Localized Topology Control for Heterogeneous Wireless Ad-hoc Networks

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Abstract—We study topology control in heterogeneous wireless ad hoc networks, where each mobile host has different maximum transmission power and two nodes are connected iff they are within the maximum transmission range of each other. We present several strategies for all wireless nodes self-maintaining sparse and power efficient topologies in heterogeneous network environment with low communication cost. The first structure is sparse and can be used for broadcasting. While the second structure keeps the minimum power consumption path, and the third structure is degree-bounded length and power spanner, constructed through a novel space partition strategy. Both the second and third structures are power efficient and can be used for unicasting. Here a structure is power efficient if the total power consumption of the least cost path connecting any two nodes in it is no more than a small constant factor of that in the original heterogeneous communication graph. All our methods use at most $O(n)$ total messages, where each message has $O(\log n)$ bits.

Keywords—Graph theory, wireless ad hoc networks, topology control, heterogeneous networks, power consumption, degree-bounded structure.

I. INTRODUCTION

A wireless Ad hoc network consists of an arbitrary distribution of radios in certain geographical area, unlike cellular wireless networks, there are no central control over it. Mobile devices can communicate via multi-hop wireless channels, a node can reach all nodes in its transmission range while two far-away nodes communicate through the messages relaying by intermediate nodes. It has a lot of promising applications, such as emergency search-and-rescue operations, meetings, law enforcement or military applications in which persons wish to quickly share information and data acquisition operations in inhospitable terrain.

Ad hoc wireless networks intrigue many challenging research problems, as it intrinsically has many special characteristics and some unavoidable limitations, compared with other wired or wireless network. An important requirement of these networks is that they should be self-organizing, i.e., transmission ranges and data paths are dynamically restructured with changing topology. Energy conservation and network performance are probably the most critical issues in ad hoc wireless networks, because wireless devices are usually powered by batteries only and have limited computing capability and memory. Also, unlike most traditional static communication devices, the wireless devices are often moving or adjusting its transmission range during the communication, which could change the network topology in some extent. Therefore, it is more challenging to design a network topology for ad hoc wireless networks.

Localized Ad hoc network topology control scheme is to let each wireless node locally adjust its transmission power and select which neighbors to communicate according to a certain strategy, while maintaining a structure that can support energy efficient routing and improve the overall network performance. It is regarded as an important low-cost and efficient soft technique to solve the energy conservation issues. In the past several years, topology control algorithms have drawn significant research interest. Centralized algorithms can achieve optimality or its approximation, which are more applicable to static networks due to the lack of adaptability to topology changes. In contrast, distributed algorithms are more suitable for mobile ad hoc networks since the environment is inherently dynamic and they are adaptive to topology changes at the cost of possible less optimality. Furthermore, these algorithms only attempt to selectively choose some neighbors of each node. The primary distributed topology control algorithms for ad hoc networks aims to maintain network connectivity, optimize network throughput with power-efficient routing, conserve energy and increase the fault tolerance.

Most Priori arts [1], [2], [3], [4], [5], [6], [7], [8] on network topology control assumed that the wireless ad hoc networks are modeled by unit disk graphs (UDG), i.e., two mobile hosts can communicate as long as their Euclidean distance is no more than a threshold. However, practically, even the homogeneous wireless ad hoc networks cannot be perfectly modeled as unit disk graphs: the maximum transmission ranges of wireless devices may vary due to various reasons such as the device differences and the small mechanical/electronic errors during the process of transmitting even the transmission powers of all devices are set the same initially. Few research efforts have addressed this issues.

A heterogeneous wireless ad hoc network is composed of a set $V$ of $n$ nodes $v_1,v_2,\ldots,v_n$, in which each wireless device $v_i$ has its own maximum transmission power $p_i^e$. Let $\epsilon_i$ be the mechanic/electronic error of a node $v_i$ in its power control. Then the maximum transmission power considered in this paper is actually $p_i = p_i^e - \epsilon_i$. Notice that it is often assumed in the literature that the power needed to support the communication between two nodes $v_i$ and $v_j$ is $\|v_i,v_j\|^\beta$, where $\beta \in [2,5]$ is a real number depending on the network environment ($\beta = 2$ in the free-space model) and $\|v_i,v_j\|$ is the Euclidean distance between $v_i$ and $v_j$. Consequently, the signal sent by a node $v_i$ can be received by all nodes $v_j$ with $\|v_i,v_j\| \leq r_i$, where $r_i^\beta \leq p_i$. Thus, for simplicity, we assume that each mobile host $v_i$ has its own transmission range $r_i$. The heterogeneous wireless ad

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heterogeneous networks. The relative neighborhood graph (RNG), defined as the intersection of two circles centered at a given point set or the unit disk graph, in which all nodes have the same transmission range. However, graphs representing communication links are rarely so completely specified as the unit disk graph. Different wireless devices may have different transmission radii due to various reasons. Consequently, two nodes can communicate directly only if they are within the transmission range of each other. A mutual inclusion graph, denoted by MG, used in wireless ad hoc networks, has an edge $uv$ if and only if $\|uv\| \leq \min(r_u, r_v)$. Here $r_u$ denotes the transmission range of a node $u$. Notice that UDG is a special case of MG. In practice, it would be more interesting to find a power efficient subgraph of MG in a localized manner. Here an algorithm $A$ is said to construct a structure $H$ from a graph $G$ locally if, every node $u$ can decide which edges $uv \in H$ using only the information of nodes within a constant number of hops of $u$.

### II. Preliminaries

#### A. Priori Arts

Many structures (such as relative neighborhood graph RNG, Gabriel graph GG, Yao structure, etc) have been proposed for topology control in homogeneous wireless ad hoc networks. Due to limited spaces, we will briefly introduce and review several basic proximity geometric structures. The relative neighborhood graph, denoted by $RNG(V)$ [9], consists of all edges $uv$ such that the intersection of two circles centered at $u$ and $v$ and with radius $\|uv\|$ do not contain any vertex $w$ from the set $V$. See Figure 1(a). The Gabriel graph [10] $GG(V)$ contains edge $uv$ if and only if $\text{disk}(u,v)$ contains no other points of $S$, where $\text{disk}(u,v)$ is the disk with edge $uv$ as a diameter. See Figure 1(b). Both $GG(V)$ and $RNG(V)$ are connected, planar, and contain the Euclidean minimum spanning tree of $V$. The intersections of $GG(V)$, $RNG(V)$ with the connected unit disk graph $UDG(V)$ are connected. Delaunay triangulation, denoted by $Del(V)$, is also used as underlying structure by several routing protocols. Here an triangle $\triangle uvw$ belongs to Delaunay triangulation $Del(V)$ if its circumcircle does not contain any node inside. It is well known that $RNG(V) \subseteq GG(V) \subseteq Del(V)$. The intersection of $Del(V)$ with the connected $UDG(V)$ has bounded length spanning ratio [11].

The Yao graph [12] with an integer parameter $k \geq 6$, denoted by $YG_k(V)$, is defined as follows. At each node $u$, any $k$ equal-separated rays originated at $u$ define $k$ cones. In each cone, choose the shortest edge $uv$ among all edges from $u$, if there is any, and add a directed link $uv$. Ties are broken arbitrarily or by ID. See Figure 1(c). The resulting directed graph is called the Yao graph. Let $YG_k(V)$ be the undirected graph by ignoring the direction of each link in $YG_k(V)$. Some researchers used a similar construction named $\theta$-graph [13], [14], the difference is that it chooses the edge which has the shortest projection on the axis of each cone instead of the shortest edge in each cone.

Fig. 1. The definitions of RNG, GG, YG. The shaded area is empty of nodes inside.

