Practical Traffic Grooming Scheme for Single-Hub SONET/WDM Rings

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Abstract

In SONET/WDM networks, one fiber supports multiple wavelengths and each wavelength supports several low rate tributary streams. Traffic grooming *is then defined as properly using SONET Add/Drop Multiplexer (ADM) to electronically multiplex and demultiplex required tributary traffic patterns with minimal resource cost (wavelengths and ADMs).*

This paper studies the traffic-grooming problem in single hub SONET/WDM networks and extends existing results. We analyze the real deployments, generalize their results, and study the practical special cases. We prove that BLSR/2 would never be more expensive than UPSR under any traffic pattern. We present the exact minimum costs of uniform traffic in both UPSR and BLSR/2. We also give approximation algorithms for optimal grooming of non-uniform traffic after showing that this problem is NP-complete. Finally, we consider how to select the line speeds if there are two different line speeds available.

Keyword: Traffic grooming, SONET/WDM ring, UPSR, BLSR/2, single-hub, bin packing.

1 Introduction

Recently as the Internet is booming, and B-ISDN services such as eConference, multimedia communication, VoIP, HDTV, VOD, start to be incentive for customers, bandwidth requirement grows rapidly. At the low end, homes are connected to digital world by existing wires. One trend is to use fast connections such as Cable Modem and DSL technique to access B-ISDN services. Another coming trend is to use fast wireless connections to connect to the digital world, rather than to use only one telephony channel.

Naturally, following the trend is the usage of optical communication technique, which has originally been used to support the high-end wide area network (WAN) for setting up high-speed MAN feeder networks and LAN access networks. In recent years, as the commercial optical communication standard, SONET/SDH coupling with WDM (Wavelength Division Multiplexing) technique has been deployed to provide B-ISDN services for customers. SONET rings are embedded in WDM rings, and one wavelength supports one SONET ring if without considering self-healing mechanism. Figure 1, Figure 2 and Figure 3 give some examples of its deployments.

Along with such an immigration are new engineering problems of network designing and planning of SONET/WDM MANs and LANs. Among those new engineering problems, we focus on the traffic-grooming problem. Notice that, in the long-haul fiber networks, each fiber needs many repeaters (say, EDFA: Erbium-doped-fiber-amplifier) and carries larger data volumes, Thus the number of available wavelengths is a rare resource. However, we often can assume that we have enough fibers to lighten for

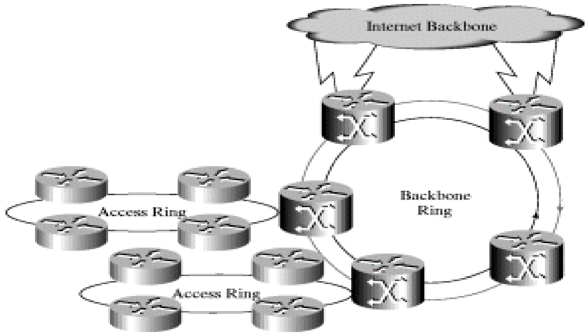

Figure 1. SONET/SDH over WDM, a likely Metropolitan deployment.

MAN and LAN deployments. In other words, we assume that wavelengths are sufficient and the terminating devices dominate the cost in this paper. Indeed, till now the bottleneck of optical communication applications lies on the O/E and E/O boundaries. Though SONET/SDH ADM provides proper and cost-efficient multiplexing/demultiplexing and O-E/E-O conversions for SONET networks, it is still expensive. So when planning to set up SONET/SDH networks in metropolitan and local areas, we will focus on how to minimize the number of SONET/SDH Add/Drop Multiplexers (ADMs).

The minimum ADM cost depends on both the underlying network architecture and the traffic pattern. Two types of SONET self-healing rings have been widely used: unidirectional path-switched rings (UPSR); two-fiber bidirectional line-switched rings (BLSR/2). The traffic could have some regular patterns such as one-to-all and all-to-all, or any irregular pattern. The traffic demands may be uniform (i.e. all traffic have the same amount of demands) or non-uniform. Each traffic demand itself is given as an integer number of low speed (tributary) streams. Alternatively, it can also be represented by its *traffic granularity*, defined as the ratio of its demand to the transmission capacity of a single wavelength. A traffic is said to be a *full-wavelength traffic*, a *sub-wavelength traffic* or a *super-wavelength traffic* if its traffic granularity is equal to one, less than one, or greater than one respectively.

