elements will be covered, which sets will provide the coverage to these selected elements, and how much each element will be charged. For set cover games when both sets and elements are selfish agents, we show that a cross-monotonic *payment*-sharing scheme does not necessarily induce a truthful mechanism.

1 Introduction

In the past, an indispensable and implicit assumption on algorithm design for interconnected computers has been that all participating computers (called *agents*) are cooperative; they will behave exactly as instructed. This assumption is being shattered by the emergence of the Internet, as it provides a platform for distributed computing with agents belonging to self-interested organizations. This gives rise to a new challenge that demands the study of *algorithmic mechanism design*, the sub-field of algorithm design under the assumption that all agents are *selfish* (*i.e.*, they only care about their own benefits) and yet *rational* (*i.e.*, they will always choose their actions to maximize their benefits).

Assume that there are n agents $\{1, 2, \cdots, i, \cdots, n\}$, and each agent i has some *private* information t_i, called its *type*. For direct-revelation mechanisms, the strategy of each agent i is to declare its type, although it may choose to report a carefully designed lie to influence the outcome of the game to its liking. For any vector $t = (t_1, t_2, \cdots, t_n)$ of reported types, the mechanism computes an output o as well as a payment p_i for each agent i. For each possible output o, agent i's preference is defined by a valuation function $v_i(t_i, o)$. The utility of agent

⋆ The research of the author was supported in part by Grant FRG/03-04/II-21 and Grant RGC HKBU2107/04E.

⋆⋆ The research of the author was supported in part by NSF under Grant CCR-0311174.

⋆⋆⋆ The research of the author was supported in part by Grant RGC HKBU2159/04E.

i for the outcome of the game is defined to be $u_i = v_i(t_i, o) + p_i$. An action a_i is called a *dominant strategy* for player i if it maximizes its utility regardless of the actions chosen by other players; a selfish agent will always choose its dominant strategy. A mechanism is *incentive compatible* (IC) if for every agent reporting its type truthfully is a dominant strategy. Another very common requirement in the literature for mechanism design is *individual rationality*: the agent's utility of participating in the outcome of the mechanism is not less than the utility of the agent if it does not participate at all. A mechanism is called *truthful* or *strategyproof* if it satisfies both IC and IR properties.

A classical result in mechanism design is the Vickrey-Clarke-Groves (VCG) mechanism by Vickrey [1], Clarke [2], and Groves [3]. The VCG mechanism applies to maximization problems where the objective function $g(o,t)$ is simply the sum of all agents' valuations. A VCG mechanism is always truthful [3], and is the only truthful implementation, under mild assumptions, to maximize the total valuation [4]. Although the family of VCG mechanisms is powerful, it has its limitations. To use a VCG mechanism, we have to compute the exact solution that maximizes the total valuation of all agents. This makes the mechanism computationally intractable for many optimization problems.

This work focuses on strategic games that can be formulated as the set cover problem. A *set cover game* can be generally defined as the following. Let $\mathcal{S} = \{S_1, S_2, \cdots, S_m\}$ be a collection of multisets (or *sets* for short) of a universal set $U = \{e_1, e_2, \cdots, e_n\}$. Element e_i is specified with an *element coverage requirement* r_i (*i.e.*, it desires to be covered r_i times). The multiplicity of an element e_i in a set S_j is denoted by $k_{j,i}$. Let $d_{\max}$ be the maximum size of the sets in $\mathcal{S}$, *i.e.*, $d_{\max} = \max_j \sum_i k_{j,i}$. Each S_j is associated with a cost c_j. For any $\mathcal{X} \subseteq \mathcal{S}$, let $c(\mathcal{X})$ denote the total cost $\sum_{S_j \in \mathcal{X}} c_j$ of the sets in $\mathcal{X}$. The outcome of the game is a *cover* $\mathcal{C}$, which is a subset of $\mathcal{S}$. Many practical problems can be formulated as a set cover game defined above. For example, consider the following scenario: a business can choose from a set of service providers $\mathcal{S} = \{S_1, S_2, \cdots, S_m\}$ to provide services to a set of service receivers $U = \{e_1, e_2, \cdots, e_n\}$.