All previous known structures are defined solely on the given point set or the unit disk graph, in which all nodes have the same transmission range. However, graphs representing communication links are rarely so completely specified as the unit disk graph. Different wireless devices may have different transmission radii due to various reasons. Consequently, two nodes can communicate directly only if they are within the transmission range of each other. A mutual inclusion graph, denoted by MG, used in wireless ad hoc networks, has an edge $uv$ if and only if $\|uv\| \leq \min(r_u, r_v)$. Here $r_u$ denotes the transmission range of a node $u$. Notice that UDG is a special case of MG. In practice, it would be more interesting to find a power efficient subgraph of MG in a localized manner. Here an algorithm $A$ is said to construct a structure $H$ from a graph $G$ locally if, every node $u$ can decide which edges $uv \in H$ using only the information of nodes within a constant number of hops of $u$.

#### B. Spanners and Stretch Factors

When constructing a subgraph of the original communication graph MG, we may need consume more power to connect some nodes since we may disconnect the most power efficient path in MG. Thus, naturally, we would require that the constructed structure approximates MG well in terms of the power consumption for unicast routing. In graph theoretical term, the structure should be a spanner. Spanners have been studied intensively in recent years [15], [16], [17], [13]. Let $G = (V, E)$ be a $n$-vertex weighted connected graph. The distance in $G$ between two vertices $u, v \in V$ is the length of the shortest path between $u$ and $v$ and it is denoted by $d_G(u,v)$. A subgraph $H = (V, E')$, where $E' \subseteq E$, is a $t$-spanner of $G$ if for every $u, v \in V$, $d_H(u,v) \leq t \cdot d_G(u,v)$. The value of $t$ is called the stretch...
factor or spanning ratio. When the graph is a geometric graph and the weight is the Euclidean distance between two vertices, the stretch factor $t$ is called the length stretch factor, denoted by $t_H(G)$. For wireless networks, the mobile devices are usually powered by battery only. We thus pay more attention to the power consumptions. The power, denoted by $p_G(u,v)$, needed to support the communication between a link $uv$ in $G$ is often assumed to be $\|uv\|^\beta$, where $2 \leq \beta \leq 5$ is a constant depending on the transmission environment. When the weight of a link $uv$ in $G$ is defined as the power to support the communication of link $uv$, the stretch factor of $H$ is called the power stretch factor, denoted by $p_H(G)$ hereafter.

Obviously, for any weighted graph $G$ and a subgraph $H \subseteq G$, we have

Lemma 1: [4] Graph $H$ has stretch factor $\delta$ if and only if for any link $uv \in G$, $d_H(u,v) \leq \delta \cdot d_G(u,v)$.

Lemma 1 implies that, to generate a spanner, we only have to make sure that every link of $G$ is approximated within a constant factor in $H$.

C. Sparseness and Bounded Degree

Assume that, in the heterogeneous network, $\frac{r_w}{r_i} \leq C$ for every node $u$ and any neighbor $v$ of $u$ in MG, where $C$ is a constant factor. In other words, the transmission range of any neighbor $v$ of a node $u$ is within a small constant factor of $r_u$.

All well-known proximity graphs ($GG(V)$, $RNG(V)$, $Del(V)$ and $YG(V)$) have been proved to be sparse graphs when network is modeled as a UDG. Recall that a sparse graph means the number of edges is linear with the number of nodes. The sparseness of all well-known proximity graphs implies that the average node degree is bounded by a constant. Moreover, we prefer the maximum node degree is bounded by a constant, because wireless nodes have limited resources and the signal interference in wireless communications. Unbounded degree (or in-degree) at node $u$ will often cause large overhead at $u$, whereas bounded degree increases the network throughput. On the other hand, bounded degree will also give us advantages when apply several routing algorithms. Therefore, it is often imperative to construct a sparse network topology with bounded node degree while it is still power-efficient. However, in all known primitive proximity graphs, Li et al. [4] showed that the maximum node degree could be as large as $n-1$. The instance consists of $n-1$ points lying on the unit circle centered at a node $u \in V$. Then each edge $uv_i$ belongs to the $RNG(V)$, $GG(V)$ and $YG_k(V)$. Thus, node $u$ has degree $n-1$ (in-degree for $YG_k(V)$) in $RNG(V)$, $GG(V)$ and $YG_k(V)$, although $YG_k(V)$ has a bounded out-degree $k$.

Recently, in homogeneous wireless ad hoc networks, some improved or combined proximity graphs [18], [19] have been proposed to build planar degree-bounded power spanner topology, which meets all preferred properties in literature for unicast routing. In heterogeneous networks, only a few research efforts [20] are published so far, in which few attention to those preferred properties has been drawn. In the following, we will first discuss the difficulties and limitations for topology control in heterogeneous networks, then present our localized strategies in detail.

III. LIMITATIONS

In heterogeneous wireless ad hoc networks, the planar topology does not necessarily exist. Figure 2 shows an example, there are four nodes $x$, $y$, $u$ and $v$ in the network, where their transmission range $r_x = r_y = \|xy\|$ and $r_u = r_v = \|uv\|$, and node $u$ are out of the transmission range of node $x$ and $y$, while node $v$ is in the transmission range of node $y$ and out of the range of $x$. The transmission ranges of $x$ and $y$ are illustrated by the dotted circles. According to the definition of MG, there are only three edges $xy$, $vy$ and $uw$ in the graph. Hence any topology control method can not make the topology planar while keeping the communication graph connected. On the other hand, it is worth to think whether we can design a new routing protocol on some pseudo-planar topologies. As will be seen later, the pseudo-planar topology GG(MG) and RNG(MG) proposed in this section has some special properties which is different than other general non-planar topologies. For instance, two intersecting triangles can not share a common edge. We leave it as a future work.

![Fig. 2. Planar topology does not exist.](image)

We also can show that the node degree in heterogeneous networks can not be bounded by a constant if the radius ratio is unbounded. Figure 3 shows such an example. In the example, a node $v$ has $p+1$ incoming neighbors $w_i$, $0 \leq i \leq p$. Assume that each node $w_i$ has a transmission radius $r_{w_i} = r_v/3^{p-i}$ and $\|w_i\| = r_{w_i}$. Obviously, $\|uw_i\| > \min(r_{w_i}, r_{w_j})$, i.e., any two nodes $w_i,w_j$ are not directly connected in MG.

![Fig. 3. Node $u$ has transmission range 1 and node $w_i$ has transmission range $r_{w_i} = \|uw_i\| = r_v/3^{p-i}$ where $0 \leq i \leq p$.](image)

Obviously, none of those edges incident on $v$ can be deleted, hence there are no topology control methods to bound the degree by a constant without violating connectivity. Consider the example illustrated by Figure 3, edges $vw_i$, $0 \leq i \leq p$, are all possible communication links. Thus, node $v$ in any connected spanning graph has degree $p+1$. 

On the other hand, we will show in section VI, in the worst case, any connected MG graph has degree $O(\log_\gamma \gamma)$ where $\gamma = \max_{v \in V} \max_{u \in \Gamma(v)} r_u$. In the example, recall that $3^p r_v = r_v$, hence $\gamma$ equals to $3^p$. Thus, $v$ has degree $\log_\gamma \gamma + 1 = O(\log_\gamma \gamma)$. In the paper, we always assume $\gamma$ is a constant, as mentioned in Section II. It is practical, since it is trivial that two wireless devices in same network have unbounded radius ratio.