The minimum ADM problem has been discussed in a number of recent works [4, 6, 7, 8, 10, 12, 13]. [6] and [8] studied optimal grooming of arbitrary full-wavelength lightpaths. [4], [12] and [13] provided grooming of uniform $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$–wavelength traffic. [7] and [10] gave some preliminary results on the traffic groomings in single-hub rings. In [10], an optimal grooming of uniform one-to-all traffic in single-hub UPSR rings was presented. Then it can be generalized to any uniform traffic. It also gave lower and upper bounds on the ADM cost of uniform all-to-all traffic with-

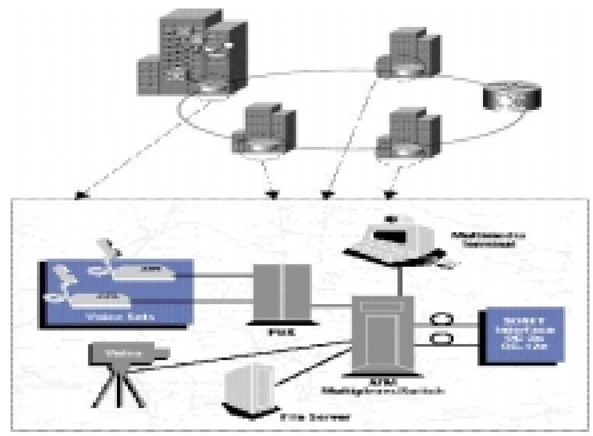

Figure 2. SONET/SDH access network (LAN) deployments. Here each ring node will support the telephony and digital services for one building. Ring nodes are connected to a hub at center office. Ring nodes will add/drop OC-3/ OC-12 tributary streams contained in wavelength channels to ATM switches. Here OC-3 (155Mbps) and OC-12 (622Mbps) are two basic ATM bit rates.

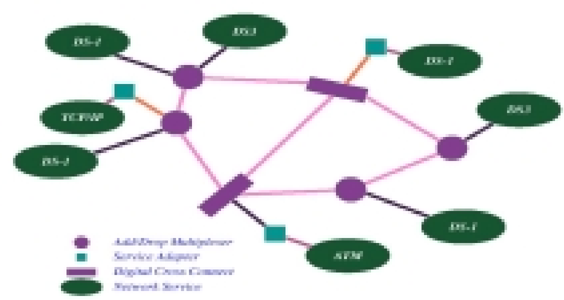

Figure 3. Typical SONET Networks. Here DS-1 is a connection with capacity of 24 linking to PBX of small companies and DS-3 is with capacity of 672 for bigger companies. TCP/IP is for Intranet such as campus networks and ATM provides statistical multiplexing for B-ISDN connections with different QoS (Quality of Service) requirements.

out a hub. In [7], lower and upper bounds on the ADM cost of uniform all-to-all traffic in both single-hub UPSR and single-hub BLSR/2 were obtained. The economics of these two types of rings were then justified by the two lower bounds. A remark on this justification is that it makes logical sense only if the lower bounds are sufficiently close the optimum. In addition, [7] also briefly discussed the criteria for using UPSR vs. BLSR rings and to mix two types of line speeds on a single SONET/WDM ring. In this paper we will further the works in [7] and [10] and provide stronger results.

The paper is structured as follows. In Section 2, we formulate the traffic grooming problem, after showing that we need only concentrate on one-to-all simplex traffic. We prove that BLSR/2 always costs no more than UPSR under any traffic, and show that the search for an optimal grooming can be confined to a narrow subset of valid groomings, referred to as *canonical groomings*. In Section 3, we construct optimal canonical groomings of uniform one-to-all traffic in both UPSR and BLSR/2 rings and derive the analytic expression of the minimum ADMs. An approximating scheme for nonuniform traffic request is presented after showing that this problem is NP-complete. We also discuss how to select the line speeds if there are two line speeds available in Section 4. We conclude our paper in Section 5.

2 Problem Formulation and Canonical Grooming

We consider a single-hub SONET/WDM ring comprising of $N+1$ nodes numbered $0, 1, \cdots, N$ clockwise, with the hub placed at node 0. The traffic demand and the transmission capacity of each wavelength are in terms of the basic low-rate (e.g., OC-3) traffic streams. Let g be the transmission capacity of a single wavelength.

We first show that all traffic grooming problems can be reduced to solving one-to-all simplex traffic grooming. We establish a reduction from grooming of any duplex traffic to grooming of one–to-all duplex traffic, and from grooming of one–to-all duplex traffic to grooming of one-to-all simplex traffic. Thus any optimal grooming of one-to-all simplex leads to an optimal grooming of one-to-all duplex and an optimal grooming of all-to-all duplex. Therefore, from then on we concentrate on only one-to-all simplex traffic.

