Interference-Aware Topology Control for Wireless Sensor Networks

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***Abstract*— Topology control has been well studied in wireless ad hoc networks. However, only a few topology control methods (*e.g.* [1]) take into account the low interference as a goal of the methods. Some researchers tried to indirectly reduce the interference by reducing the transmission power or by devising low degree topologies, but none of those protocols can guarantee low interference. In this paper we present several algorithms to construct network topologies such that the maximum (or average) link (or nodal) interference of the topology is either minimized or approximately minimized. The algorithms and definitions introduced in this paper are not based on any geometry information about the nodes and they work for any graph models of wireless communication. The theoretical results are corroborated by simulation studies.**

I. Introduction

Energy conservation is one of the critical issues in designing wireless ad hoc or sensor networks. Various aspects of the networking will affect the energy consumption of the wireless networks, such as the medium access control (MAC) protocols, the routing protocols, and so on. Topology control, a layer between MAC and routing protocol, provides another dimension to save the energy consumption of the wireless networks. In the literature, most of the research in the topology control is about adjusting the transmission power, or designing some *sparse* network topologies that can result in more efficient routing methods. However, less attention is paid to minimize the interference caused by these structures when routing is performed on top of them. Notice that, if a topology has a large interference, then either many signals sent by nodes will collide (if no collision avoidance MAC is used), or the network may experience a serious delay at delivering the data for some nodes.

Interference plays a very important role in several applications [2]. For example, consider the basic problem of transmitting data from a server to a client terminal over a wireless channel. Due to possible channel interference, the transmitted data may be corrupted in transit and data must be repeatedly retransmitted until it is received correctly at the terminal. We thus need to specially consider interference-aware topology control. One might consider using multi channel communication to avoid interference. If there is enough channels available at nodes, there would be no interference, but in practice the number of channels is limited and an interference aware topology is always desired.

In wireless ad hoc or sensor networks, typically a wireless device can selectively decide which nodes to communicate either by adjusting its transmission power, or by only maintaining the communication links with some special nodes within its transmission range. Maintaining a small number of communication links will also speed up the routing protocols in addition to possibly alleviate the interferences among simultaneous transmissions, and also to possibly save the energy consumption. The question in topology control we have to deal with is how to design a network structure such that it ensures attractive network features such as low-stretch factor (so-called spanning ratio), linear number of links, and more importantly, low interference. In recent years, there was a substantial amount of research on topology control for wireless ad hoc networks [3], [5], [6], [7], [8], [9]. A common implicit assumption in traditional topology control methods is that *low node degree implies small interference*, which is not always true, as shown in [1]. Notice that, in practice, almost all topology control methods will select shorter links and avoid longer links. However, even selecting "short" links only cannot guarantee that the interference of the resulting topology is within a constant factor of the optimum. Further, even if each node only communicates with its nearest neighbor, the resulting communication graph may still have an interference arbitrarily, up to $O(n)$ factor, larger than the optimum. Burkhart *et al.* [1] first raised a fundamental question "*Does topology control reduce interference?*". They showed that traditional topology control methods will not always produce a subgraph whose interference is within a constant factor of the optimum. Burkhart *et al.* [1] proposed several methods to construct topologies whose maximum link interference is minimized while the topology is connected or is a spanner for Euclidean length.

In this paper, we continue the investigation on the topology control with small interference along this direction. We firmly show that *topology control does reduce interference* under various measurements of *interference*. We will address how to minimize the average link interference and also introduce two models for node interference and for each introduced model we will study how to minimize the maximum and the average interference.

The main contributions of this paper are as follows. First of all, we define various criteria to measure the interference quality of a structure. Under these interference quality criteria, we give efficient centralized algorithms to construct network topologies such that the maximum link (or node), or the

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average interference of the topology is either minimized or approximately minimized. We also study how to construct topology locally with small interference while it is power efficient for unicast routing. Although the study of the maximum or average interference of the structure captures the worst case possible performance of the structure, it may not reflect the average performances of a structure for some randomly deployed networks. We then further study the average performances (in terms of their interference qualities) for several widely used structures such as RNG and EMST. We show that these structures have large maximum node interference even for randomly deployed networks. Surprisingly, we found that the average interference of these structures for randomly deployed networks is bounded by some constants. Our theoretical studies are corroborated in our simulations.

The remainder of the paper is organized as follows. In Section II, we specifically discuss what network model is used in this paper, and how we define the interference of a topology. In Section III, we propose several methods to construct various topologies such that the maximum link interference or the average link interference of the topology is minimized. In Section IV, we proposed several methods to construct various topologies such that the maximum node interference or the average node interference of the topology is minimized. Localized methods are presented in Section V to construct topologies with low interference with additional properties. In Section VI, we study the performances of some widely used topology control structures. Our simulation results are reported in Section VII. We conclude our paper in Section VIII and also point out some future works

II. Preliminaries

A. Network Model

We consider a wireless sensor network with all nodes distributed in a two dimensional plane. Assume that all wireless nodes have distinctive identities and each wireless node u has a maximum transmission power P_u. We only consider undirected (symmetric) communication links meaning that a message sent by a node u over a link uv can be acknowledged by the receiver v over link vu. In other words, link uv exists if and only if the nodes u and v can communicate with each other directly when they use their maximum transmission power. Let V be the set of all n wireless nodes and E be the set of symmetric links uv. We use $G = (V, E)$ to denote the original communication graph when all nodes using their maximum transmission power. It is required that the graph G is connected if all nodes use their maximum power, otherwise devising a topology that preserves the connectivity is impossible. For each node u, we use $T(u, p)$ to denote the region where a node can receive the signal from u correctly when node u transmits at a power level p. Typically, it is assumed that $T(u, p)$ is a disk centered at u. In addition, we use $I(u, p)$ to denote the region where a node will have interference when it receives the signal from a node other than u and node u is also transmitting at a power level p.

Consider node u sending a message to one of its neighbors node v, the consumed energy for this communication is composed of three parts: (1) the energy used by node u to prepare the outgoing signal, (2) the energy needed to compensate the path loss of the signal from u to v, and (3) the energy needed by node v to process the incoming signal from node u. In the literature, the following path loss model is widely adopted: the signal strength received by a node v is p_1/r^{α}, where p_1 is the signal strength at one meter, r is the distance of node v from the source node u, and α is a path loss gradient, which is a constant between 2 and 5 depending on the transmission environment. Consequently, we define the energy cost p_{uv} for each link as $p_{uv} = c_1 + c_2 \cdot \|uv\|^{\alpha}$, where c_1, and c_2 are some constants depending on the electronic characteristics and the antenna characteristics of the wireless devices. The specific model of p_{uv} is not crucial for the results presented in this paper as long as p_{uv} is a monotone increasing function of the distance $\|uv\|$.