IV. Heterogeneous Sparse Structure

In this section, we propose a strategy for all nodes self-forming a sparse structure, called RNG(MG), based on the relative neighborhood graph structure, whose total number of links is $O(n)$. We add a link $uv \in MG$ to RNG(MG) if there is no another node $w$ inside $lune(u,v)$ and both links $uv$ and $vw$ are in MG. Here $lune(u,v)$ is the intersection of $disk(u,||uv||)$ and $disk(v,||uv||)$. The algorithm will be similar to Algorithm 2, thus we omit it here. Notice that the total communication cost of constructing RNG(MG) is $O(n \log n)$ bits, assuming that the radius and ID information of a node can be represented in $O(\log n)$ bits. In addition, the structure RNG(MG) is symmetric: if a node $u$ keeps a link $uv$, node $v$ will also keep link $uv$. Thus, a node $u$ does not have to tell its neighbor $v$ whether it keeps a link $uv$ or not.

It is not difficult to prove that structure RNG(MG) is connected by induction. On the other hand, same as the case in homogeneous networks (i.e., UDG mode), RNG(MG) does not have a bounded length stretch factor, nor constant bounded power stretch factor, and does not have bounded node degree. In this paper, we will show that RNG(MG) is a sparse graph: it has at most 6n links at most.

In the following, we define a new structure, called ERNG(MG). Assume that each node $v$ knows its maximum transmission radius $r_v$. Let $N(u) = \{v \mid r_v \geq r_u\}$. A node $u$ processes a link $uv$ from MG if $r_u \geq r_v$, i.e., $v \in N(u)$. Node $u$ removes a link $uv$, where $v \in N(u)$, if there is another node $w \in N(u)$ inside $lune(u,v)$ with both links $uw$ and $vw$ are in MG. All the links $uv$ kept by all nodes form the structure ERNG(MG).

Algorithm 1: Constructing-ERNG

1. In the beginning, each node $u$ locally broadcasts a HELLO message with $ID_u$, $r_u$ and its position $(x_u,y_u)$ to all nodes in its transmission range. Note that $r_u = p_u^{1/3}$ is its maximum transmission range. Each node $u$ initiates sets $E_{MG}(u)$ and $E_{ERNG}(u)$ to be empty. Here $E_{MG}(u)$ is the set of links of MG known to $u$ so far and $E_{ERNG}(u)$ is the set of links of ERNG known to $u$ so far.

2. At the same time, each node $u$ processes the incoming messages. Assume that node $u$ gets a message from some node $v$. If $||uv|| \leq \min\{r_u, r_v\}$, then node $u$ adds a link $uv$ to $E_{MG}(u)$. If $r_v \geq r_u$, then node $u$ removes a link $uv$. If $r_\nu \geq r_u$, then node $u$ performs the following procedures. Node $u$ checks if there is another link $uv \in E_{MG}(u)$ with the following additional properties: 1) $w \in lune(u,v)$, 2) $r_w \geq r_u$, and 3) $||uv|| \leq \min\{r_w, r_v\}$. If such a link $uv$, then add $uv$ to $E_{ERNG}(u)$.

For any link $uv \in E_{ERNG}(u)$, node $u$ checks if the following conditions hold: 1) $v \in lune(u,v)$, and 2) $||uv|| \leq \min\{r_w, r_v\}$. If the conditions hold, then remove link $uv$ from $E_{ERNG}(u)$.

3. Node $u$ repeats the above steps until no new HELLO messages received.

4. For each link $uv \in E_{ERNG}(u)$, node $u$ informs node $v$ to add link $uv$.

5. All links $uv \in E_{ERNG}(u)$ are the final links in ERNG(MG) incident on $u$.

We then prove that the structure ERNG has at most 6n links.

Lemma 2: Structure ERNG(MG) has at most 6n links.

Proof: Consider any node $u$. We will show that $u$ keeps at most 6 directed links emanated from $u$. Assume that $u$ keeps more than 6 directed links. Obviously, there are two links $uw$ and $u\nu$ such that $\angle u\nu w < \pi/3$. Thus, $uv$ is not the longest link in triangle $\triangle uvw$. Without loss of generality, we assume that $||uw||$ is the longest in triangle $\triangle uvw$. Notice that the existence of link $uw$ implies that $||uw|| \leq \min\{r_u, r_w\}$. Consequently, $||uv|| \leq ||uw|| \leq \min\{r_u, r_w\}$. From the fact that the link $r_u \geq r_v$, we know $||uv|| \leq \min\{r_u, r_w\}$. Hence, link $uv$ does exist in the original communication graph MG, it implies that link $uw$ cannot be selected to ERNG. This finishes our proof.

Similar to Lemma 2, we can prove the following lemma.

Lemma 3: Structure RNG(MG) has at most 6n links.

Proof: Imagine that each link $uv$ has a direction as follows: $\overrightarrow{uv}$ if $r_u \leq r_v$. Then similar to Lemma 2, we can prove that each node $u$ only keeps at most 6 such imagined directed links.

Similarly, we can define a structure $EGG(MG)$, which contains an edge $uv$ if $r_u \leq r_v$ and there is no node $w$ with the following properties: 1) $r_u \leq r_w$, 2) $w$ is inside the disk $disk(u,v)$.

V. Heterogeneous Power Spanner

In the section, we give a strategy for all nodes self-forming a power spanner structure, called GG(MG), based on the Gabriel graph structure. We add a link $uv \in MG$ to GG(MG) if there is no another node $w$ inside $disk(u,v)$ and both links $uw$ and $vw$ are in MG. Our localized construction method works as follows.

Algorithm 2: Constructing-GG

1. In the beginning, each node $u$ locally broadcasts a message with $ID_u$, $r_u$ and its position $(x_u,y_u)$ to all nodes in its transmission range. Note that $r_u = p_u^{1/3}$ is its maximum transmission range. Let $E_{MG}(u)$ and $E_{GG}(u)$ be the set of links known to $u$ from MG and GG respectively. Each node $u$ initiates both $E_{MG}(u)$ and $E_{GG}(u)$ as empty.

2. At the same time, each node $u$ processes the incoming messages. Assume that node $u$ gets a message from some node $v$. If $||uv|| \leq \min\{r_u, r_v\}$, then node $u$ adds a link $uv$ to $E_{MG}(u)$.

Node $u$ checks if there is another link $uv \in E_{MG}(u)$ with the following two additional properties: 1) $w \in disk(u,v)$,
and 2) $||wv|| \leq \min\{r_w, r_v\}$. If no such link $uw$, then add $uv$ to $E_{GG}(u)$. For any link $uw \in E_{GG}(u)$, node $u$ checks if the following two properties hold: 1) $v \in disk(u, w)$, and 2) $||uv|| < \min\{r_u, r_v\}$. If the conditions hold, then remove link $uw$ from $E_{GG}(u)$.

3. Node $u$ repeats the above steps until no new messages received.

4. All links $uv$ in $E_{GG}(u)$ are the final links in $GG(MG)$ incident on $u$.

We first show that Algorithm 2 builds the structure $GG(MG)$ correctly. For any link $uv \in G(MG)$, clearly, we cannot remove them in Algorithm 2. For a link $uw \not\in G(MG)$, assume that a node $w$ is inside $disk(u, v)$ and both links $uw$ and $vw$ belong to MG. If node $u$ gets the message from $w$ first, and then gets message from $v$, clearly, $uv$ cannot be added to $E_{GG}(u)$. If node $u$ gets the message from $v$ first, then node $u$ will remove link $uw$ from $E_{GG}(u)$ (if it is there) when $u$ gets the information of node $w$.

It is not difficult to prove that structure $GG(MG)$ is connected by induction. In addition, since we remove a link $uv$ only if there are two links $uw$ and $vw$ with $w$ inside $disk(u, v)$, it is easy to show that the power stretch factor of $GG(MG)$ is 1. In other words, the minimum power consumption path for any two nodes $v_1$ and $v_2$ in MG is still kept in $GG(MG)$. Remember that here we assume the power needed to support a link $uw$ is $||uv||^\beta$, for $\beta \in [2, 5]$.