2.1 Reduction to One-to-All Simplex Traffic

Assume that the traffic between any pair of nodes is full-duplex and the traffic demand between node i and j is r_{ij}. As the traffic stream between any pair of nodes must be routed through the hub, any traffic pattern can be treated as a number of duplex requests between the hub and all other nodes. To be more specific, in the equivalent one-to-all duplex traffic, the traffic demand between node i and the hub is

$$r_i = \sum_{j \neq i} r_{ij}$$

for all $1 \leq i \leq N$. Thus it is sufficient for us to consider only one-to-all duplex traffic.

In the following we take a further step of reduction. Let $ADM_d\,(r_1, \cdots, r_N)$ be the minimum ADM cost of a one-to-all duplex traffic, in which the demand between node i and the hub is r_i for $1 \leq i \leq N$. Let $ADM_s\,(r_1, \cdots, r_N)$ be the minimum ADM cost of a one-to-all simplex traffic, in which the demand from the hub to node i is r_i for $1 \leq i \leq N$. Obviously, in either UPSR or BLSR,

$$ADM_d\,(r_1, \cdots, r_N) \geq ADM_s\,(r_1, \cdots, r_N) \qquad (1)$$

as the one-to-all duplex traffic is a superset of one-to-all simplex traffic. On the other hand, any grooming of the simplex traffic naturally gives rise to a grooming of the corresponding duplex traffic with the same cost in the following way. Let w be any wavelength used in the grooming of the simplex traffic, and let r_i^w be the portion of the demand from the hub to node i carried in wavelength w. Now consider the following grooming of the duplex traffic: we use the same set of wavelengths used in the grooming of the simplex traffic, each wavelength w carries r_i^w units of demand from the hub to node i and r_i^w units of demand from node i to the hub for all $1 \leq i \leq N$. It's easy to see that such grooming is a valid solution and it uses the same number of ADMs. Thus

$$ADM_d\,(r_1, \cdots, r_N) \leq ADM_s\,(r_1, \cdots, r_N)\,. \qquad (2)$$

From Equation 1 and Equation 2, we have

$$ADM_d\,(r_1, \cdots, r_N) = ADM_s\,(r_1, \cdots, r_N)\,.$$

The following lemma summarizes this reduction.

Lemma 2.1 *The minimum ADM cost of any one-to-all duplex traffic is same as that of the corresponding one-to-all simplex traffic.*

2.2 Problem Formulation

So from now on, we will only concentrate on the grooming of one-to-all simplex pattern. Thus for single hub SONET/SDH (over WDM) networks, the *traffic grooming problem* has the following formulation.

Instance: Given a single hub SONET/WDM ring comprising of $N+1$ nodes, and the hub is placed at node 0. The traffic request from node i to the hub 0 is r_i, for $1 \leq i \leq N$. Also assume that one wavelength supports g tributary streams.

Solution: A valid assignment of tributary streams in wavelengths to the traffic set.

Objective: Minimize the number of needed ADMs.

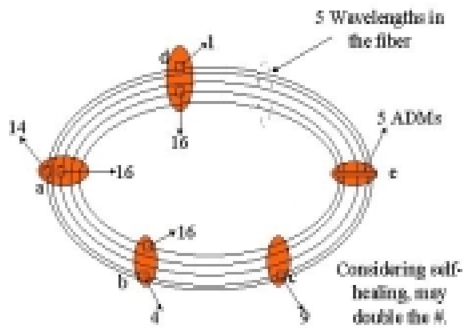

Figure 4. The optimal grooming scheme for the given instance. Here for this instance, the communication capacity requests are: $a-e$ 30 OC-3's, $b-e$ 20 OC-3's, $c-e$ 9 OC-3's, $d-e$ 17 OC-3's. Each wavelength channel supports $g = 16$ OC-3's. Assume hub e is a node at some center office.

For example, assume in Figure 2, four buildings a, b, c, d need capacity 30 OC-3's, 20 OC-3's, 9 OC-3's, 17 OC-3's to connect to the center office node e respectively. Then the optimal solution is shown in Figure 4 and uses 12 SONET/SDH ADMs (if considering self-healing we have to double this number).

2.3 UPSR vs. BLSR/2

In [7], the economics of single-hub UPSR and single-hub BLSR/2 are justified by comparing the lower bounds on the minimum ADM cost of uniform all-to-all duplex traffic, which is essentially the minimum ADM cost of corresponding uniform one-to-all simplex traffic according to the reductions made in the previous section. Logically, the conclusion drawn from such comparison is reasonable only if the lower bounds are sufficiently close to the optimum. Furthermore, the conclusion may still not be persuasive by just considering uniform requests. In this section, we prove that under any type of traffic, the single-hub BLSR/2 costs no more than the single-hub UPSR. The argument applies to any traffic pattern.