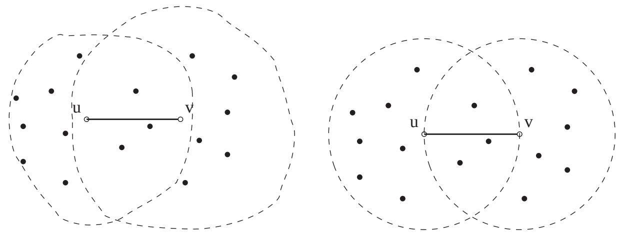
We also assume that each wireless device can adjust its transmission power to any value p_u from 0 to its maximum transmission power P_u or to a given sequence of discrete transmission powers. Furthermore, in the literature it is often assumed that each wireless device u can adjust its transmission power for every transmission depending on the intended receiver v (*i.e.*, node u will use the minimum transmission power available to reach next-hop node v). Some researchers assume that, given a undirected network topology H, each wireless device will only adjust its transmission power to the minimum power such that it can reach its farthest neighbor in H. In this paper, we will consider all possible power adjustments.

B. What Is Interference?

As mentioned earlier, the ultimate goal of the topology control is to conserve the energy consumption of the wireless networks. To achieve this goal most of topology control algorithms consider adjusting the transmission power of nodes, bounding the number of wireless nodes a node has to communicate, or bounding the power spanning ratio of the structure, while minimizing the inherent interference of the structure which enables simultaneous parallel transmissions and in turn decreases the number of retransmissions is ignored. Then a natural question is "*What is the interference of a structure*?". In this subsection, we will discuss different models of defining the interference of a structure.

The interference model proposed in [10] is based on the current network traffic. However, it requires a priori information about the traffic in a network, which is often not available when designing the network topology due to the fact that the amount of the network traffic is often random and depends on the applications. Thus, when we design a network topology to minimize the "interference", we prefer a static model of interference that depends solely on the distribution of the wireless nodes and, maybe, their transmission ranges.

Notice that, symmetric links are often preferred in wireless communications, *i.e.*, a link uv exists in the communication graph if they can communicate with each other directly. Using this observation, Burkhart *et al.* [1] defined the interference of a link uv as the number of nodes covered by two disks centered at u and v with radius $\|uv\|$, *i.e.*, they assume that the transmission region of every node is a disk centered at this node and the transmission power is dynamically adjusted to p_{uv} by the send u for each individual next-hop node v. See Figure 1 (b) for an illustration. Let $D(u, r)$ denote



(a) Our model (b) Priori art

Fig. 1. The interference of link uv based on coverage.

the disk centered at node u with radius r. Specifically, they define the coverage of a link uv as $cov(uv) = \{w \mid w\, is\, covered\, by\, D(u, \|uv\|)\ or\ D(v, \|uv\|)\}$, *i.e.*, the set of all nodes that could be affected by u or v when they communicate with each other using exactly the minimum power needed to reach each other.

In this paper, we also consider the interference to be proximity-based: the signal sent by a node u with power p_u will only interfere the nodes inside some region, denoted by $I(u, p_u)$. Consider a link uv and assume that the node u needs to use power p_u to be able to send message to node v and node v needs to use power p_v to be able to send message to node u. Then we define the coverage of the link uv as follows:

$$cov(uv) = \{w \mid w \in I(u, p_u)\ or\ w \in I(v, p_v)\}.$$

Note that this model works for both continuous and discrete power models. In continuous model, p_u and p_v are equal to $p_{uv} = c_1 + c_2 \cdot \|uv\|^\alpha$ and in discrete power model p_u and p_v are the smallest power level that nodes u and v need to be able to communicate with each other. Here, $cov(uv)$ represents the set of all nodes that could be affected by node u or v when they communicate with each other using exactly the minimum power needed to reach each other. We call this interference model as Interference based on Coverage (IC) model, and will use $IC(uv)$ to denote the interference of a link uv under this model, *i.e.*, $IC(uv) = |cov(uv)|$, the cardinality of $cov(uv)$. See left figure of Figure 1 for an illustration. This model is chosen since whenever a link uv is used for a send-receive transaction all nodes in $cov(uv)$ will be affected. In the remainder of the paper, we always use $IC(uv)$ to denote the interference of a link in both models.

The network is then represented by a undirected *weighted* graph, $G = (V, E, W)$, with n vertices representing wireless nodes, m edges representing communication links, and the weight of a link uv being $IC(uv)$. After assigning weights to all links, we call the graph the *interference graph*. Thus, given a subgraph H of the original communication graph G of n wireless devices, the maximum interference, denoted as $MIC(H)$, of this structure H is defined as $\max_{e \in H} IC(e)$, and the average interference, denoted as $AIC(H)$, of this structure H is defined as $\sum_{e \in H} IC(e)/m_H$, where m_H is the number of links of H.

Notice that the interference model used in [1] and the model defined in previous discussions implicitly assume that the node u will send message to v and node v will send message to u at the same time. We argue that when u sends data to node v, typically node v only has to send a very short acknowledge message to u. The communication then becomes one way by ignoring this small acknowledge message from v. Clearly, when v is receiving message from node u, the nodes "nearby" node v cannot send any data, otherwise interference occurs. Practically speaking, the transmission by another node w causes the interference with the transmission from node u to node v if the signal to interference and noise ratio (SINR) of the signal received by node v is below a certain threshold [1] of node v when node w transmits at a given power. To simplify the analysis of SINR, we assume that the transmission of a node w causes such interference if node v is inside the interference region of w. In other words, we say an interference occurs when v is inside the transmission region of sender u and inside the interference region of another node w, and both node u and node w transmit signal simultaneously. The number of such nodes w is the total number of nodes whose transmission will cause the interference to the signal received by node v.

Given a subgraph H of the original graph G, the power range of each node u is defined as the minimum power p_u that node u needed to reach all its neighbors in H, *i.e.*, $p_u(H) = \max_{uv \in H} p_{uv}$. Here p_{uv} is the minimum power that node u needs to send a message to node v.

Considering a node w, the transmission of node w may cause interference to *all* nodes inside its interference region. Thus, to alleviate the interference, we would like to minimize the number of nodes inside the transmission region of node w by setting its transmission power p_w appropriately. We call such interference model as Interference based on Sender (IS) model and will use $IS_H(w)$ to denote the interference of a node w under a given network topology H, which is defined as the cardinality of the set $\{w \mid p_{wv} \le p_w(H)\}$. The maximum interference of a structure H, denoted as $MNIS(H)$, is defined as $\max_{w \in V} IS_H(w)$, and the average interference of H, denoted as $ANIS(H)$, is defined as $\sum_{w \in V} IS_H(w)/n$.

In addition to this sender based model, one could also argue for the following receiver based model. Considering a node v, when node v is inside the interference region of multiple nodes, only one such node can send message to v at any give time. Thus, to alleviate the interference, we would like to minimize the number of nodes whose interference region contains the node v by setting their transmission power appropriately. We call such interference model as Interference based on Receiver (IR) model and will use $IR_H(v)$ to denote the interference of a node v under a given network topology H. The interference number $IR_H(v)$ of a node v is then defined as the cardinality of the set $\{u \mid p_{uv} \le p_u(H)\}$. The maximum interference of this structure H, denoted as $MNIR(H)$, is defined as $\max_{u \in V} IR_H(u)$, and the average interference of this structure H, denoted as $ANIR(H)$, is defined as $\sum_{u \in V} IR_H(u)/n$.

[1]The threshold of node v depends on the sensitivity of the antenna of node v, the modulation technique of the signal, and other factors.