On the other hand, same as the case in homogeneous networks (i.e., UDG mode), $GG(MG)$ is not a length spanner, and does not have bounded node degree. Furthermore, it is unknown whether $GG(MG)$ is a sparse graph. Recently, it was proven in [21] that $GG(MG)$ has at most $O(n^{8/3} \log \gamma)$ edges where $\gamma = \max r_u/r_v$.

Notice that the structures defined as follows cannot guarantee the connectivity. In the first structure, (called $LGG_k(MG)$ in [21]) we do not add a link $uv \in MG$ to $GG(MG)$ if there is another node $w$ inside $disk(u, v)$. In the second structure, (called $LGPG_k(MG)$ in [21]) we do not add a link $uv \in MG$ to $GG(MG)$ if there is another node $w$ inside $disk(u, v)$, and either link $uw$ or link $vw$ is in MG.

VI. Heterogeneous Degree-Bounded Spanner

Undoubtedly, as described in preliminaries, we always prefer a structure has more nice properties, such as degree-bounded (stronger than sparse), power spanner etc. Naturally, we could extend the previous known degree-bounded spanner, such as the Yao related structures, from homogeneous networks to heterogeneous networks. Unfortunately, simple extension of the Yao structure from UDG to MG even does not even guarantee the connectivity. Figure 4 illustrates such an example. Here $r_u = r_v = ||uw||$, $r_w = ||uw||$, $r_x = ||vx||$, and $||uv|| < ||uw||$, $||uv|| < ||vw||$, $||vx|| < ||uw||$, and $||vx|| < ||vx||$. In addition, $v$ and $w$ are in the same cone of node $u$, and nodes $x$ and $u$ are in the same cone of node $v$. Thus, the original MG graph contains links $uw$, $uw$ and $vx$ only and is connected. However, when applying Yao structure on all nodes, node $u$ will only have information of node $v$ and $w$ and it will keep link $uw$. Similarly, node $w$ keeps link $uw$; node $v$ keeps link $vx$; and node $x$ keeps link $vx$. In other words, only link $vx$ and $uv$ are kept by Yao method. Thus applying Yao structure disconnects node $v$, $x$ from the other two nodes $u$ and $w$. Consequently, we need more sophisticated extensions of the Yao structure to MG to guarantee the connectivity of the structure.

A. Extended Yao Graph

![Fig. 4. Simple extension of Yao structure does not guarantee the connectivity.](image)

**Algorithm 3: Constructing-EYG**

1. In the beginning, each node $u$ divides the disk $disk(u, r_u)$ centered at $u$ with radius $r_u$ by $k$ equal-sized cones centered at $u$. We generally assume that the cone is half open and half-close. Let $C_{i}(u), 1 \leq i \leq k$, be the $k$ cones partitioned. Let $C_{i}(u), 1 \leq i \leq k$, be the set of nodes $v$ inside the ith cone $C_{i}(u)$ with a larger or equal1 radius than $u$. In other words,

$$C_{i}(u) = \{v \mid v \in C_{i}(u), \ and \ r_v \geq r_u\}.$$  

Initially, $C_{i}(u)$ is empty.

2. Each node $u$ broadcasts a message with $ID_u, r_u$ and its position $(x_u, y_u)$ to all nodes in its transmission range.

3. At the same time, each node processes the incoming broadcast messages. Assume it gets a message from some node $v$. If $v$ is inside the ith cone $C_{i}(u)$ of node $u$ and $r_v \geq r_u$, then set $C_{i}(u) = C_{i}(u) \cup \{v\}$. If $v \not\in disk(u, r_u)$, $v$ is not considered here.

4. Node $u$ chooses a node $v$ from each cone $C_{i}(u)$ so that the link $uv$ has the smallest $ID(uv)$ among all links $uv$, with $v_j$ in $C_{i}(u)$, if there is any.

5. Finally, each node $u$ informs all 1-hop neighbors of its chosen links through a broadcast message.

Let $\tilde{EYG}_k(MG)$ be the union of all chosen links. In other words, the above method computes the Extended Yao graph $\tilde{EYG}_k(MG)$ for MG. Since the symmetric communications are required, let $EYG_k(MG)$ be the undirected graph by ignoring the direction of each link in $\tilde{EYG}_k(MG)$. Graph $EYG_k(MG)$ is the final network topology. Since node $u$ chooses a node $v$ inside $disk(u, r_u)$ with $r_v \geq r_u$, link $uv$ is indeed a bidirectional link, i.e., $u$ and $v$ are within the

1This is the main difference between this algorithm and the simple extension of Yao structure discussed before, in which it considers all nodes $v$ that $u$ can get signal from.
transmission range of each other. Additionally, this strategy could avoid the possible disconnection by simple Yao extension we mentioned before.

Obviously, each node only broadcasts twice: one for broadcasting its ID, radius and position; and the other one for broadcasting the selected neighbors. Remember that it selects at most $k$ neighbors. Thus, each node sends messages at most $O((k+1) \cdot \log n)$ bits. Here, we assume that the node ID and its position can be represented using $O(\log n)$ bits for a $n$-node wireless network.

Before we study the properties of this structure, we have to define some terms first. Assume that each node $v_i$ of MG has a unique identification number $ID_{v_i} = i$. The identity of a bidirectional link $uv$ is defined as $ID(uv) = (\|uv\|, ID_u, ID_v)$. Note that we use the bidirectional links instead of the directional links in the final topology to guarantee connectivity. In other words, we require that both node $u$ and node $v$ can communicate with each other through this link. In this paper, all proofs about connectivity or stretch factors take the notation $wuv$ and $vuw$ as same, which is meaningful. Only in the topology construction algorithm or proofs about bounded-degree, $w$ is different than $v$: the former is initiated and built by $u$, whereas the latter is by node $v$. Sometimes we denote a directional link from $v$ to $u$ as $\overline{vu}$ if necessary. Then we can order all bidirectional links (at most $n(n-1)$ such links) in an increasing order of their identities. Here the identities of two links are ordered based on the following rule: $ID(uv) > ID(pq)$ if (1) $\|uw\| > \|pq\|$ or (2) $\|uw\| = \|pq\|$ and $ID_u > ID_p$ or (3) $\|uw\| = \|pq\|, u = p$ and $ID_v > ID_q$.

Correspondingly, the rank of each link $uv$, denoted by $rank(uv)$, is its order in the sorted bidirectional links. Notice that, we actually only have to consider the links in MG. We then show that the constructed network topology is a connected length and power spanner.

Theorem 4: The length stretch factor of the Yao graph $EYG_k(MG)$, $k > 6$, is at most $\ell = \frac{1}{1-2\sin\left(\frac{\pi}{k}\right)}$.

Proof: From Lemma 1, it is sufficient to show that for any nodes $u$ and $v$ with $\|uv\| \leq \min(r_u, r_v)$, i.e. $w \in MG$, there is a path connecting $u$ and $v$ in $EYG_k(MG)$ with length at most $\ell\|uv\|$. We construct a path $u \rightsquigarrow v$ connecting $u$ and $v$ in $EYG_k(MG)$ as follows.

Assume that $r_u \leq r_v$. If link $uv \in EYG_k(MG)$, then set the path $u \rightsquigarrow v$ as the link $uv$. Otherwise, consider the disk $(u, r_u)$ of node $u$. Clearly, node $u$ will get information of $v$ from $v$ and node $v$ will be selected to some $C_i(u)$ since $r_v \geq r_u$. Thus, from $w \notin EYG_k(MG)$, there must exist another node $w$ in the same cone as $v$, which is a neighbor of $u$ in $EYG_k(MG)$. Then set $u \rightsquigarrow v$ as the concatenation of the link $uw$ and the path $w \rightsquigarrow v$. Here the existence of path $w \rightsquigarrow v$ can be easily proved by induction on the distance of two nodes. Notice that the angle $\theta$ of each cone section is $\frac{\pi}{k}$. When $k > 6$, then $\theta < \frac{\pi}{5}$. It is easy to show that $\|wv\| < \|uv\|$. Consequently, the path $u \rightsquigarrow v$ is a simple path, i.e., each node appears at most once.