Theorem 2.2 *Given any set of traffic demands, the single-hub BLSR/2 costs no more than the single-hub UPSR.*

PROOF. Consider any grooming of the given set of demands in UPSR. Let w be any wavelength used in working ring in the UPSR, and let r_i^w be the portion of the demand from the hub to node i carried in wavelength w. Now consider the following grooming in the BLSR/2: each wavelength w is used in both rings of the BLSR/2, and in each ring the wavelength w carries $r_i^w/2$ units of demand from the hub to node i for all $1 \leq i \leq N$. It's easy to see that such grooming is a valid solution and it uses the same number of ADMs as in UPSR. Thus the theorem is true. ◻

In Section 3.1, we will quantize the exact cost difference if the given traffic is uniform.

2.4 Canonical Grooming

In [10], it claimed that the search of optimal grooming of uniform traffic in UPSR can be confined to those canonical groomings defined by us later. We give a formal proof of the claim and generalize this property to arbitrary traffic pattern with arbitrary traffic demands in both UPSR and BLSR/2.

Given a set of demands $\{r_1, \cdots, r_N\}$ in a UPSR and the wavelength capacity g, a grooming is said to be a *canonical grooming* if at each node $1 \leq i \leq N$, its demand is carried in $\left\lceil \frac{r_i}{g} \right\rceil$ wavelengths, among which $\left\lfloor \frac{r_i}{g} \right\rfloor$ wavelengths each carries g units of demands to node i, and the remaining one, if there is any, carries $r_i \bmod g$ units of demands to node i.

Given a set of demands $\{r_1, \cdots, r_N\}$ in a BLSR/2 and the wavelength capacity g, a grooming is said to be a *canonical grooming* if at each node $1 \leq i \leq N$, its demand is carried in $\left\lceil \frac{r_i}{\frac{g}{2}} \right\rceil = \left\lceil \frac{2r_i}{g} \right\rceil$ wavelengths (counting each wavelength used in both directions as two), among which $\left\lfloor \frac{r_i}{\frac{g}{2}} \right\rfloor = \left\lfloor \frac{2r_i}{g} \right\rfloor$ wavelengths each carries $\frac{g}{2}$ units of demands to node i, and the remaining one, if there is any, carries $r_i \bmod \frac{g}{2}$ units of demands to node i.

The next lemma states that when looking for optimal traffic grooming, we can pay attention to only these canonical groomings.

Lemma 2.3 *Given any set of demands in UPSR or BLSR/2, there is a canonical grooming with minimum ADM cost.*

PROOF. We prove the lemma by transforming any given optimal grooming into a canonical grooming with the same cost in a number of steps. The procedure at each step is as follows. Suppose that the current optimal grooming is not canonical. Then at some node i, two portion of its demands, $0 < f_2 \leq f_1 < g$, are carried in two wavelengths w_1 and w_2 respectively. We consider two cases.

Case 1: $f_1 + f_2 \leq g$. We use an unused wavelength to carry the two portion of demands f_1 and f_2 instead of using w_1 and w_2. Then in the new wavelength two ADMs are used to carry these $f_1 + f_2$ portion. But the two ADMs used at node i in the wavelengths w_1 and w_2 are removed. So the ADM cost does not increase.

Case 2: $f_1 + f_2 > g$. We swap all traffic except f_1 carried in wavelength w_1 with the $g - f_1$ portion within f_2 in wavelength w_2. In the resulting grooming, wavelength w_1 carries the full g units of demands to node i, and wavelength w_2 carries $f_1 + f_2 - g$ units of demands to node i. The total ADM cost remains the same.

Notice that the number of wavelengths carrying portion traffic of node i is decreased by 1 in both cases. It's easy to see that one canonical grooming can be reached after a finite number of such procedures. The resulting canonical grooming has the minimum ADM cost and thus is optimal. □

In the next section, we will apply Lemma 2.3 to find the minimum ADM cost of uniform traffic and non-uniform traffic by designing optimal or suboptimal canonical groomings.

3 Practical Solutions

3.1 Uniform Traffic Grooming

In this section, we present the optimal canonical grooming of uniform traffic in both single-hub UPSR and single-hub BLSR/2. We assume that the traffic demand from each node to the hub is r.

We first consider the optimal grooming of uniform traffic in single-hub UPSR.