- ⋆ With a fixed cost c_j, each service provider S_j can provide services to a fixed subset of service receivers.
- ⋆ There may be a limit $k_{j,i}$ on the number of units of service that a service provider S_j can provide to a service receiver e_i.
- ⋆ Each service receiver e_i may have a limit r_i on the number of units of service that it desires to receive (and is willing to pay for).

A mechanism of the game is to determine an optimal (or approximately optimal) outcome of the game, according to a pre-defined objective function. We design various mechanisms that are aware of the fact that the service receivers and/or the service providers are selfish and rational. In addition to truthfulness, we aim to achieve the following objectives, which are sometimes at odds with each other and thus require proper tradeoffs.

- ⋆ **Economic Efficiency** A mechanism is α-efficient if its output is no worse is than α times the optimal solution with respect to the objective function.

- ⋆ **Budget Balance** Let $C(S)$ be the total cost incurred by providing services to all agents in S. If $\xi_i(S)$ is the cost charged to each agent $i \in S$, the cost-sharing method is β-budget-balanced if $\sum_{i \in S} \xi_i(S) \geq \beta \cdot C(S)$, for some $0 < \beta < 1$.
- ⋆ **Fair Cost-Sharing** We also need to make the cost-sharing method fair so that it encourages agents to participate. Besides the well accepted measures such as *cross-monotonicity* (*i.e.*, the cost share of an agent should not go up if more players require the service), we also consider a less-studied measure, called *fairness under core* (*i.e.*, the cost shares paid by any subset of agents should not exceed the minimum cost of providing the service to them alone), which is derived from game theory concepts [5].
- ⋆ **No Positive Transfers (NPT)** The cost shares are non-negative.
- ⋆ **Voluntary Participation (VP)** The utility of each agent is guaranteed to be non-negative if an element reports its bid truthfully.
- ⋆ **Consumer Sovereignty (CS)** When an agent's bid is large enough, and others' bids are fixed, the agent will get the service.

We first consider the case where the elements to be covered are selfish agents; each e_i has a privately known valuation $b_{i,r}$ of the r-th unit of service to be received. We show that the truthful cost-sharing mechanism designed by a straightforward application of a cross-monotonic cost-sharing scheme is not α-efficient for any $\alpha > 0$. We present another truthful mechanism such that the total valuation of the elements covered is at least $\frac{1}{d_{\max}}$ times that of an optimal solution. This mechanism, however, may have free-riders: some elements do not have to pay at all and are still covered. We then present an alternative truthful mechanism without free-riders and it is at least $\frac{1}{d_{\max} \ln d_{\max}}$-efficient. When the sets are also selfish agents with privately known costs, we show that the cross-monotonic *payment*-sharing scheme does not induce a truthful mechanism; a set could lie about its cost to improve its utility. The positive side is that the mechanism is still truthful for elements.

Previously, Devanur *et al.* [7] studied the truthful cost-sharing mechanisms for set cover games, with elements considered to be selfish agents. In a game of this type, each element will declare its bid indicating its valuation of being covered, and the mechanism uses the greedy algorithm [8] to compute a cover with an approximately minimum total cost. Li *et al.* [6] extended this work by providing a truthful cost-sharing mechanism for multi-cover games. They also designed several cost-sharing schemes to fairly distribute the costs of the selected sets to the elements covered, for the case that both sets and elements are unselfish (*i.e.*, the will declare their costs/bids truthfully). The case of set cover games where sets are considered as selfish agents was also considered. Immorlica *et al.* [9] provided bounds on approximate budget balance for cross-monotone cost-sharing scheme for the set cover games.

2 Preliminaries

Typically, the objective function of a game is defined to be the total valuation of the agents selected by the outcome of the game. In set cover games, when sets are considered to be agents (*e.g.*, [6]), maximizing the total valuation of all selected agents is equivalent to minimizing the total cost of all selected sets. However, if the elements are considered to be agents, the objective becomes to maximize the total valuation of all elements (*i.e.*, the sum of all bids covered). Correspondingly, we need to solve the following optimization problem:

Problem 1. Each element e_i is associated with a *coverage requirement* r_i and a set of bids $B_i = \{b_{i,1}, b_{i,2}, \cdots, b_{i,r_i}\}$ such that $b_{i,1} \geq b_{i,2} \geq \cdots \geq b_{i,r_i}$. An *assignment* $\mathcal{C}$ is defined as the following: i) $\mathcal{C} \subseteq \mathcal{S}$; ii) a bid $b_{i,r}$ can be assigned to at most one set $S_{\pi(i,r)} \in \mathcal{C}$; iii) for any $S_j \in \mathcal{C}$, the *assigned value* $\nu_j(\mathcal{C}) = \sum_{\pi(i,r)=j} b_{i,r}$ is no less than c_j (S_j is "affordable"); iv) $\kappa_{j,i} \leq k_{j,i}$, where $\kappa_{j,i}$ is the number of bids of e_i assigned to S_j; v) if the number γ_i of assigned bids of e_i is less than r_i, then the assigned bids must be the first γ_i bids (with the greatest bid values) of e_i. The *total value* $V(\mathcal{C}) = \sum_{S_j \in \mathcal{C}} \nu_j(\mathcal{C})$ is the sum of all assigned bids in $\mathcal{C}$. The problem is to find an assignment with the maximum total value.

This problem is NP-hard. In fact, the weighted set packing problem, which is NP-complete, can be viewed as a special case of this problem. Therefore, the VCG mechanism cannot be used here if polynomial-time computability is required. In the rest of the paper, we concentrate on designing approximately efficient and polynomial-time computable mechanisms.

All our methods follow a round-based greedy approach: in each round t, we select some set S_{j_t} to cover some elements. After the s-th round, we define the *remaining required coverage* r'_i of an element e_i to be $r_i - \sum_{t'=1}^{s} \kappa_{j_{t'},i}$. For any $S_j \notin \mathcal{C}_{grd}$, the *effective coverage* $k'_{j,i}$ of e_i by S_j is defined to be $\min\{k_{j,i}, r'_i\}$. The *effective value* (or *value* for short) v_j of S_j is therefore $\sum_{i=1}^{n} \sum_{r=1}^{k'_{j,i}} b_{i,r_i - r'_i + r}$ and it is *affordable* after s-th round if $v_j \geq c_j$.

One scheme is to select a set S_j as long as it is still affordable, and assign all appropriate bids to S_j. However, in this case an element may find it profitable to lie about its bid, as we will show in Section 3. An alternative scheme is to pick a set only if it is *individually affordable*, as defined as the following:

Definition 1. *A set S_j is individually affordable by d bids if it contains at least d bids each with a value no less than $\frac{c_j}{d}$, for some $d > 0$.*

Consequently, only the d largest bids are assigned to S_j, for the maximum d such that S_j is individually affordable by d bids. Notice that here an implicit assumption is that each set S_j can selectively provide coverage to a subset of elements contained by S_j. This is to prevent anybody from taking "free rides." The *modified value* $\tilde{v}_j$ of S_j is defined to be the total value of these bids. The following lemma gives upper bounds on the total value lost by enforcing individually affordable sets:

Lemma 1. *For any set $S_j \in \mathcal{S}$, i) if S_j is individually affordable, the modified value $\tilde{v}_j$ is no less than $\frac{1}{\ln d_{\max}}$ fraction of its value v_j; ii) if S_j is not individually affordable, its value is no more than $\ln d_{\max}$ times the cost c_j of S_j.*

3 Set Cover Games with Selfish Receivers

In this section we first study the case where only elements are selfish.

An obvious solution to designing a truthful mechanism for single-cover set cover games is to use a cross-monotone cost-sharing scheme based on a theorem proved in [10]: a cross-monotone cost-sharing scheme implies a group-strategyproof mechanism when the cost function is submodular, non-negative, and non-decreasing. A cost function C is submodular if $C(T_1)+C(T_2) \geq C(T_1 \cup T_2) + C(T_1 \cap T_2)$. A cost function C is non-decreasing if $C(T_1) \leq C(T_2)$ for any $T_1 \subseteq T_2$. A cost-sharing scheme is group-strategyproof if, for any group of agents who collude in revealing their valuations, if no member is made worse off, then no member is made better off. For set cover games, it is not difficult to show by example that the following cost functions are *not* submodular: the cost $c(\mathcal{C}_{opt})$ defined by the optimal cover $\mathcal{C}_{opt}$ of a set of elements, and the cost defined by the traditional greedy method (*i.e.*, in every round we select the set S_j with the minimum ratio of cost c_j over the number of elements covered by S_j and not covered by sets selected before)[3]. Even if a cost function is submodular, sometimes it may be NP-hard to compute this cost, and thus we cannot use this cost function to design a truthful mechanism. It was shown in [6] that there is a cost function that is indeed submodular: for each element $e_i \in T$, we select the set S_j with the minimum cost that covers e_i. Notice that, if it is a multi-cover set cover game, each set S_j is only eligible to cover an element e_i $k_{j,i}$ times. Let $\mathcal{C}_{lcs}(T)$ be all sets selected to cover a set of elements T. Then $c(\mathcal{C}_{lcs})$ is submodular, non-decreasing, and non-negative.