We then prove by induction that the path $u \rightsquigarrow v$ has total length at most $\ell\|uv\|$. Obviously, if there is only one edge in $u \rightsquigarrow v$, $d(u \rightsquigarrow v) = \|uv\| < \ell\|uv\|$. Assume that the claim is true for any path with $l$ edges. Then consider a path $u \rightsquigarrow v$ with $l + 1$ edges, which is the concatenation of edge $uv$ and the path $w \rightsquigarrow v$ with $l$ edges, as shown in Figure 5 where $\|wv\| = \|xv\|$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{The length stretch factor of $EYG_k(MG)$ is at most $\frac{1}{1-2\sin\left(\frac{\pi}{k}\right)}$.}
\end{figure}

Notice, from induction, $d(w \rightsquigarrow v) \leq \ell\|uv\|$. Then, let $\varphi = \angle uvw$ and $\alpha = \angle uvw$, we have

$$
\frac{\|uv\|}{\|vx\|} = \frac{\sin(\angle uvw)}{\sin(\angle vxw)} = \frac{\sin(\frac{\varphi}{2} + \frac{\pi}{2})}{\sin(\frac{\varphi}{2} + \frac{\pi}{2} + \alpha)} = \frac{\cos(\frac{\varphi}{2})}{\cos(\frac{\varphi}{2} + \alpha)} \leq 1
$$

Consequently,

$$
\frac{\|wv\|}{\|vx\|} = \frac{\cos(\frac{\varphi}{2} + \alpha)}{\cos(\frac{\varphi}{2} + \alpha + \beta)} \leq \frac{\cos(\frac{\varphi}{2} - \frac{\pi}{2})}{\cos(\frac{\varphi}{2} + \frac{\pi}{2})} = \frac{1}{1 - 2\sin\left(\frac{\pi}{k}\right)}
$$

The first inequality is because $0 \leq \alpha \leq \pi - \frac{\pi}{k}$ and the second inequality is because $0 \leq \varphi \leq \frac{\pi}{2}$. Consequently, $d(u \rightsquigarrow v) = \|uv\| + d(w \rightsquigarrow v) < \ell\|uv\| + \ell\|wv\| = \ell\|uv\|$, where $\ell = \frac{1}{1-2\sin\left(\frac{\pi}{k}\right)}$. That is to say, the claim is also true for the path $w \rightsquigarrow v$ with $l + 1$ edges.

Thus, the length stretch factor of $EYG_k(MG)$ is at most $\ell = \frac{1}{1-2\sin\left(\frac{\pi}{k}\right)}$. This finishes the proof.

Theorem 5: The power stretch factor of the extended Yao graph $EYG_k(MG)$, $k > 6$, is at most $\rho = \frac{1}{1-2\sin\left(\frac{\pi}{k}\right)}$.

Proof: The proof is similar to that in UDG [4], [5] except the induction procedure. For the completeness of presentation, we shall give the detail here.

From Lemma 1, it is sufficient to show that for any nodes $u$ and $v$ with $\|uv\| \leq \min(r_u, r_v)$, i.e. $w \in MG$, there is a path connecting $u$ and $v$ in $EYG_k(MG)$ with power consumption at most $\rho$. We construct a path $u \rightsquigarrow v$ connecting $u$ and $v$ in $EYG_k(MG)$ as we did in Theorem 4. We then prove by induction, on the number of its edges, that the path $u \rightsquigarrow v$ has power cost, denoted by $p(u \rightsquigarrow v)$, at most $\rho\|uv\|^2$.

In the procedure of induction, if $r_u \leq r_v$ then we induct on path $w \rightsquigarrow v$, otherwise we induct on path $v \rightsquigarrow w$. In fact, here $w \rightsquigarrow v$ is same as $v \rightsquigarrow w$ since the path is bidirectional for communication. Directional link is only considered in building process and is meaningless when we talk about the path. This induction rule is applied throughout the remainder of the paper.
Obviously, if there is only one edge in $u \rightsquigarrow v$, $p(u \rightsquigarrow v) = ||uv||^3 < \rho ||uv||^2$. Assume that the claim is true for any path with $l$ edges. Then consider a path $u \rightsquigarrow v$ with $l + 1$ edges, which is the concatenation of edge $uw$ and the path $w \rightsquigarrow v$ with $l$ edges. We consider two cases.

Case 1: the angle $\angle uvw$ is not acute. See Figure 6 (a). We have $||uv||^2 + ||vw||^2 \leq ||uw||^2$. Notice that $\frac{||uw||}{||uv||} \leq 1$ and $\frac{||uw||}{||vw||} \leq 1$. It implies that

$$
\left( \frac{||uw||}{||uv||} \right)^3 + \left( \frac{||uw||}{||vw||} \right)^3 \leq \left( \frac{||uw||}{||uw||} \right)^2 + \left( \frac{||uw||}{||uw||} \right)^2 \leq 1
$$

Therefore,

$$
||uw||^3 + ||vw||^3 \leq ||uw||^2
$$

for any $\beta \geq 2$. Since $||uw|| < ||uv||$, we can apply induction on the path $w \rightsquigarrow v$ also. Therefore, $p(w \rightsquigarrow v) \leq \rho ||uw||^2$ by induction. Then $p(u \rightsquigarrow v) = ||uw||^3 + p(w \rightsquigarrow v) \leq ||uw||^3 + \rho ||uw||^2 \leq \rho ||uw||^2$.

Case 2: the angle $\angle uvw$ is acute. See Figure 6 (b). We bound the length $||uv||$ respecting to $||uw||$. Notice that $||uv|| \leq ||uw||$ and $\angle uvw < \theta$. The maximum length of $uv$ is achieved when $||uv|| = ||uw||$ because the angle $\angle uvw$ is acute. Therefore $||uv|| \leq 2 \sin \frac{\theta}{2} ||uw|| = 2 \sin \frac{\theta}{2} ||uw||$. By induction, we have

$$
p(u \rightsquigarrow v) = ||uw||^3 + p(w \rightsquigarrow v) \leq ||uw||^3 + \rho ||uw||^2 \\
\leq ||uw||^3 + \rho \cdot \left( 2 \sin \frac{\pi}{k} \right)^3 ||uw||^2 = \rho ||uw||^2
$$

This finishes the proof.

B. Novel Space Partition

Partitioning the space surrounding a node into $k$ equal-sized cones enables us to bound the node out-degree using the Yao structure. Using the same space partition, Yao-Yao structure [4], [5] produces a topology with bounded in-degree when the networks are modeled by UDG. Yao-Yao structure (for UDG) is generated as follows: a node $u$ collects all its incoming neighbors $v$ (i.e., $v \in Y\hat{G}_k(V)$), and then selects the closest neighbor $v$ in each cone $C_j(u)$. Clearly, Yao-Yao has bounded degree at most $k$. They also showed that another structure YaoSink [4], [5] has not only the bounded node degree but also constant bounded stretch factors. The network topology with bounded degree can increase the communication efficiency. However, these methods [4], [5] may fail when the networks are modeled by MG: they cannot even guarantee the connectivity, which is verified by following discussions.