C. Related Works on Topology Control

Due to the limited power and memory, a wireless node prefers to only maintain the information of a subset of neighbors it will communicate, which is called *topology control*. In recent years, there is a substantial amount of research on topology control for wireless ad hoc or sensor networks [3], [4], [5], [6], [7]. These algorithms are designed for different objectives such as minimizing the maximum link length (or node power) while maintaining the network connectivity [5]; bounding the node degree [7]; bounding the spanning ratio [3], [4]; constructing planar spanner locally [3]. Here a subgraph H of a graph G is a length (or power) spanner of G if, for any two nodes, the length (or power) of the shortest-path connecting them in H is no more than a constant factor of the length of the shortest-path connecting them in the original graph G. Planar structures are used by several localized routing algorithms [11]. In [12], Li *et al.* proposed the first localized algorithm to construct a bounded degree planar spanner. Recently, Li, Hou and Sha [13] proposed a novel local MST-based method for topology control and broadcasting. In [8], [9], Li *et al.* proposed several new localized methods with $O(n)$ messages to construct structures that approximate the Euclidean minimum spanning tree (EMST).

However, none of these structures proposed in the literature can *theoretically* bound the ratio of the interference of the constructed structure over the interference of the respected optimum structure. Several papers studied the throughput of a wireless ad hoc network by considering the impact of the interference. In their seminal paper [14], Gupta and Kumar studied the throughput of wireless networks under two models of interference: a protocol model that assumes interference to be an all-or-nothing phenomenon and a physical model that considers the impact of interfering transmissions on the signal-to-noise ratio. In [15], Kodialam and Nandagopal considered the problem of computing optimal throughput for a given wireless network with a given traffic pattern. They assume a limited model of interference in which the only constraint is that node may not transmit and receive simultaneously. They model the problem as a graph coloring problem.

In [16], Jain and Padhye *et al.* considered the issue of interference when calculating the maximum throughput by a wireless network. They showed that a key issue impacting performance is wireless interference between neighboring nodes. In other words, by employing an interference aware routing protocol there is opportunity for achieving throughput gains. A fundamental issue in multi-hop wireless networks is that performance degrades sharply as the number of hops traversed increases. For example, in a network of nodes with identical and omnidirectional radio ranges, going from a single hop to 2 hops halves the throughput of a flow because wireless interference dictates that only one of the 2 hops can be active at a time. They used a conflict graph to model the effects of wireless interference. The conflict graph indicates which groups of links mutually interfere and hence cannot be active simultaneously.

Notice that, in this paper, we separate the interference from the traffic pattern of the network. We are mainly interested in quantifying the interference quality of a network topology, and given an optimizing criterion, how to find the (approximately) best network topology. In the literature, the work that is closest to ours is a creative research by Burkhart *et al.* [1]. They proposed centralized methods to select a connected spanning subgraph while the maximum interference of selected links is minimized. They also proposed centralized and novel localized methods to select subgraphs with additional requirement that the subgraph is an Euclidean length spanner of the original communication graph. In this paper, we not only consider the link interference model but also propose a more natural interference model defined for each node. In addition, Burkhart *et al.* [1] concentrated their effort on minimizing the maximum link interference of the final structure while we will study not only how to minimize this worst link performance of the structure, but also how to minimize the average performance of all links (or nodes) of the final structure. Furthermore, we also study the performance of some widely used structures for randomly deployed networks. Recently Rickenbach *et al.* [17] studied the receiver-centric interference model and give an algorithm that can achieve a $\sqrt[4]{\Delta}$-approximation ratio of the optimal connectivity perserving topology in the general highway model.

III. Link Based Interference

In this section, we study the interference-aware topology control in terms of the link interference to preserve some network properties such as connectivity.

A. Minimizing the Maximum Interference

Problem 1: The Min-Max link interference with a property $\mathcal{P}$ problem (abbreviated as MMLIP) is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum interference $MIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Essentially, in [1], Burkhart *et al.* gave a centralized method to construct a connected topology that minimizes the maximum interference. They also introduced centralized and localized methods for the the Min-Max link interference with a property *bounded Euclidean spanning ratio*. In their algorithm (called LIFE) edges are sorted by their weights (interference) in ascending order. Starting from the edge with minimum weight, in each iteration of the algorithm an edge uv is processed. If nodes u and v are already connected in the induced graph, the edge uv is just ignored and otherwise it will be added to the topology. The algorithm continues till a connected graph is constructed. Clearly, the time complexity of this approach is $O(m \log m + hn)$, where h is the number of links in the final structure H. If a t-spanner structure is needed, they [1] add a link uv if the shortest path connecting u and v using previously added "short" links has length larger than t times the length of link uv; otherwise, link uv will not be added. Clearly, the time complexity of this approach is $O(m \log m + h(h + n \log n))$.

A graph property $\mathcal{P}$ is called *polynomially verifiable* if we can test whether any given graph H has this property $\mathcal{P}$ in polynomial time in the size of the graph H. A graph property $\mathcal{P}$ is called *monotonic* if a graph H has this property $\mathcal{P}$ then all graph containing H has this property $\mathcal{P}$. For example, the connectivity property, the bounded spanning ratio property, and the k-connectivity property are all polynomially verifiable and monotonic. Assume that we are given any *polynomially verifiable* and *monotonic* property $\mathcal{P}$. The following binary search based approach is then straightforward to solve problem MMLIP.

Algorithm 1 Min-Max Link Interference with property $\mathcal{P}$

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Compute the interference for all links.
Sort the weight (i.e., interference number) of all links in
ascending order. Let w_1, w_2, ..., w_m be the sorted list of
link weights. Let U = m and L = 1.
repeat
    Let i = ⌊(L+U)/2⌋ and w = w_i.
    Test if the structure H formed by all links with weight
    ≤ w has the property P. If it does, then U = i,
    otherwise, then L = i.
until U = L
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Using range search method, we can compute the interference number of all links in time $O(m \log m)$. Assume that the time complexity to test whether a given structure H (with n vertices and at most m links) has a property $\mathcal{P}$ takes time $\beta_{\mathcal{P}}(m, n)$. It is easy to show that the above binary search based approach has time complexity $O(m \log m + \beta_{\mathcal{P}}(m, n) \cdot \log n)$. For example, to test whether a structure is connected can be done in time $O(m)$, which implies that the finding connected structure with minimum interference can be done in time $O(m \log m + m \log n) = O(m \log n)$. Testing whether a given structure H is a t-spanner of the original graph G can be done in time $O(n(n \log n + m)) = O(n^2 \log n + mn)$, which implies that finding a structure minimizing the interference with *t-spanner* property can be done in time $O(m \log m + n^2 \log^2 n + mn \log n) = O(n \log n(m + n \log n))$ using a binary search based approach described by Algorithm 1. The following theorem is obvious and the proof is thus omitted. Notice that the above analysis is not tight. We are aware of more rigorous methods that can improve the time complexity of Algorithm 1 for some special properties. The details are omitted here due to space limit.

Theorem 1: For a given property $\mathcal{P}$, Algorithm 1 gives the optimum solution for MIN-MAX link interference problem.

B. Minimizing the Average Interference

The maximum interference of the structure captures the worst link on the structure, however, it does not capture the overall performance of the structure in terms of the interference. In this section, we design algorithms that will minimize the *average* interferences of the structure while preserving some additional property $\mathcal{P}$.