Given the cost function $c(\mathcal{C}_{lcs})$, it was shown in [6] that the cost-sharing method $\xi_i(T)$, defined as $\xi_i(T) = \sum_{S_j \in \mathcal{C}_{lcs}(T)} \frac{\kappa_{j,i} \cdot c_j}{\sum_a \kappa_{j,a}}$, is budget-balanced, cross-monotone and a $\frac{1}{2n}$-core. Here $\kappa_{j,i}$ is the number of bids of e_i assigned to S_j. For a single-cover set cover game, based on the method described in [10], given the single bid $b_{i,1}$ by each element e_i, we can define a mechanism $M(\xi)$ as follows.

Algorithm 1 Mechanism for single cover games via cost-sharing.

1: $S^0 = U$; $t = 0$;
2: **repeat**
3: $\quad S^{t+1} = \{e_i \mid b_{i,1} \geq \xi_i(S^t)\}$; $t = t + 1$;
4: **until** $S^{t-1} = S^t$
5: The output of mechanism $M(\xi)$ is $\tilde{U}(\xi, b) = S^t$,
6: The charge by $M(\xi)$ to an element e_i is $\xi_i(\tilde{U}(\xi, b))$.

The following theorem is directly implied by the result in [10].

Theorem 1. *The cost-sharing mechanism $M(\xi)$ is group-strategyproof, budget-balanced, and meets NPT, CS, and VP.*

[3] Notice that the greedy method we will present later is different from this traditional greedy set cover method.

However, this mechanism is not *efficient* at all. We can construct an example to show that it cannot be α-efficient for any $\alpha > 0$. Next, in Algorithm 2, we describe a new greedy algorithm that computes for a single cover game an approximately optimal assignment $\mathcal{C}_{grd}$. Starting with $\mathcal{C}_{grd} = \emptyset$, in each round t' the algorithm adds to $\mathcal{C}_{grd}$ a set $S_{j_{t'}}$ with the maximum effective value.

Algorithm 2 Greedy algorithm for single cover games.

```
C_grd ← ∅.
For all S_j ∈ S, x compute effective value v_j.
while S ≠ ∅ do
   pick set S_t in S with the maximum effective value v_t.
   C_grd ← C_grd ∪ {S_t}, S ← S \ {S_t}.
   for all e_i ∈ S_t do
      π(i, 1) ← t; remove e_i from all S_j ∈ S.
   for all S_j ∈ S do
      update effective value v_j.
      If v_j < c_j, then S ← S \ {S_j}.
```

The following theorem establishes an approximation bound for the algorithm.

Theorem 2. *Algorithm 2 computes an assignment* $\mathcal{C}_{grd}$ *with a total value* $V(\mathcal{C}_{grd}) \geq \frac{1}{d_{\max}} \cdot V(\mathcal{C}_{opt})$.

Obviously, Algorithm 2 satisfies the monotone property defined in [11]: when an element e_i was selected with a bid $b_{i,1}$, then it will always be selected with a bid $\bar{b}_{i,1} > b_{i,1}$. This monotone property implies that there is always a truthful cost-sharing mechanism using Algorithm 2 to compute its output. Further, Algorithm 2 is a round-based greedy method that satisfies the cross-independence property defined in [11]. Thus, the payment to each element can always be computed in polynomial time. We include the description of this mechanism in the full version of this paper [13].