First, let us consider $r \bmod g = 0$. Then the optimal canonical grooming is unique in the sense that each wavelength carry g units of demands exclusively to some node. Thus each node contributes $2 \cdot \frac{r}{g} = \frac{2r}{g}$ ADMs: half at the node itself and half at the hub. So the total ADM cost in the working fiber is $N \cdot \frac{2r}{g} = \frac{2Nr}{g}$. The total ADM cost is then $\frac{4Nr}{g}$ if protection fiber is also considered.

Now we assume that $r \bmod g > 0$. In any canonical grooming, at each node there are $r - r \bmod g$ portion of demands carried in $\left\lfloor \frac{r}{g} \right\rfloor$ wavelengths exclusively. These demands use $2N \left\lfloor \frac{r}{g} \right\rfloor$ ADMs in the working fiber. In any optimal grooming, the remaining demands at each node, referred to as *residue demands*, must use a minimum ADM cost. This can be achieved in the same way as in [10]. We partition the N nodes into $\left\lceil \frac{N}{\left\lfloor \frac{g}{r \bmod g} \right\rfloor} \right\rceil$ groups of at most $\left\lfloor \frac{g}{r \bmod g} \right\rfloor$ nodes. The residue demands of nodes in each group are carried in a single wavelength. These residue demands totally require $N + \left\lceil \frac{N}{\left\lfloor \frac{g}{r \bmod g} \right\rfloor} \right\rceil$ ADMs in the working fiber. Thus the total ADMs used in the working fiber is

$$N \left\lceil \frac{r}{g} \right\rceil + N \left\lfloor \frac{r}{g} \right\rfloor + \left\lceil \frac{N}{\left\lfloor \frac{g}{r \bmod g} \right\rfloor} \right\rceil .$$

Let

$$F(g, r, N) = \begin{cases} \frac{2Nr}{g} & \text{If } r \bmod g = 0, \\ N \left\lceil \frac{r}{g} \right\rceil + N \left\lfloor \frac{r}{g} \right\rfloor + \left\lceil \frac{N}{\left\lfloor \frac{g}{r \bmod g} \right\rfloor} \right\rceil & \text{otherwise.} \end{cases}$$

Then the minimum ADM cost in the working fiber is $F(g, r, N)$, and the total ADM cost is $2F(g, r, N)$.

Similarly, the minimum ADM cost in BLSR/2 is $F(\frac{g}{2}, r, N)$. The optimum canonical grooming can be constructed in the similar way.

The next theorem summarizes the above discussions.

Theorem 3.1 *The minimum ADM costs of uniform traffic demand with rate r in UPSR and BLSR/2 are $2F(g, r, N)$ and $F(\frac{g}{2}, r, N)$ respectively.*

In Section 2.3, we have proved the BLSR/2 always costs no more than UPSR under any traffic patten. When the traffic is uniform, this can be verified by the inequality

$$F(\frac{g}{2}, r, N) \leq 2F(g, r, N).$$

Notice that the cost difference of UPSR and BLSR/2 is $2F(g, r, N) - F(\frac{g}{2}, r, N)$ for uniform traffic.

3.2 Non-uniform Traffic Grooming

3.2.1 General Approach

It was proved in [10] that the optimal grooming of non-uniform *sub-wavelength* traffic grooming for UPSR is NP-complete. By applying Lemma 2.3, we can prove that the optimal grooming of arbitrary non-uniform traffic is NP-complete in both UPSR and BLSR/2. The reduction is also made from the well-known bin-packing problem.

Lemma 3.2 *Bin Packing problem reduces to single hub traffic grooming problem.*

PROOF. Due to the canonical lemma, assume each request r_i requires $r_i \bmod g$ tributary streams lying in one fractional wavelength and all other tributary streams lying in $\left\lfloor \frac{r_i}{g} \right\rfloor$ wavelengths exclusively. Each wavelength supports g tributary streams. So we have to solve a bin packing problem: to pack N objects into as few as possible bins, where the i-th object requires capacity of $r_i \bmod g$ and the bin size is g. The inverse reduction also holds. □

In the next, we present approximation algorithms to groom non-uniform traffic for UPSR. Given a traffic demands $r_1, \cdots, r_N$, a canonical grooming is constructed as follows. At each node i, we carry $r_i - r_i \bmod g$ portion of demands in $\left\lfloor \frac{r_i}{g} \right\rfloor$ wavelengths exclusively. And we use a bin-packing approximation algorithm to groom all residue demands $r_i \bmod g$, for $1 \leq i \leq N$.