Assume that we already construct a connected directed structure $EY\hat{G}_k(MG)$. Let $I(v) = \{ w \mid uv \in EY\hat{G}_k(MG) \}$. In other words, $I(v)$ is the set of nodes that have directed links to $v$ in $EY\hat{G}_k(MG)$. Let $I_x(v) = I(v) \cap C_i(u)$, i.e., the nodes in $I(v)$ located inside the $i$th cone $C_i(v)$. Yao-Yao structures will pick the closest node $w$ in $I_x(v)$ and add undirected link $uw$ to Yao-Yao structure. Previous example in Figure 3 illustrates the situation that Yao-Yao structure is not connected. In the example, a node $v$ has $p + 1$ incoming neighbors $w_i$, $0 \leq i \leq p$. Assume that each node $w_i$ has a transmission radius $r_{w_i} = r_v/3^{p-i}$ and $||uw_i|| = r_{w_i}$. Obviously, $||w_iw_j|| \geq \min(r_{w_i}, r_{w_j})$, i.e., any two nodes $w_i, w_j$ are not directly connected in MG. It is easy to show that the Yao structure $EY\hat{G}_k(MG)$ only has directed links $w_iw'_j$. Obviously, node $v$ will only select the closest neighbor $w_1$ to the Yao-Yao structure, which disconnects the network. This example can also show that the structure based on Yao-Sink [4], [5] is also not connected for heterogeneous wireless ad hoc networks.

Thus, selecting the closest incoming neighbor in each cone $C_i$ is too aggressive to guarantee the connectivity. Observe that, in Figure 3, to guarantee the connectivity, when we delete a directed link $w_iw'_j$, we need to keep some link, say $w_jv$, such that $w_jw_i$ is a link in MG. Thus, we want to further partition the cone into a limited number of smaller regions and we will keep only one node in each region, e.g., the closest node. Clearly, to guarantee that other nodes in the same region are still connected to $v$, we have to make sure that any two nodes $w_i, w_j \in I(v)$ that co-exist in a same small region are directly connected in MG. Consequently, if the number of regions is bounded by a constant, a degree-bounded structure could be generated. In the remainder of this subsection, we will introduce a novel space partition strategy satisfying the above requirement.

Method 1: Partition-EYG

For each node $v$, let $\gamma_v = \max_{w \in I(v)} \frac{1}{r_w}$. Remember that all nodes in $I(v)$ have transmission radius at most $r_v$. Let $h$ be the positive integer satisfying $2^{h-2} < \gamma_v \leq 2^{h-1}$. We then discuss in detail our partition strategy of the cones, which is illustrated by Figure 7. Each node $v$ divides each cone centered at $v$ into limited number of triangles and caps, where $||va_i|| = ||vb_i|| = \frac{1}{\gamma_v} r_v$ and $c_i$ is the midpoint of the segment $a_i b_i$, for $1 \leq i \leq h$. Notice that
this partition can be conducted by node $v$ locally since it can
collect the transmission radius information of nodes in $I(v)$. The triangles $\triangle va_1b_1$, $\triangle a_i b_i c_{i+1}$, $\triangle a_{i+1} c_{i+1} c_{i+1}$, $\triangle b_{i-1} c_{i+1}$, $\triangle b_{i-1} c_i c_{i+1}$, for $1 \leq i \leq h - 1$, and the cap $a_h b_h$ form the
final space partition of each cone. For simplicity, we call such a
triangle or the cap as a region. We then prove that this
partition indeed guarantees that any two nodes in any
same region are connected in MG.

Thus,

$$\min(a_{i+1}c_{i+1}),$$

and

$$\max(a_{i+1}c_{i+1}).$$

This finishes the proof.

C. Extended Yao-Yao Graph

In this section, using the space partition discussed in
Section VI-B, we present our method to locally build a
sparse network topology with bounded degree for hetero-
genous wireless ad hoc network. Here we assume that
$\gamma = \max_{v \in E} \gamma_v$ is bounded, where $\gamma_v = \max_{w \in I(v)} \frac{r_w}{r_v}$, and
$I(v) = \{w \mid \overrightarrow{vw} \in EY_y^{(M)}(MG)\}$. But the
union of all chosen links is the final network topol-
ogy, denoted by $\overrightarrow{EY}_G^{(M)}(MG)$. We call itextended Yao-Yao graph. Let $\overrightarrow{EY}_G^{(M)}$ be the undirected graph by ignor-
ing the direction of each link in $\overrightarrow{EY}_G^{(M)}$.

Theorem 7: The out-degree of each node $v$ in $\overrightarrow{EY}_G^{(M)}$, $k \geq 6$, is bounded by $k$ and the in-degree is bounded by
$(3\log_2 \gamma_v + 2)k$, where $\gamma_v = \max_{w \in I(v)} \frac{r_w}{r_v}$.

Proof: It is obvious that the out-degree of node $v$ is bounded by $k$ because the out-degree bound of $\overrightarrow{EY}_G^{(M)}$ is $k$ and this algorithm does not add any di-
rected link.

For the in-degree bound, as shown in Figure 7, obvi-
ously, the number of triangle regions in each cone is $3h - 2$.
Remember that $2^{h-2} \leq \gamma_v \leq 2^{h-1}$, which im-
plies $h = 1 + \log_2 \gamma_v \leq 2^{h-1}$, which implies $h = 1 + \log_2 \gamma_v$. Thus, considering the cap region also, the in-degree of node $v$ is at most $(3\log_2 \gamma_v + 2)k$.

This finishes the proof.
Here $\gamma = \max_{v} \gamma_v$. Obviously, the maximum node degree in graph $EYY_k(MG)$ is bounded by $(3\log_2 \gamma + 3)\kappa$.

Notice that the extended Yao-Yao graph $EYY_k(MG)$ is a subgraph of the extended Yao graph $EYG_k(MG)$, thus, there are at most $k \cdot n$ edges in $EYY_k(MG)$. Thus, the total communications of Algorithm 4 is at most $O(k \cdot n)$, where each message has $O(\log u)$ bits. It is interesting to see that the communication complexity does not depend on $\gamma$ at all.

Fig. 9. (a) In $EYG_k(MG)$, star formed by links toward to $v$. (b) Node $v$ chooses the shortest link in $EYG_k(MG)$ toward itself from each region to produce $EYY_k(MG)$. (c) The sink structure at $v$ in $EYY_k(MG)$.

**Theorem 8:** The graph $EYY_k(MG)$, $k \geq 6$, is connected if MG is connected.

**Proof:** Notice that it is sufficient to show that there is a path from $u$ to $v$ for any two nodes with $uv \in MG$. Remember the graph $EYG_k(MG)$ is connected, therefore, we only have to show that $\forall uv \in EYG_k(MG)$, there is a path connecting $u$ and $v$ in $EYY_k(MG)$. We prove this claim by induction on the ranks of all links in $EYG_k(MG)$.

If the link $uv$ has the smallest rank among all links of $EYG_k(MG)$, then $uv$ will obviously survive after the second step. So the claim is true for the smallest rank.

Assume that the claim is true for all links in $EYG_k(MG)$ with rank at most $r$. Then consider a link $uv$ in $EYG_k(V)$ with $\text{rank}(uv) = r + 1$ in $EYG_k(MG)$. If $uv$ survives in Algorithm 4, then the claim holds. Otherwise, assume that $r_u < r_v$. Then directed edge $wu$ cannot belong to $EYG_k(MG)$ from Algorithm 3. Thus, directed edge $uv$ is in $EYG_k(MG)$. In Algorithm 4, directed edge $uv$ can only be removed by node $v$ due to the existence of another directed link $vw$ with a smaller identity and $w$ is in the same region as $u$. In addition, the angle $\angle wwu$ is less than $\frac{\theta}{2}(k \geq 6)$. Therefore we have $\|wu\| < \|uv\|$. Notice that here $wu$ is not guaranteed to be in $EYG_k(MG)$. We then prove by induction that there is a path connecting $u$ and $v$ in $EYY_k(MG)$. Assume $r_w \leq r_u$. The scenario $r_w > r_u$ can be proved similarly. There are two cases here.