Problem 2: The MIN-AVERAGE link interference with a property $\mathcal{P}$ problem (abbreviated as MALIP) is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the average interference $AIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

When the given property $\mathcal{P}$ is just the connectivity of structure, one may conjecture that the minimum spanning tree (with the link interference as the link weight) minimizes the average interference among all connectivity-preserving structures. Unfortunately, a network example illustrated by Figure 2 shows that this is not true.

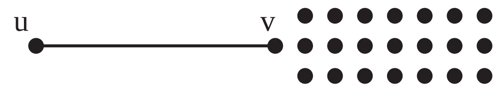


Fig. 2. The average interference of MST is not optimum. There are $n-2$ nodes uniformly distributed in a grid with side-length ϵ. The average interference of MST is close to 5 while the structure connecting all grid links has an average interference clost to 4.

Hereafter, we assume that the property $\mathcal{P}$ is to preserve network connectivity. Obviously, the minimum average interference is no more than the average interference of MST. Let z be the maximum link interference of MST. First of all, we will prove the following lemma.

Lemma 2: The optimum structure with minimum $AIC(H)$ will not use any link with interference larger than z.

Proof: We prove it by contradiction. Assume that the optimum structure H does use a link uv with interference larger than z. If removing link uv will not disconnect the network H, obviously we can remove link uv and get a better structure with smaller average interference in consequence since the average interference of H is less than z and link uv has interference larger than z.

Then consider the case that removing link uv will disconnect the network H. Since $uv \notin MST$, there is a path $\Pi_M ST(u, v)$ in MST that connects u and v. We then remove uv from H and add all links in $\Pi_M ST(u, v)$ to H (some may already belong to H). It is easy to show that the new structure will have a smaller average interference than H. ■

Notice that any structure preserving connectivity will have to use some link with interference z from the definition of MST. A key observation for building a structure with minimum AIC is as follows:

1) all links with interference smaller than minimum AIC will be used in the optimum structure since otherwise we can decrease the AIC by using these unused links with smaller interference.
2) a link uv with interference at least of the minimum AIC is used in the optimum structure only when it is in MST since more such links will increase AIC (the detailed proof is omitted here due to space limit).

Then the following algorithm for building a structure minimizing the average link interference is straightforward.

Note that we will construct the minimum spanning tree of the interference graph, which is different from the Euclidean minimum spanning tree. Actually, the Euclidean MST (*i.e.* where the weight of each edge is the Euclidean length of the edge) can be $\Omega(n)$ times worse than the optimum for minimizing AIC. The example illustrated by Figure 5 in [1] (although they used this example for different purposes) can be used to show that the Euclidean MST is asymptotically

Algorithm 2 Minimize AIC Perserving Connectivity

Compute the interference for all links.
Sort the interference number of all links in the ascending order. Let $w_1, w_2, \cdots, w_m$ be the sorted list of link interference and e_i be the corresponding link with interference w_i.
Build the MST of the interference graph.
Let H be MST and T be the total link interference of H and $m = n - 1$ be the number of links in H. Let $i = 1$.
repeat
  If link $e_i \in MST$, set $i = i + 1$.
  If link $e_i \notin MST$ and $w_i < \frac{T}{m}$, add link e_i to H and set $m = m + 1$, $i = i + 1$, $T = T + w_i$.
until $w_i \geq \frac{T}{m}$

the worst structure. For that example, both the maximum interference and the average interference of the Euclidean MST are $\Theta(n)$, while in the optimum structures, both the maximum and the average link interference are $O(1)$. Thus, Euclidean MST is $\Omega(n)$ times worse than the optimum for both criteria. Notice that $\Theta(n)$ is actually the worst possible ratio for any structure: the worst maximum interference is at most n and the best maximum interference is at least $O(1)$.

IV. Node Interference

In this section we study the node centric interference instead. We will consider two different models here. The first model is based on all incident links' interference and the second model is based on the number of nodes that are in the transmission region of a node.

A. Node Interference via Link

Given a network topology H, a node u will then only communicate using links in H. If node u communicates with a neighbor v over link $uv \in H$, node u may experience the interference from $IC(uv)$ number of nodes. We then would like to know what is the worst interference number experienced by node u, *i.e.*, we are then interested in $IC(u) = \max_{uv \in H} IC(uv)$. In this model the interference of each node u is the maximum link interference of all links incident to it.

Definition 1: Node Interference Via Link: Given a structure H, the interference of a node u, denoted as $IC_H(u)$, is defined as the maximum interference of all links incident on u, *i.e.*, $IC_H(u) = \max_{uv \in H} IC(uv)$. Then the maximum node interference of a structure H is defined as $MNIC(H) = \max_{u \in V} IC_H(u)$, and the average node interference of a structure is defined as $ANIC(H) = \sum_{u \in V} IC_H(u)/n$.

1) Minimizing the Maximum Interference:

Problem 3: The Min-Max node interference via link with a property $\mathcal{P}$ problem (abbreviated as MMNILP) is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum node interference $MNIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

It is easy to show that minimizing the maximum node interference via link problem MMNILP is equivalent to the minimizing the maximum link interference problem MMLIP which we discussed in Section III-A, so we just focus of the MMLIP problem.

2) Minimizing the Average Interference:

Similarly, we can also minimize the average node interference of the structure.

Problem 4: The Min-Average node interference via link with a property $\mathcal{P}$ problem (abbreviated as MANILP) is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the average node interference $ANIC(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Solving the Min-Average node interference with a property $\mathcal{P}$ is not easy and since the simple form of this problem by requiring a connectivity property is similar to the min-average power symmetric connectivity, which is well-known to be NP-Hard. Thus, instead of trying to solve it optimally, we give a good approximation algorithm to achieve the connectivity property. The following theorem proves that the MST (of the interference graph G) is a 2-approximation for the Min-Average node interference with connectivity.

Theorem 3: MST is a 2-approximation for MANILP.

Proof: Consider any spanning tree T and let $I(T)$ denote the average node interference of graph T and let $W(T)$ denote the total weight of the links of graph T. Note that here the weight of each link is the interference of that link. Since the weight of each edge is assigned to at most two nodes, $n \cdot I(T) \leq 2W(T)$. On the other hand, consider the spanning tree as a tree rooted at some nodes. or any leaf node u, the interference of the link that connects u to its parent is the interference that is assigned to node u; for any internal node v, the interference assigned to node v is less than or equal to the interference of the link between node v and its parent in the tree; and the interference assigned to root is some value greater than zero. Thus, the total interference of the nodes is greater than the total interference of the links of the tree and we have $W(T) < n \cdot I(T)$. Now let OPT be the optimum structure. Clearly OPT is a spanning tree (*i.e.*, cycles can be removed, if there is any, without increasing the average interference). We have $n \cdot I(MST) \leq 2W(MST)$. Since MST is the minimum weight spanning tree, $W(MST) \leq W(OPT)$ and $W(OPT) < n \cdot I(OPT)$. Consequently, $I(MST) < 2I(OPT)$. This finishes the proof. ■

The MST based heuristic also works if the weight of each edge is some quality such as the power needed to support the link, the delay of the link, or the SINR. Again, we can show that the Euclidean MST can be $\Omega(n)$ times worse than the optimum. Since the maximum interference is at most $O(n)$, obviously $\Theta(n)$ is the worst possible ratio. It is surprising that Euclidean MST is asymptotically the *worst* structure for problem MMNILP and MANILP (also MMLIP and MALIP), while the MST of the interference graph is asymptotically the *best* structure for these problems.