Then we analyze the performance of the bin packing method as follows. Let Opt be the minimum ADM cost. We divide the total ADM cost to two portions: the first portion is the cost of ADMs shared by more than one node at the hub; the second portion, denoted by C_{base}, is the total cost of rest of ADMs (i.e., devoted to exactly one node at the hub or used at each node). It is simple to show that

$$C_{base} = \sum_{i=1}^{N} \left\lceil \frac{r_i}{g} \right\rceil + \sum_{i=1}^{N} \left\lfloor \frac{r_i}{g} \right\rfloor .$$

Then the minimum ADM cost at the hub required by the residue demands is $Opt - C_{base}$. Consider any approximation algorithm $\mathcal{A}$ for the bin-packing problem. Assume that $\mathcal{A}$ has approximation ratio α. Then the ADM cost at the hub required by the resulting grooming of the residue demands is at most $\alpha\,(Opt - C_{base})$, if we apply $\mathcal{A}$ to groom the residue demands. So the total ADM cost of the grooming constructed in this way is

$$C_{base} + \alpha\,(Opt - C_{base}) = \alpha \cdot Opt - (\alpha - 1)\,C_{base}.$$

Then the approximation ratio of this scheme is

$$\frac{\alpha \cdot Opt - (\alpha - 1)\,C_{base}}{Opt} = \alpha - (\alpha - 1)\frac{C_{base}}{Opt}.$$

Notice that the number of ADMs used at the hub is at most the total ADMs used at all non-hub nodes. Hence, we have $C_{base} \geq Opt - C_{base}$. So the number of ADMs used by the above scheme is within $\frac{\alpha+1}{2}$ factor of the optimum. The following theorem summarizes the above analysis.

Theorem 3.3 *Using an α-approximation bin-packing algorithm to groom the residue demands, the ADM cost by the canonical grooming is within $\frac{\alpha+1}{2}$ factor of the optimum.*

There are a number of bin packing approximation algorithms developed [3]. The off-line First-Fit-Decreasing (FFD) bin packing method first sorts the input objects in the decreasing order, and then assigns the bins sequentially for objects. The assigned bin is the first bin that still can fit the current object. It gives a $\frac{11}{9}$ approximation for the minimal number of bins used [3]. In turn it gives a $\frac{10}{9}$ approximation non-uniform traffic grooming algorithm as following. First we sort the residual demands $r_i \bmod g$ at all non-hub nodes decreasingly, and assign it to the first ADM with sufficient spare capacity.

3.2.2 Special Cases

For real world application, the number of tributary streams one wavelength can support is limited by SONET protocol. For example, besides SONET historically supports T1, E1, T3 and other streams, now in ISDN networks it is generally used to support OC-12 (ATM base rate 622Mbps) and OC-3 (ATM base rate 155 Mbps) by wavelength channels with speed OC-48 and OC-192. Thus one OC-48 can support 4 OC-12's (i.e., $g = 4$) and 16 OC-3's (i.e., $g = 16$). One OC-192 supports 16 OC-12's (i.e., $g = 16$) and 64 OC-3's (i.e., $g = 64$).

Now we study how to groom non-uniform traffic when the traffic demands at each node is integer. Let $d_i = r_i \bmod g$ be the residual demand at node i, where r_i is integer traffic demand at node i. So for several specific g's, we consider how to solve the integer bin packing problem exactly. At following paragraphs we give the optimal solutions for $g = 2, 4, 8$ and the proof of optimality is omitted. We also find that some solutions are very similar (but not totally the same) with that given by the FFD scheme, which suggests that FFD is really a good heuristic for SONET traffic grooming problem. Recall that we had proved that FFD gives a grooming whose cost is no more than $\frac{10}{9}$ factor of the optimum.

The Case $g = 2$. We consider a bin packing problem where each bin has capacity $g = 2$ and each object has volume 1. Assume we have k nodes with residual 1, then we exactly need $\lceil \frac{k}{2} \rceil$ ADMs.

The Case $g = 4$. In this case, we have residue $d_i \in \{1, 2, 3\}$. Assume we have n_1 nodes with residue 1, n_2 nodes with residue 2, n_3 nodes with residue 3 among all nodes. The following steps give an optimal solution:

1. First we need n_3 ADMs for those nodes with residue $d_i = 3$. We also can use these ADMs to carry the traffic of $min(n_1, n_3)$ nodes whose residue is 1.
2. Now we need $\lceil \frac{n_2}{2} \rceil$ ADMs for those nodes with residue 2. We may also fill at most 2 nodes with residue 1 if n_2 is odd and there is any nodes with residue 1 and remaining unfilled.
3. Now if $n_1 > n_3 + 2(n_2\%2)$, we need $\left\lceil \frac{n_1 - n_3 - 2(n_2\%2)}{4} \right\rceil$ ADM's for remaining nodes with residue 1.