However, Algorithm 2 and and its induced cost-sharing mechanism together may produce an output such that the payment by a certain element is 0. To avoid this zero payment problem, we use a slightly different algorithm to determine the outcome of the game. Our modified greedy method (described in Algorithm 3) instead only selects individually affordable sets. When a set S_j is added into $\mathcal{C}_{grd}$, the algorithm only assigns to S_j the largest d bids, such that S_j is individually affordable with d bids, for the maximum such d. Using the same argument, we can show that there is a polynomial-time computable and truthful cost-sharing mechanism using Algorithm 3.

On the approximate efficiency of the modified greedy algorithm, we have

Theorem 3. *When only individually affordable sets are allowed to be picked, the assignment* $\mathcal{C}_{grd}$ *computed by Algorithm 3 has a total value that is: 1) no less than* $\frac{1}{d_{\max}} \cdot V(\mathcal{C}_{opt})$*, if the optimal assignment* $\mathcal{C}_{opt}$ *also allows only individually*

Algorithm 3 Improved greedy algorithm for single cover games.

```
C_grd ← ∅.
For all S_j ∈ S, compute the modified value ṽ_j.
while S ≠ ∅ do
  pick set S_t in S with the maximum modified value ṽ_t.
  C_grd ← C_grd ∪ {S_t}, S ← S \ {S_t}.
  d_t ← the largest d such that the set S_t is individually affordable by d largest
  unsatisfied bids.
  for all e_i ∈ S_t do
    if b_{i,1} is one of the largest d_t unsatisfied bids in S_t then
      π(i, 1) ← t; remove e_i from all S_j ∈ S.
  for all S_j ∈ S do
    update the modified value ṽ_j.
    If ṽ_j < c_j, then S ← S \ {S_j}.
```

affordable sets; 2) no less than $\frac{1}{2d_{\max}} \cdot V(\mathcal{C}_{opt})$*, if the optimal assignment* $\mathcal{C}_{opt}$ *allows sets that are not individually affordable, but all sets in* $\mathcal{S}$ *are individually affordable initially.*

Theorem 2 and Theorem 3 can easily be extended to the case of multi-cover. However, when it comes to computing payments, there is a problem: in the multi-cover case, an element can lie in different ways, and it may not be of its best interest if it achieves the maximum utility in the first bid (or the last bid). In that case, how can we compute payments efficiently?

To overcome the computational complexity of computing payments, we design another mechanism using a different greedy algorithm to compute the outcome of the game. This algorithm is the same as Algorithm 3 of [6]. In [6] it is shown that this mechanism produces an outcome with a total cost no more than $\ln d_{\max}$ times the total cost of an optimal outcome. We claim that the outcome is also approximately efficient with respect to the total valuation of the assigned (covered) bids. Further, due to the monotone property, this mechanism is truthful.

Theorem 4. *Algorithm 3 of [6] defines a budget-balanced and truthful mechanism. Further, it is* $\frac{1}{d_{\max} H_{d_{\max}}}$*-efficient, if all sets are individually affordable initially.*

4 Set Cover Games with Selfish Providers and Receivers

So far, we assume that the cost of each set is publicly known or each set will truthfully declare its cost. In practice, it is possible that each set could also be a selfish agent that will maximize its own benefit, *i.e.*, it will provide the service only if it receives a payment by some elements (not necessarily the elements covered by itself) large enough to cover its cost. In [6], Li *et al.* designed several truthful payment schemes to selfish sets such that each set maximizes its utility when it truthfully declares its cost and the covered elements will pay whatever a charge computed by the mechanism. They also designed a payment sharing scheme that is budget-balanced and in the core.

To complete the study, in this section, we study the scenario when both the sets and the elements are individual selfish agents: each set S_j has a privately known cost c_j, while each element e_i has a privately known bid $b_{i,r}$ for the r-th unit of service it shall receive and is willing to pay for it only if the assigned cost is at most $b_{i,r}$. It is well-known that a cross-monotone *cost* sharing scheme implies a truthful mechanism [10]. Unfortunately, since the sets are selfish agents, it is impossible to design any cost-sharing scheme here, and the best we can do is to design some payment sharing scheme. It was shown in [12] that a cross-monotone payment sharing scheme does *not* necessarily induce a truthful mechanism by using multicast as a running example: a relay node could lie its cost upward or downward to improve its utility.