B. Sender Centric Interference

Notice that, when a topology H is used for routing, each wireless node typically adjusts its transmission power to the minimum that can reach its farthest neighbor in H. Considering this power level, we say that the interference of each node u is the number of nodes inside its transmission range. Let r_u denote the transmission range of node u then the sender-centric interference is defined as follows:

Definition 2: SENDER-CENTRIC NODE INTERFERENCE: Given a structure H, the sender-centric interference of a node u is number of nodes inside its transmission range, *i.e.*,

$$IS_H(u) := |\{v \mid p_{uv} \le p_u\}|.$$

The maximum node interference of a structure is then defined as $MNIS(H) = \max_{u \in V} IS_H(u)$, and the average node interference of a structure is defined as $ANIS(H) = \sum_{u \in V} IS_H(u)/n$.
Remember that p_{uv} is the minimum power needed by node u to send message directly to v, and $p_u(H)$ is the minimum power by node u to reach all its neighbors in a structure H.

1) Minimizing the Maximum Interference:

Problem 5: The MIN-MAX node interference with a property $\mathcal{P}$ problem (abbreviated as MMNISP) is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the maximum node interference $MNIS(H)$ of structure H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

Consider node u and let $N(u)$ be the number of neighbors of node u when node u adjusts its transmission range to maximum. Node u can adjust its transmission range to have exactly k neighbors ($0 \le k \le N(u)$) inside its transmission region. In other words, each node u can set its interference to any value between 0 and $N(u)$ by using the appropriate transmission range. Having this property, solving the MIN-MAX node interference with a property $\mathcal{P}$ problem is only a simple binary search.

Algorithm 3 Min-Max Node Interference with Property $\mathcal{P}$.

Let $U = n - 1$ and $L = 1$.
repeat
 Let $i = \lfloor \frac{L+U}{2} \rfloor$ and let H_i be the graph formed by connecting each node u to its first i-shortest links. Notice that, if u has less than i neighbors in the original graph, then u will only connect to all its $N(u)$ neighbors.
 Test if the structure H_i has the property $\mathcal{P}$. If it does, then $U = i$, otherwise, then $L = i$.
until $U = L$.

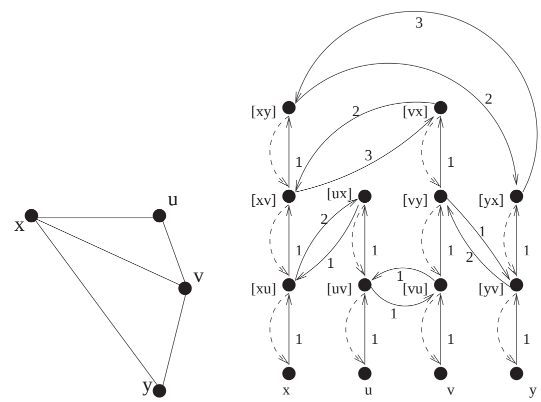
Assume Algorithm 3 gives an interference value i. Since setting the interference of each node to a value less than i cannot preserve the property $\mathcal{P}$. The following theorem is then obvious.

Theorem 4: Algorithm 3 produces the optimum solution for the MIN-MAX Node Interference with a property $\mathcal{P}$.

2) Minimizing the Average Interference:

Problem 6: The MIN-AVERAGE node interference with a property $\mathcal{P}$ problem (abbreviated as MANISP) is to construct a subgraph H of a given communication graph $G = (V, E)$ such that the average node interference $TNIS(H)$ of H achieves the minimum among all subgraphs of G that have a given property $\mathcal{P}$.

We conjecture that solving problem MANISP is NP-Hard. We leave the proof of this statement or the counter-proof as future work. Here we give an efficient heuristic to find a structure that is practically good. Our heuristic involves transforming the original communication graph G to a new graph G' and then solve some problem on the graph G'. We then transform the solution of that problem back as a solution to the original problem MANISP on G. Given a



(a) original network (b) transformed graph

Fig. 3. Transform a network into another graph for minimizing average interference.

communication graph G (*e.g.*, illustrated by Figure 3 (a)), we construct a directed graph $G' = (V', E', W')$ as follows. For each edge uv of G, we introduce two additional vertices $[uv]$ and $[vu]$. Each node u, sorts its neighbors $v_1, v_2, ..., v_k$ in ascending order of distances from u. Then we connect node u to node $[uv_1]$ using directed link $u[uv_1]$ and we assign weight 1 to it; we also define a directed link $[uv_1]u$ and we assign weight 0 to link $[uv_1]u$. We also connect vertices $[uv_i]$ and $[uv_{i+1}]$ using two directed links $[uv_i][uv_{i+1}]$ and $[uv_{i+1}][uv_i]$ ($1 \le i < k$) and assign weight 1 to all those links $[uv_i][uv_{i+1}]$ and we assign weight 0 to all links $[uv_{i+1}][uv_i]$ ($1 \le i < k$). All pairs $[uv], [vu]$ are connected also. Assume node u is the p^{th} nearest neighbor of node v and node v is the q^{th} nearest neighbor of node u. Then we assign weight p to the edge $[uv][vu]$ and weight q to $[vu][uv]$. Figure 3 depicts the original graph and the transformed graph. All dashed edges have weight 0. Now we start from any node $u \in V$ and we solve the min-cost multicast problem to all other nodes $v \in V$. It is easy to show that the min-cost multicast problem in G' is equal to the min-average node interference graph in G.

We then introduce a greedy based algorithm for this multicast problem in the directed graph G'. The algorithm starts with an empty set of *processed nodes*, denoted by A, and picks a random node u and puts it in the set A. We define the distance between a node v that does not belong to set A and set A as the shortest path starting from a node in set A to v. Then in each iteration the node that is the closest to the set A is added to set A and the distances of nodes to the set A are updated. The algorithm continues till all nodes of G are in A. Let H_u be the final structure constructed when node u is first put to the set A.

To find the best structure possible, we will construct the structures H_{v_i} for all nodes $v_i \in V$ and then find the structure with the minimum average nodal interference. We use H_1 to denote this heuristic hereafter.

The approach used in this algorithm is like the Prim's algorithm. The set of nodes V is divided into two sets S and $V-S$, a random node is put in S and in each iteration the node *closest* to the set S is added to it till $S=V$. Now we have to define the distance between a node $v \in V-S$ and the set S. Consider edge uv such that $u \in S$ and $v \in V-S$, if this edge is added then the interference of nodes u and v might increase, we define this incremental interference as the weight of edge uv, and like Prim's algorithm the distance of node v from the set S is the weight of the shortest edge connecting v to S. Whenever an edge uv is added, the adjustable transmission range of nodes u and v is updated if necessary.

Beside the above heuristic H_1, we propose another heuristic, denoted by H_2, to solve this problem. This heuristic is only slightly different and similar to the Kruskal's method computing MST. We start from n components and each component has exactly one node. In each iteration two components that are the closest to each other are merged. Edge weights are defined the same way and the distance between two components is defined as the weight of the shortest edge connecting them. The algorithm continues till there is only one component left. Our simulation results show that this simple trick slightly improves the performance.