Consequently, we need exactly

$$n = \begin{cases} n_3 + \lceil \frac{n_2}{2} \rceil, \; if \; n_1 \leq n_3 + 2(n_2\%2) \\ n_3 + \lceil \frac{n_2}{2} \rceil + \left\lceil \frac{n_1 - n_3 - 2(n_2\%2)}{4} \right\rceil, \; otherwise \end{cases}$$

ADMs to groom the residual traffic.

The Case $g = 8$**.** Now the residue $d_i \in \{1, 2, 3, 4, 5, 6, 7\}$ and assume that we have n_1 1's, n_2 2's,...,n_7 7's. The following steps give an optimal solution:

1. First it is obvious that the nodes with residue more than 4 can not share ADMs among them. Therefore, we need n_7 ADMs for those nodes with residue 7, n_6 ADMs for those nodes with residue 6, n_5 ADMs for those nodes with residue 5. Also $\lceil \frac{n_4}{2} \rceil$ ADMs are needed for those nodes with residue 4. Assume the set of ADMs used above are denoted by A_7, A_6, A_5, A_4 respectively.

2. For ADMs from A_7, we can only fill nodes with residue 1 and can fill at most $min(n_1, n_7)$ such nodes. Update $n_1 \leftarrow \max(0, n_1 - n_7)$, i.e., the number of nodes with residue 1 and remain unfilled.

3. For ADMs from A_6, if we have $n_1 > 0$, we can only select nodes with residue 1 or 2 to share ADMs with those nodes having residue 6. We prefer to select nodes with residue 2 first since the nodes with residue 1 give the most freedom for future filling. Therefore, we select $min(n_6, n_2)$ nodes with residue 2 and fill into ADMs from A_6. If there are ADMs from A_6 remaining unfilled (i.e., $n_6 > n_2$), nodes with residue 1, if there is any, are used to share those ADMs. In other words, we select another $min(2(n_6 - n_2), n_1)$ nodes with residue 1 and fill into ADMs from A_6 if there is any. Update n_2 and n_1 accordingly.

4. Then consider ADMs from A_5. We may select nodes with residue 1, 2 or 3 to share ADMs that have been used by one node with residue 5. For each such ADM, there are many ways to select nodes with residue less than 4 to share it. We select nodes in the following order: first is node with residue 3; then pair of nodes with residue 1 and 2; followed by only nodes with residue 2; finally are nodes with residue 1. Update the number n_1, n_2, n_3 accordingly.

5. We then study how to groom nodes with residue 4. Divide all nodes with residue 4 into pairs and assign each pair an ADM. We may need an extra ADM if n_4 is odd. Therefore, we only need solve the remainder instance: n_1 nodes with residue 1, n_2 nodes with residue 2, n_3 nodes with residue 3, and a possible half-full ADM if originally n_4 is odd.

6. For these remainder instance, we use the idea of FFD to solve it. Divide all nodes with residue 3 into $\lfloor \frac{n_3}{2} \rfloor$ pairs. Then carry the traffic of each pair by an ADM. Assume the set of ADM used at this step is A_3.

7. Fill as many as possible nodes with residue 2 into ADMs of A_3 and maybe an non-full ADM from A_4 if there is one. Update the number n_2 accordingly.

8. For these remaining nodes with residue 2, we use $\lfloor \frac{n_2}{4} \rfloor$ ADMs, each of which carries the traffic of 4 nodes with residue 2. Notice that we need an extra ADM if $4 \nmid n_2$. After this step, all ADMs can not be shared by nodes with residue 2.

9. Fill as many as possible nodes with residue 1 into ADMs that are not full. Update the number n_1 accordingly.

10. For those remaining nodes with residue 1, we use $\lceil \frac{n_1}{8} \rceil$ ADMs to carry their traffic.

Observe that the basic schemes of all above methods are same as the FFD method: Order the residue in decreasing order and assume that all ADMs are lined sequentially; assign each node an ADM with the largest spare capacity.

4 Select Speeds with Two Line Speeds Available

In the previous discussions, we assume that all SONET rings have the same line speed. However, if we allow the SONET rings to have different line speeds, we have to partition the traffic from each node into the SONET rings of different line speeds. After the partition, the traffic grooming algorithms developed in the previous sections can be applied to the rings of any particular line speed. Thus a solution has two components, the partition of the traffic, and the groomings of the traffic in rings of each speed. Both components affect the overall cost. So efficient algorithms or criteria should be developed to find traffic partitions which may lead to the minimum ADM cost. This section is intended to address this problem. In [9], the authors studied it by more detailed analysis.