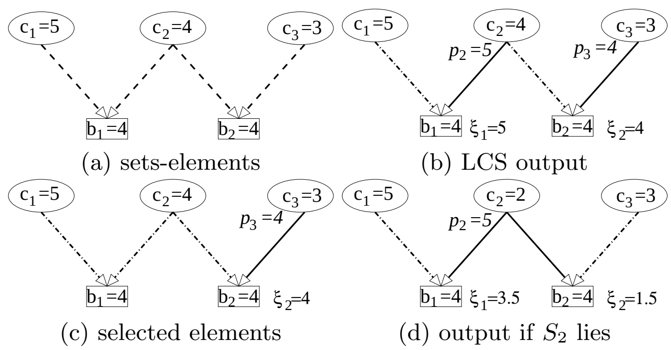
Given a subset of elements $T \subseteq U$ and their coverage requirement r_i for $e_i \in T$, a collection of multisets $\mathcal{S}$, and each set $S_j \in \mathcal{S}$ with cost c_j, let M_S be a truthful mechanism that will determine which sets from $\mathcal{S}$ will be selected to provide the coverage to *all* elements T, and the payment p_j to each set S_j. We assume that the mechanism is normalized: the payment to an unselected set S_j is always 0. Based on two monotonic output methods, the traditional greedy set cover method (denoted as GRD) and the least cost set method (denoted as LCS) for each element, Li *et al.* [6] designed two truthful mechanisms for set cover games. Let $E(S_j, c, T, M_S)$ be the set of elements covered by S_j in the output of M_S. In the remaining of the paper, we assume that the mechanism M_S satisfies the property that if a set S_j increases its cost then the set of elements covered by S_j in the output of M_S will *not* increase, *i.e.*, $E(S_j, c|^j d, T, M_S) \subseteq E(S_j, c, T, M_S)$ for $d > c_j$. This property is satisfied by all methods currently known for set cover games.

Let $\xi_{i,j}(T)$ be the shared payment by element e_i for its jth copy when the set of elements to be covered is T, given a truthful payment scheme M_S to all sets. Following the method described in [10], given the set U of n elements and their bids $B_1, \cdots, B_n$ we can compute the outcome $\tilde{U}(\xi, B)$ as the limit of the following inclusion monotonic sequence: $S^0 = U$; $\quad S^{t+1} = \{e_i \mid b_{i,j} \geq \xi_{i,j}(S^t)\}$. Notice that here we have to recompute the payments to all sets, and thus the shared payments by all elements, when the set of elements to be covered is changed from S^t to S^{t+1}. In other words, we define a mechanism $M_E(\xi)$ associated with the payment sharing method ξ as follows: the set of elements to be covered is $\tilde{U}(\xi, B)$, the charge to element e_i is $\xi_{i,j}(\tilde{U}(\xi, B))$ if $e_i \in \tilde{U}(\xi, B)$; otherwise its charge is 0. Based on the truthful mechanism using LCS as output for set cover games, Li *et al.* [6] designed a payment sharing mechanism that is budget-balanced, cross-monotone, and in the core.

Hereafter, we assume that for the payment-sharing scheme ξ, the payment p_j to the set S_j is only shared among the elements, *i.e.*, $E(S_j, c, T, M_S)$, covered by S_j. This property is satisfied by the payment-sharing methods studied in [6] for set cover games.

Theorem 5. *For set cover games with selfish sets and elements, a truthful mechanism M_S to sets and a cross-monotone payment sharing scheme ξ imply that in mechanism M_E each set S_j cannot improve its utility by lying upward its cost.*

Unfortunately, for set cover games, we show that a truthful mechanism M_S to sets and a cross-monotone payment sharing scheme ξ do *not* induce a truthful mechanism M_E for each element. Figure 1 illustrates such an example when LCS is used as the output, a set s_j can lie its cost downward to improve its utility from 0 to $p_j - c_j$. A similar example can be constructed when the traditional greedy method is used as the output. When set S_2 is truthful, although LCS will select it to cover element e_1 with payment $p_2 = 5$, but the corresponding sharing by e_1 is $\xi_1 = 5$, which is larger then its bid $b_{1,1} = 4$. Consequently, set S_2 will not be selected and element e_1 will not be covered (see Figure 1 (c)). On the other hand, if S_2 lies its cost downward to $\overline{c}_2 = 2$, its payment is still $p_2 = 5$, but now, since it covers elements e_1 and e_2, the shared payments by e_1 and e_2 become $\xi_1 = 3.5$ and $\xi_2 = 1.5$. Thus, the set S_2 becomes affordable by elements e_1 and e_2.