V. Localized Approaches

In the previous sections, we discussed in detail several centralized methods for topology control to minimize the interference while preserving some property $\mathcal{P}$. Although these centralized methods can find the optimum or near optimum structures for wireless ad hoc networks, but they may be too expensive to be implemented in wireless ad hoc networks in some circumstances.

A. Preserving Spanning Property

In this section, we shift our attention to localized topology control methods to minimize the interference, with an additional requirement such as the final topology being a hop spanner, length spanner or power spanner. Here we always assume that the desired spanning ratio is given. If the structure is required to be t-length spanner, as shown in [1], for each link uv we only need the information of $(t/2)\cdot\|uv\|$ neighborhood (*i.e.* nodes whose distance to node u or to node v is less than $(t/2)\cdot\|uv\|$). Similarly for k-hop spanner it suffices to gather the information of $\lceil k/2 \rceil$ hops of nodes u and v (*i.e.* nodes which are at most $\lceil k/2 \rceil$ hops away from node u and node v). Here we say that a structure H is a t-spanner for power consumption if for any pair of nodes u and v, the minimum power of all paths connecting them in H is no more than t times the minimum power of the best path connecting them in the original communication graph. Remember that, the power needed to support a link $e=(x,y)$, denoted by $p(e)$, is $c_1 + c_2 \cdot \|xy\|^\alpha$. The total power of a path Π, denoted by $v_0 v_1 \cdots v_k$, connecting u and v is $p(\Pi)=\sum_{i=0}^{k-1} p(v_i v_{i+1}) = k\cdot c_1 + c_2 \cdot \sum_{i=0}^{k-1}\|v_i v_{i+1}\|^\alpha$. Here u is node v_1 and v is node v_k. Let $u \to_H v$ be the path connecting u and v using links in H with the minimum total power consumption, denoted by $p(u \to_H v)$. Formally speaking, a structure H is a t-power-spanner of original graph G if

$$\max_{u,v\in V} \frac{p(u \to_H v)}{p(u \to_G v)} \le t.$$

In the remainder of the paper, we assume that the maximum transmission range of every node is R_0 (*i.e.*, the maximum transmission power of every node is $c_1 + c_2 R_0^\alpha$).

Lemma 5: Consider any structure H that is a t-power-spanner. For any link uv in the original graph G, the t-power spanner path $u \to_H v$ has an Euclidean length at most $t\cdot\mathcal{A}\cdot(c_1+c_2\|uv\|^\alpha)$, where $\mathcal{A}=\frac{c_2^{1/\alpha}(\alpha-1)^{1+1/\alpha}}{\alpha c_1^{1-1/\alpha}}$ is a constant.

Proof: Remember that the power cost of using a link uv is $c_1 + c_2\|uv\|^\alpha$. We define the *mileage* of this model as $\max_{0<x}\frac{x}{c_1+c_2 x^\alpha}$. In other words, milage is the maximum distance a message can be sent using unit amount of energy. It is easy to see that $x=\sqrt[\alpha]{\frac{c_1}{(\alpha-1)c_2}}$ achieves the maximum mileage for this energy model. Clearly the maximum mileage is $\frac{c_2^{1/\alpha}(\alpha-1)^{1+1/\alpha}}{\alpha c_1^{1-1/\alpha}}$. Hereafter, we use $\mathcal{A}$ to denote such mileage.

We then show that the least power path $u \to_H v$ has an Euclidean length, say x, within some constant factor of the Euclidean length $\|uv\|$. From the definition of mileage, we know that the total power of the path $u \to_H v$ is at least $\frac{x}{\mathcal{A}}$. Since it is a t-power-spanner path for uv, we have $x/\mathcal{A} \le t(c_1+c_2\|uv\|^\alpha)$. In other words, $x \le t\cdot\mathcal{A}\cdot(c_1+c_2\|uv\|^\alpha)$. ■

This lemma implies that node u can locally decide whether a link uv will be kept in a t-power spanner H by using only the information of nodes within distances $\frac{x}{2}+\|uv\|$ to node u. It also implies that the minimum power path for any link uv uses only local neighborhood nodes as long as the mileage (the maximum ratio of the length of a link over the power needed to support the direct communication of this link) is bounded from above by a constant.

Then similar to [1], we can construct a network topology H such that the maximum interference is minimized while the structure H is a t-power spanner of the original communication graph. For the completeness of the presentation, we still include the algorithm here. Algorithm 4 is presented from the point view of a node u. The proof of the correctness of Algorithm 4 is similar to that of [1], and thus omitted due to space limit.

B. Preserving Connectivity

In Section V-A, we discussed how to achieve the bounded-spanning-ratio property in a localized manner. Most of applications in wireless networks only require the final topology to be connected. Here we suggest two simple localized interference-aware topologies to preserve connectivity. The first method is based on local minimum spanning tree (*LMST*) [8] where given a weighted undirected graph a spanning tree is built in a localized manner. We call our method *I-LMST* (Interference

Algorithm 4 Min-Max Link Interference with a t-power spanner

1: Each node u collects the information of nodes with distance $\frac{t\cdot\mathcal{A}\cdot(c_1+c_2R_0^{\alpha})}{2}+R_0$. Let $N(u,t)$ be the set of such collected nodes.
2: Sort the interference number in ascending order of all links formed by nodes in $N(u,t)$. Let $w_1, w_2, \cdots, w_m$ be the sorted list of link weights. Let $U=m$ and $L=1$.
3: **repeat**
4: Let $i=\lfloor\frac{L+U}{2}\rfloor$ and $w=w_i$.
5: For each physical link uv, test if the structure H formed by all links with interference $\leq w$ has a path with total power at most $t\cdot(c_1+c_2\|uv\|^2)$. If it does, then $U=i$, otherwise, then $L=i$.
6: **until** $U=L$

based LMST) which is the local minimum spanning tree where the weight of each edge uv is $IC(uv)$ as defined in Section II-B. The I-LMST provides answers close to optimum in random graphs and we will study the effectiveness of I-LMST in Section VII. The second structure is I-RNG. Note that interference based topologies introduced here, such as I-MST, I-LMST, and I-RNG can be built without any geometry information of the nodes while the Euclidean based topologies require location service.

Algorithm 5 Interference Based One-Hop Local MST

1: Every node u assigns to each link uv a weight $IC(uv)$ and then broadcasts $IC(uv)$ to its one-hop neighbors.
2: Every node u collects the weight information $I(v_1v_2)$, where v_1 and v_2 are its one-hop neighbors.
3: Every node u computes the minimum spanning tree $MST(N_1(u))$ of its neighbors $N_1(u)$, including u itself. The weight of each link xy is $IC(xy)$ here.
4: For every link $uv\in MST(N_1(u))$, node u sends a message $Propose(u,v)$ to node v informing node v about the existence of edge uv in $MST(N_1(u))$
5: If node u receives the message $Propose(v,u)$ and $uv\in MST(N_1(u))$ then u adds node v to the list of its neighbors in final topology $LMST_1^-$. In other words, if $uv\in MST(N_1(u))$ and $uv\in MST(N_1(v))$ then uv belongs to I-LMST$_1^-$.

Theorem 6: I-LMST$_1^-$ contains MST and thus is connected.