Case 1: the link $wu$ is in $EYG_k(MG)$. Notice that rank of $wu$ is less than the rank of $wv$. Then by induction, there is a path $w \rightarrow \cdots \rightarrow u$ connecting $w$ and $u$ in $EYY_k(MG)$. Consequently, there is a path (concatenation of the undirected path $w \rightarrow \cdots \rightarrow u$ and the link $wu$) between $u$ and $v$.

Case 2: the link $wu$ is not in $EYG_k(MG)$. Then, from proof of Theorem 4, we know that there is a path $\Pi_{EYG_k(w,u)} = q_1q_2\cdots q_m$ from $w$ to $u$ in $EYG_k(MG)$, where $q_1 = w$ and $q_m = u$. Additionally, we can show that each link $q_i q_{i+1}, 1 \leq i < m$, has a smaller rank than $wu$, which is at most $r$. Here $\text{rank}(q_i q_{i+1} < \text{rank}(w,u)$ because the selection method in Algorithm 3.

And $\text{rank}(q_i q_{i+1}) < \text{rank}(w,u), 1 < i < m$, because $\|q_i q_{i+1}\| < \|q_i u\| < \|q_{i-1} u\| < \cdots < \|q_1 u\| = \|wu\|$. Then, by induction, for each link $q_i q_{i+1}$, there is a path $q_i \rightarrow \cdots \rightarrow q_{i+1}$ survived in $EYY_k(MG)$ after Algorithm 4. The concatenation of all such paths $q_i \rightarrow \cdots \rightarrow q_{i+1}, 1 \leq i < m$, and the link $wu$ forms a path from $u$ to $v$ in $EYY_k(MG)$.

This finishes the proof. Although $EYY_k(MG)$ is a connected structure, it is unknown whether it is a power or length spanner. We leave it as a future work.

D. Extended Yao-Sink Graph

In [4], [5], the authors applied the technique in [15] to construct a sparse network topology in UDG, Yao and sink graph, which has a bounded degree and a bounded stretch factor. The technique is to replace the directed star consisting of all links toward a node $v$ by a directed tree $T(v)$ with $v$ as the sink. Tree $T(v)$ is constructed recursively. To apply this technique into MG, we need extend it by a more sophisticated way. In the remainder of this section, we discuss how to locally construct a bounded degree structure with bounded power stretch factor for heterogeneous wireless ad hoc networks. Our method works as follows.

**Algorithm 5: Constructing-EYG**

1. Each node finds the incident edges in the Extended Yao graph $EYG_k(MG)$, as described in Algorithm 3. Each node $v$ will have a set of incoming nodes $I(v) = \{u \mid \overrightarrow{wu} \in EYG_k(MG)\}$.

2. Each node $v$ partitions the $k$ cones centered at $v$ using the partitioning method described in Method 1. Notice that for partitioning, node $v$ uses parameter $\gamma_v$ in Method 1, which can be easily calculated from local information. Figure 9(a) illustrates such a partition.

3. Each node $v$ chooses a node $u$ from each region $\Omega$. Let $\Omega_u(v)$ be the region $\Omega$ partitioned by node $v$ with node $u$ inside, so that the link $uv$ has the smallest $ID(w)$ among all links computed in the first step in the region $\Omega_u(v)$. In other words, in this step, it constructs $EYY_k(MG)$.

4. For each region $\Omega_u(v)$ and the selected node $u$, let $S_\Omega(u) = \{w \mid w \neq u, w \in \Omega_u(v) \cap I(v)\}$, i.e., the set of incoming neighbors of $v$ (other than $u$) in the same region as $u$. For each node $u$, node $v$ uses the following function
Tree\((u, S_\Omega(u))\) (described in Algorithm 6) to build a tree \(T(u)\) rooted at \(u\). We call \(T(u)\) a sink tree and call the union of all links chosen by node \(v\) the sink structure at \(v\). Figure 9(c) illustrates a sink structure at \(v\), which is composed of all trees \(T(u)\) for \(u\) selected in the previous step.

5. Finally, node \(v\) informs nodes \(x\) and \(y\) for each selected link \(xy\) in the sink structure rooted at \(v\).

The union of all chosen links is the final network topology, denoted by \(EY_G^k(MG)\). We call such structure as the Extended Yao-Sink graph. Notice that, sink node \(v\), not \(u\), constructs the tree \(T(u)\) and then informs the end-nodes of the selected links to keep such links if already exist or add such links otherwise.

**Algorithm 6: Constructing-Tree Tree\((u, S_\Omega(u))\)**

1. If \(S_\Omega(u)\) is empty, then return.
2. Otherwise, partition the disk centered at \(u\) by \(k\) equal-sized cones: \(C_1(u), C_2(u), \ldots, C_k(u)\).
3. Find the node \(w_i \in S_\Omega(u) \cap C_i(u), 1 \leq i \leq k\), with the smallest \(ID(w_i, u)\), if there is any. Link \(w_i, u\) is added to \(T(u, S_\Omega(u))\) and node \(w_i\) is removed from \(S_\Omega(u)\).
4. For each node \(w_i\), call Tree\((w_i, S_\Omega(u) \cap C_i(u))\) and add the created edges to \(T(u, S_\Omega(u))\).

Notice that the above Algorithm 6 is only performed by a node \(v\). We then prove that the constructed structure \(EY_G^k(MG)\) indeed has bounded degree (thus sparse), and is power efficient.

**Theorem 9:** The maximum node degree of the graph \(EY_G^k(MG)\) is at most \(k^2 + 3k + 3k \cdot \lceil \log_2 \gamma \rceil\).

**Proof:** Initially, each node has at most \(k\) out-degrees after constructing graph \(EY_Gk(MG)\). In the algorithm, each node \(v\) initiates only one sink structure, which will introduce at most \(3\lceil \log_2 \gamma \rceil + 2\) \(k\) in-degrees. Additionally, each node \(x\) will be involved in Algorithm 6 for at most \(k\) sink trees (once for each directed link \(xy \in EY_Gk(MG)\)). For each sink tree involvement, Algorithm 6 assigns at most \(k\) links incident on \(x\). Thus, at most \(k^2\) new degrees could be introduced here. Then the theorem follows.

Since the total number of edges is at most \((k^2 + 3k + 3k \cdot \lceil \log_2 \gamma \rceil) \cdot n\), the total communication cost of our method is \(O(\log_2 \gamma \cdot n)\). Here each message has \(O(\log n)\) bits.

**Theorem 10:** The length stretch factor of the graph \(EY_G^k(MG), k > 6\), is at most \((\frac{1}{1-2\sin(\frac{\pi}{k})})^2\).

**Proof:** We have proved that \(EY_G^k(MG)\) has length stretch factor at most \((\frac{1}{1-2\sin(\frac{\pi}{k})})^2\). We thus only have to prove that, for each link \(vw \in EY_G^k(MG)\), there is a path connecting them in \(EY_G^k(MG)\) with length at most \((\frac{1}{1-2\sin(\frac{\pi}{k})})\|vw\|\). If link \(vw\) is kept in \(EY_G^k(MG)\), then this is obvious. Otherwise, assume \(r_w \leq r_v\), then directed link \(vw\) belongs to \(EY_G^k(MG)\). Then, there must have a node \(u\) in the same region (partitioned by node \(v\)) as \(w\) node. Using the same argument as Theorem 4, we can prove that there is a path connecting \(v\) and \(w\) in \(T(u)\) with length at most \((\frac{1}{1-2\sin(\frac{\pi}{k})})\|vw\|\). It implies that the length stretch factor of \(EY_G^k(MG)\) is at most \((\frac{1}{1-2\sin(\frac{\pi}{k})})^2\). Similarly, we have:

**Theorem 11:** The power stretch factor of the graph \(EY_G^k(MG), k > 6\), is at most \((\frac{1}{1-2\sin(\frac{\pi}{k})})^2\).