(a) sets-elements (b) LCS output (c) selected elements (d) output if S_2 lies

Fig. 1. An example that a set can lie its cost to improve its utility when LCS is used.

We leave it as future work to study whether there exists a truthful mechanism to select selfish sets to cover selfish elements using the combination of a truthful mechanism for sets, and a good payment-sharing method for elements.

5 Conclusion

Strategyproof mechanism design has attracted a significant amount of attentions recently in several research communities. In this paper, we focused the set cover games when the elements are selfish agents with privately known valuations of being covered. We presented several (approximately budget-balanced) truthful mechanisms that are approximately efficient. See [13] for more details about the algorithms and the analysis. Mechanism 1 is based on a cross-monotone cost-sharing scheme and thus is budget-balanced and group-strategyproof. However, in the worse case it cannot be α-efficient for any $\alpha > 0$. The second mechanism is based on Algorithm 2 and its induced cost-sharing mechanism and it produces an output that has a total valuation at least $\frac{1}{d_{\max}}$ of the optimal. However, this mechanism may charge an element 0 payment. The third mechanism, based on Algorithm 3, avoids this zero payment problem, but it is only $\frac{1}{2d_{\max}}$-efficient under some assumptions. We conducted extensive simulations to study the actual total valuations of three mechanisms. In all our simulations, we found that the first mechanism (based on cost-sharing) and the second mechanism have similar efficiencies in practice. As expected, the third mechanism always produces an output that has less total valuations than the other two methods since it only picks sets that are individually affordable.

When the service providers (*i.e.* sets) are also selfish, we show that a cross-monotonic *payment*-sharing scheme does not necessarily induce a truthful mechanism. This is a sharp contrast to the well-known fact [10] that a cross-monotonic *cost*-sharing scheme always implies a truthful mechanism.

References

1. Vickrey, W.: Counterspeculation, auctions and competitive sealed tenders. Journal of Finance **16** (1961) 8–37
2. Clarke, E.H.: Multipart pricing of public goods. Public Choice **11** (1971) 17–33
3. Groves, T.: Incentives in teams. Econometrica **41** (1973) 617–631
4. Green, J., Laffont, J.J.: Characterization of satisfactory mechanisms for the revelation of preferences for public goods. Econometrica **45** (1977) 427–438
5. Osborne, M.J., Rubinstein, A.: A course in game theory. The MIT Press (1994)
6. Li, X.Y., Sun, Z., Wang, W.: Cost sharing and strategyproof mechanisms for set cover games. In: Proceedings of the 22nd Int. Symp. on Theoretical Aspects of Compt. Sci. Volume 3404 of LNCS (2005) 218–230
7. Devanur, N., Mihail, M., Vazirani, V.: Strategyproof cost-sharing mechanisms for set cover and facility location games. In: Proceedings of the 4th ACM EC. (2003) 108–114
8. Chvátal, V.: A greedy heuristic for the set-covering problem. Mathematics of Operation Research **4** (1979) 233–235
9. Immorlica, N., Mahdian, M., Mirrokni, V.S.: Limitations of cross-monotonic cost-sharing schemes. In: Proceedings of the 16th Annual ACM-SIAM SODA. (2005)
10. Moulin, H., Shenker, S.: Strategyproof sharing of submodular costs: budget balance versus efficiency. Economic Theory **18** (2001) 511–533
11. Kao, M.Y., Li, X.Y., Wang, W.: Toward truthful mechanisms for binary demand games: A general framework. In: Proceedings of the 6th ACM EC. (2005).
12. Wang, W., Li, X.Y., Sun, Z., Wang, Y.: Design multicast protocols for non-cooperative networks. In: Proceedings of the 24th IEEE INFOCOM. (2005).
13. Sun, Z., Li, X.Y., Wang, W.,: Mechanism Design For Set Cover Games When Elements Are Agents. Full version of this paper at http://www.cs.iit.edu/~xli/paper/Conf/AAIM_SetCover.pdf.