Proof: Consider node u building minimum spanning tree locally. Assume node v is a neighbor of u. It suffices to show that: $uv\notin MST(N_1(u))\Rightarrow uv\notin MST$. If $uv\notin MST(u)$ then there is a path from u to v in the neighborhood of u and the weight of uv is more than the weight of every edge in the path. Now we have a cycle with uv being the longest edge of this cycle, thus uv does not belong to the global MST. ■

Obviously, to build LMST, some communications are needed to collect the interference numbers of links v_1v_2 for all pairs of one-hop neighbors of u. We then introduce another simple localized topology called I-RNG (Interference-based RNG), that uses less communications but with more links in the topology. Like I-LMST, the weight of each link uv in I-RNG is $IC(uv)$ and the topology is defined analog to the traditional RNG: a link uv is removed if there is a node w such that $IC(uv)>IC(uw)$ and $IC(uv)>IC(vw)$. We will study the effectiveness of I-RNG in Section VII.

Algorithm 6 Interference Based Relative Neighborhood Graph

1: Every node u computes the interference number $IC(uv)$ for each of its one-hop neighbors.
2: Node u removes a link uv if there is a node w such that $rank(uv)>rank(uw)$ and $rank(uv)>rank(vw)$. Here the rank of a link xy is defined as $rank(xy)=(IC(xy),\max(x,y),\min(x,y))$.
3: All other links kept form the final topology.

VI. Performances on Randomly Deployed Networks

In the previous sections, we studied how to design topologies with low maximum or average interferences in the worst case. Worst case performance analysis provides us the insight how bad these methods could behave. However, the worst case does happen rarely in practice. Another important performance analysis is average performances analysis, which gives us insight how a structure will perform generally. In this section, we will show that the most commonly used structures in the literature could have arbitrarily large maximum node interferences, but their average interferences are often bounded by a small constant.

For average performance analysis, we consider a set of wireless nodes distributed in a two-dimensional unit square region. The nodes are distributed according to either the uniform random point process or homogeneous Poisson process. A point set process is said to be a *uniform random point process*, denoted by $\mathcal{X}_n$, in a region Ω if it consists of n independent points each of which is uniformly and randomly distributed over Ω. The standard probabilistic model of *homogeneous Poisson process* is characterized by the property that the number of nodes in a region is a random variable depending only on the area (or volume in higher dimensions) of the region. In other words,

- The probability that there are exactly k nodes appearing in any region Ψ of area A is $\frac{(\lambda A)^k}{k!}\cdot e^{-\lambda A}$.
- For any region Ψ, the conditional distribution of nodes in Ψ given that exactly k nodes in the region is *joint uniform*.

Given a set V of wireless nodes, several structures (such as relative neighborhood graph RNG, Gabriel graph GG, Yao structure, etc) have been proposed for topology control in wireless ad hoc networks. The *relative neighborhood graph*, denoted by $RNG(V)$, consists of all edges uv such that the intersection of two circles centered at u and v and with radius $\|uv\|$ do not contain any node w from the set V. The *Gabriel graph* [18] $GG(V)$ contains an edge uv if and only if the disk using link uv as diameter, denoted by $disk(u,v)$, contains no other nodes of V. We will study the expected maximum node interference and the expected average node interference for

Euclidean Minimum Spanning Tree (EMST), Gabriel Graph (GG) and the Relative Neighborhood Graph (RNG). The proof of the following theorems and lemmas are succinct and details are omitted due to space limit.

Theorem 7: For a set of nodes produced by a Poisson point process with density n, the expected maximum node interferences (thus link interferences) of EMST, GG, RNG and Yao structures are at least $\Theta(\log n)$.

Proof: Let d_n be the longest edge of the Euclidean minimum spanning tree of n points placed independently in 2-dimensions according to standard poisson distribution with density n. In [19], they showed that $\lim_{n\to\infty} P_r(n\pi d_n^2 - \log n \le \alpha) = e^{-e^{-\alpha}}$. Notice that the probability $P_r(n\pi d_n^2 - \log n \le \log n)$ will be sufficiently close to 1 when n goes to infinity, while the probability $P_r(n\pi d_n^2 - \log n \le -\log\log n)$ will be sufficiently close to 0 when n goes to infinity. That is to say, with high probability, $n\pi d_n^2$ is in the range of $[\log n - \log\log n, 2\log n]$.

Given a region with area A, let $m(A)$ denote the number of nodes inside this region by a Poisson point process with density δ. Then $P_r(m(A) = k) = \frac{e^{-\delta A}(\delta A)^k}{k!}$. It is well-known that the expected number of nodes lying inside a region with area A is δA. For a Poisson process with density n, let uv be the longest edge of the Euclidean minimum spanning tree, and $d_n = \|uv\|$. Then, the expected number of nodes that fall inside $\mathcal{D}(u, d_n)$ is $E(m(\pi d_n^2)) = n\pi d_n^2$, which is larger than $\log n$ almost surely, when n goes to infinity. That is to say, the expected maximum interference of Euclidean MST is $\Theta(\log n)$ for a set of nodes produced according to a Poisson point process. Consequently, the expected maximum node interference of any structure containing EMST is at least $\Omega(\log n)$. Thus, the expected maximum node interference of GG, RNG and Yao are at least $\Omega(\log n)$. ■

The above theorem shows that all commonly used structures for topology control in wireless ad hoc networks generally have a large maximum node interference even for *randomly* deployed nodes. Our following analysis will show that the average interference of all nodes of these structures is small.

Theorem 8: For a set of nodes produced by a Poisson point process with density n, the expected average node interferences (thus link interferences) of EMST and RNG are bounded from above by some constants.

Proof: Consider a set V of wireless nodes produced by Poisson point process. Given a structure G, let $I_G(u_i)$ be the node interference caused by a node u_i, *i.e.*, the number of nodes inside the transmission region of node u_i. Here the transmission region of node u_i is a disk centered at u_i whose radius is the length r_i of the longest incident links of G at node u_i. Hence, the expected average node interference is

$$E(\frac{\sum_{i=1}^n I_G(u_i)}{n}) = \frac{1}{n}E(\sum_{i=1}^n I_G(u_i)) = \frac{1}{n}\sum_{i=1}^n E(I_G(u_i))$$
$$= \frac{1}{n}\sum_{i=1}^n E(m(\pi r_i^2)) = \frac{1}{n}\sum_{i=1}^n E((n\pi r_i^2)) = \sum_{i=1}^n E(\pi r_i^2)$$
$$\le 2\sum_{e_i\in G} E(\pi e_i^2).$$

The last inequality follows from the fact that r_i is the length of some edge in G and each edge in G can be used by at most two nodes to define its radius r_i.