**VII. Experiments**

In this section we measure the performance of the proposed heterogeneous network topologies by conducting extensive simulations. In our experiments, we randomly generate a set \(V\) of \(n\) wireless nodes with random transmission range for each node. We then construct the mutual inclusion communication graph \(MG(V)\), and test the connectivity of \(MG(V)\). If it is connected, we construct different localized topologies: \(GG(MG), EGG(MG), RNG(MG), ERNG(MG), EY_Gk(MG), EYY_k(MG)\) and \(EY_G^k(MG)\). Then we measure the sparseness (the average node degree), the power efficiency and the communication cost of building these topologies. In the experimental results presented here, the wireless nodes are distributed in a \(2000m \times 2000m\) square field. Each wireless node has a transmission radius randomly selected from \([300m, 1300m]\). The number of wireless nodes is \(30\), where \(i\) is varied from 1 to 10. For each set of \(i\), we randomly generate 100 set of \(30\) nodes. All structures proposed in this paper are generated for each set of nodes. The number of cones is set to 7 when we construct \(EY_G^k(MG), EYY_k(MG)\) and \(EY_G^k(MG)\). Figure 11 illustrates all seven different topologies for the MG graph illustrated by the first figure of Figure 11 for one such random node set.

**Node Degree**

First of all, we want to test the sparseness of each network topology proposed in this paper. Notice that, we can theoretically proved that \(RNG(MG)\) and \(ERNG(MG)\) both have at most \(6n\) links; structure \(EY_Gk(MG)\) has at most \(k \cdot n\) links, where \(k\) is the number of cones divided; structure \(EYY_k(MG)\) also has at most \(k \cdot n\) links since \(EY_Yk(MG) \subseteq EY_Gk(MG)\); structure \(EY_G^k(MG)\) also has at most \(k \cdot n\) links since the sink structure for each node \(u\) has exactly the number of links as the links toward \(u\) in the directed structure \(EY_G^k(MG)\). We do not how many links graph \(GG(MG)\) and \(EGG(MG)\) could have.

Although almost all proposed structures are sparse theoretically, all of them could have unbounded node degree. The node degree of the wireless networks should not be too large. Otherwise a node with a large degree has to communicate with many nodes directly. This potentially increases the signal interference and the overhead at this node. The node degree should neither be too small: a small node degree usually implies that the network has a lower fault tolerance and it also tends to increase the overall network power consumption as longer paths may have to be taken. Thus, the node degree is an important performance metric for the wireless network topology. Figure 12 illustrates the average node degree of different topologies. Notice that graph \(RNG(MG)\) always has the smallest average node degree in our simulations and structure \(EY_G^k(MG)\) always has the largest average node degree. We also found that the average node degree becomes almost stable when the number of nodes increases, i.e., the network becomes denser.
Figure 11. Different sparse topologies generated from the same MG.

Figure 12. Average node degree of different topologies.

Figure 13. Maximum node degree of Yao-based structures.

Figure 14 illustrates the length spanning ratio of these structures. As the theoretical results suggest, we found that RNG(GG) has a much larger length spanning ratio compared with other structures. It is surprising to see that structure ERNG(MG) also has a much smaller spanning ratio than RNG(MG). Notice that we do know that ERNG(MG) has smaller spanning ratios than RNG(GG) since ERNG(MG) ⊆ RNG(MG). Also notice that struc-
ture $EYG_k(MG)$, as the theoretical results suggest, has the smallest length spanning ratio among all structures proposed here.

For wireless ad hoc networks, we want to keep as less links as possible while still keep relatively power efficient paths for every pair of nodes. Figure 15 illustrates the power spanning ratio of these structures. Here we assume that the power needed to support a link $uv$ is $\|uv\|^2$. As we expected, structures $GG(MG)$ and $EGG(MG)$ keep the most power efficient path for every pair of nodes, i.e., their power spanning ratios are exactly one. We found that all structures have power spanning ratio almost one, and again $RNG(MG)$ and $ERNG(MG)$ do have the largest power spanning ratios in our simulations.

**Communication Cost of Construction**

It is not difficult to see that structures $GG(MG)$, $RNG(MG)$, and $EYG_k(MG)$ can be constructed using only $n$ messages by assuming that each node can tell its neighbors its maximum transmission range, and its geometry position information in one single message. Each node $n$ can uniquely determine all the links $uv$ in these three structures after knowing all its one hop neighbors in $MG$. Structures $EYG_k(MG)$, and $EYG_k^*(MG)$ can be constructed using only $k \cdot n + n$ messages since the final structures have at most $kn$ links. Similarly, structure $ERNG(MG)$ can be constructed using at most $7n$ messages. We do not know any theoretical bound about the number of messages needed to construct $EGG(MG)$ since each node $u$ has to inform its neighbors the links selected by $u$ for $EGG(MG)$. We measured the actual average number of messages needed to construct these structures. We only measure the average number of messages per wireless node for structures $EGG(MG)$, $ERNG(MG)$, $EYG_k(MG)$, and $EYG_k^*$ since every node only has to spend one message for other three structures $GG(MG)$, $RNG(MG)$, and $EYG_k(MG)$). Figure 16 illustrates the communication cost. We found that structure $EYG_k^*$ is the most expensive to construct although it has the most favorable properties theoretically (bounded length, power spanning ratio and bounded node degree). It is almost as expensive as constructing $EYG_k$ to construct structure $EYG_k(MG)$.

![Fig. 14. Average length spanning ratio of different topologies.](image)

![Fig. 15. Average power spanning ratio of different topologies.](image)

![Fig. 16. Average communication cost of building different topologies.](image)

**VIII. Conclusion**

In this paper, we studied topology control in heterogeneous wireless ad hoc networks, where each mobile host has different maximum transmission power and two nodes are connected if they are within the maximum transmission range of each other. We presented several strategies for all wireless nodes self-maintaining sparse and power efficient topologies in heterogeneous network environment with low communication cost. All structures $GG(MG)$, $RNG(MG)$, $EYG_k(MG)$, $EYG_k^*(MG)$, and $EYG_k^*(MG)$ are connected if MG is connected, while $EYG_k(MG)$ and $EYG_k^*(MG)$ have constant bounded power and length stretch factors. Additionally, we showed that $EY_k(MG)$ and $EYG_k^*(MG)$ have bounded node degrees $O(\log_2 \gamma)$, where $\gamma = \max_{v \in V} \max_{u \in \mathcal{I}(v)} \left( \frac{n}{d(u,v)} \right)$. In the worst case any connected graph will have degree at least $O(\log_2 \gamma)$ for heterogeneous wireless ad hoc networks. In other words, the structures constructed by our method are almost optimum. Our algorithms are all localized and have communication cost at most $O(\log_2 \gamma \cdot n)$, where each message has $O(\log n)$
It remains an open problem whether graph $E_{Y_k}(MG)$ is a length or power spanner. It is also unknown how many links $GG(MG)$ could have in the worst case although we show that it is definitely less than $O(n^{8/5} \log_2 \gamma)$. Some other future works are what are the conditions that we can build a structure with some other properties for $MG$, such as planar or low weight. Notice that it is easy to show we cannot build a planar topology for any given heterogeneous wireless ad hoc networks.

References