To simplify the problem, we assume that there are only two line speeds g_1 and g_2 with $g_2 = 4g_1$ as did in [7]. We also adopt the same cost model used in [7]: the cost of an ADM of speed g_1 is 1, and the cost of an ADM of speed g_2 is 2.5. Assume the demand between node i and the hub is r_i for $1 \leq i \leq n$. Then any traffic partition can be represented by a n-dimensional vector

$$f = (f_1, \cdots, f_n)$$

where $0 \leq f_i \leq r_i$ is the amount of the traffic between node i and the hub placed to a low-speed ring. For any traffic partition, we can groom the traffic carried in low-speed rings and the traffic carried in high-speed rings separately. A simple approach presented in [7] is that for each traffic demand with value r, assign $f_i = r \bmod g_2$. The performance of this approach comparing to the optimal assignment was not discussed in [7]. In this section, more general solutions will be developed and their optimality is proven in [9].

For uniform traffic demands, we provide optimal traffic partition and grooming, which is summarized in Tab. 1. For

Range of all r's	$(f_1, f_2, \cdots, f_n)$
$(0, 1\frac{1}{2}]$	$f_i = r, \forall i$
$(1\frac{1}{2}, 2]$,$n = 2k$	$f_i = 0, \forall i$
$(1\frac{1}{2}, 2]$,$n = 2k+1$	$f_i = 0, \forall i \neq j; f_j = r$
$(2, 2\frac{1}{2}]$	$f_{2i-1} = 0, f_{2i} = 2r - 4$
$(2\frac{1}{2}, 4]$	$f_i = 0, \forall i$

Table 1. Selecting line speeds for traffic grooming for UPSR.

Range of all r's	$(f_1, f_2, \cdots, f_n)$
$(0, \frac{3}{4}]$	$f_i = r, \forall i$
$(\frac{3}{4}, 1]$,$n = 2k$	$f_i = 0, \forall i$
$(\frac{3}{4}, 1]$,$n = 2k+1$	$f_i = 0, \forall i \neq j; f_j = r$
$(1, 1\frac{1}{4}]$	$f_{2i-1} = 0, f_{2i} = 2r - 2$
$(1\frac{1}{4}, 2]$	$f_i = 0, \forall i$

Table 2. Selecting line speeds for traffic grooming for BLSR.

non-uniform traffic demands, optimal or suboptimal solutions have been developed depending on the range of all demands. If all demands are at most 1.5, then all of them are carried in low-speed rings. If all traffic demands are greater than 1.5 but less than 2, then with even n, all of them are carried in high-speed rings and the total cost of ADMs in the working ring only is $3.75n$; with odd n, all of them except an arbitrary one are carried in high-speed rings and the total cost of ADMs in the working ring only is $3.75n + 1.5$. Such costs remain the same as long as all demands are greater than 1.5 but less than 2. If all traffic demands are greater than 2.5, all of them are carried in high-speed rings and the total cost of ADMs in the working ring only is $5n$. Such cost also remain the same as long as all demands are greater than 2.5. When all traffic demands are greater than 2 but less than 2.5, the solution is a little complicated. We first pair up the n nodes. If n is odd, some node is stand-alone and its whole traffic is carried in a high-speed ring. For each pair of nodes i and j, we use a high-speed ring to carry the whole traffic from node i and the remaining capacity is used to carry the traffic from node j. For detail of the optimality proof, the reader is referred to [9].

If node i has demand $r_i > g_2$ then from the canonical grooming, we know that $\left\lfloor \frac{r_i}{g_2} \right\rfloor g_2$ traffic will be carried by $\left\lfloor \frac{r_i}{g_2} \right\rfloor$ high speed rings exclusively. Hence, in the above discussions, we only consider the case when $r_i \leq g_2$ for node i. The above argument is restricted to UPSR. However, it can be extended to BLSR as well. For uniform traffic demands, we summarize optimal traffic partition and grooming in Tab. 2. For more detail of the analysis and proof, the reader is referred to [9].

5 Conclusions

In this paper we consider how to groom both uniform and non-uniform traffic to minimize the number of ADMs in the single-hub UPSR and BLSR/2. We give optimal grooming of uniform traffic. The optimal grooming of non-uniform is shown to be NP-complete, hence we present a $\frac{10}{9}$ approximation algorithm. When the traffic demands is integer number of basic low rate, we give the optimal solution for non-uniform traffic grooming of single hub ring network for some practical cases. We also study the optimal mixture of a number of different line speeds to minimize the overall cost with two different line speeds available. In this paper, we specifically study the two line speeds case with cost ratio 2.5. For uniform traffic demands, we give the optimal traffic partition and grooming for both UPSR and BLSR. Near optimal solutions are also given for non-uniform traffic demands for both UPSR and BLSR.

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