Let e_i, $1 \le i \le n-1$ be the length of all edges of the EMST of any n points inside a unit disk. It was proven in [20] that $\sum_{e_i\in EMST} e_i^2 \le 12$. Thus, the expected average node interference of EMST is

$$E(\frac{\sum_{i=1}^n I_{EMST}(u_i)}{n}) \le 2\sum_{e_i\in EMST} E(\pi e_i^2) \le 24\pi.$$

For RNG graph, similar to the proof of [20], we can show that $\sum_{e_i\in RNG} e_i^2 \le 8\pi/\sqrt{3}$. This implies that $E(\frac{\sum_{i=1}^n I_{RNG}(u_i)}{n}) \le 2\sum_{e_i\in RNG} E(\pi e_i^2) \le 16\pi^2/\sqrt{3}$. ■

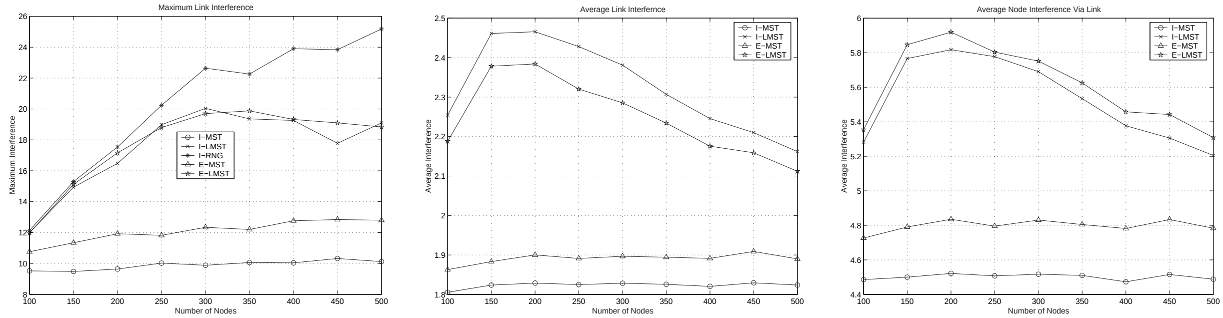
VII. Simulation Studies

In our simulations, the network is modeled by unit disk graph (although our algorithms work for any graph model). We put different numbers of nodes that are randomly placed in a $250m \times 250m$ square region and the maximum transmission range of each node is set to $35m$. Since this is the first that studies interference aware topology control, except for MMLIP which had been introduced in [1], we compared our algorithm with the well-known topologies like MST and RNG. It is known that MST cannot be built in a localized manner so it is not suitable for wireless ad hoc environment, but fortunately there is a localized version of MST (so-called LMST) available. We also considered with RNG topology, since it can be built inexpensively and locally. Traditional RNG, LMST [8], [13] are based on Euclidean distance between nodes and might not be suitable for low interference. Thus in Section V-B we defined slightly different topologies called I-RNG and I-LMST. To distinguish between the Euclidean-based and Interference-based topologies we call the former topologies E-LMST and E-RNG where “E” stands for “Euclidean”.

We first studied the performances of various structures in terms of link interference. Figure 4(a) compares the performance of I-MST and E-MST and also their localized versions I-LMST, E-LMST. We also considered I-RNG. Although it does not perform well, it uses much less communications than I-LMST. Note that I-LMST does not always give results better than E-LMST. When the required property is connectivity, we found that E-MST gives answers slightly worse than I-MST. The localized version of these two topologies, I-LMST and E-LMST perform slightly worse than the centralized versions but they are more suitable for wireless ad hoc network environment. See Figure 4(b) for an illustration.

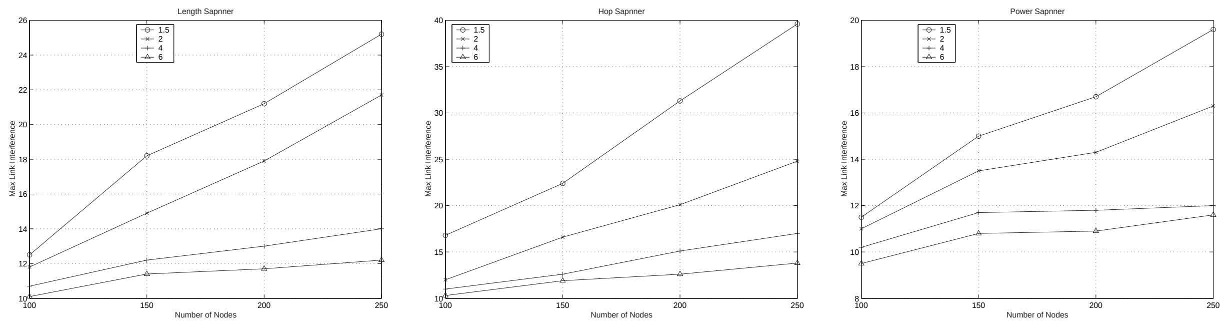
Then we study the performance of the optimum structures when different spanning ratio requirements are posted. Our simulation results are plotted in Figure 5. A critical observation is that the maximum interference does increase with the increasing of network density as we showed theoretically.

We then studied the nodal interference derived from all its incident communication links. In Figure 4(c) the performances of different topologies for MANILP problem when the required property is connectivity are compared. I-MST is a 2-approximation, E-MST performs slightly worse than I-MST and as we expected the localized versions of there two



(a) MMLIP problem (b) MALIP Problem (c) MANILP Problem

Fig. 4. Performances of various structures for a number of link-interference related problems.



(a) length spanner (b) hop spanner (c) power spanner

Fig. 5. Minimize the maximum link interference with different spanning ratio requirements.

topologies perform poorer than the centralized versions due to the fact that they do not use global information.

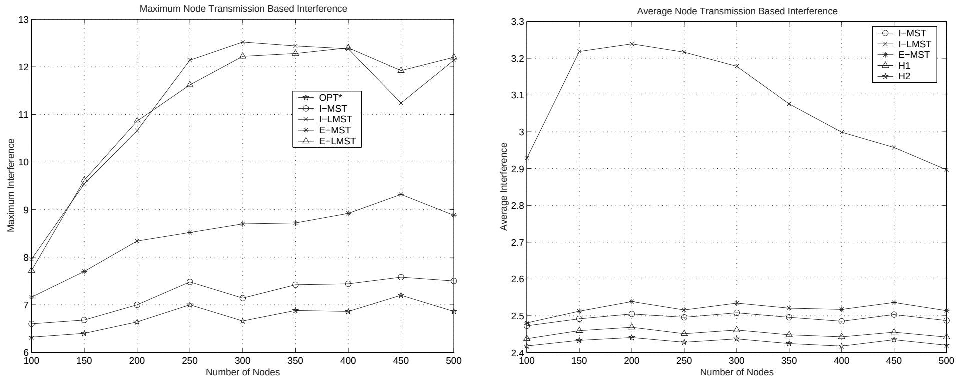
At last, we studied the performances of structures in terms sender-centric interference. First we consider the MMNISP problem when the required property is connectivity. As mentioned in Section IV-B.1, Algorithm 3 gives the optimum answer, but this algorithm is centralized, thus it is not suitable for wireless ad hoc environment. Here we compared it with localized algorithms and also some other centralized topologies as shown in Figure 6(a).

Although I-MST does not give the optimum answer, but it performs fairly well and again E-MST performs not as good as I-MST. The localized versions of these two topologies are also drawn. Note that I-MST, E-MST, I-LMST, and E-LMST are all based on link interference (*i.e.*, the weight of each link is the interference of that link) and not node transmission based interference. The reason that these topologies still provide good results compared to the optimum solution is the fact that they choose small edges. Then we study the performances of the optimum structures when different spanning ratio requirements are posted. Our simulation results are plotted in Figure 7.

For MANISP problem, we gave localized heuristics in Section IV-B.2. Figures 6(b), 6(c) compare our heuristic with some other localized topologies. Two heuristics $H1$ and $H2$ perform better than other topologies studied in this paper. See Figure 6(b) for an illustration. We also noticed that $H2$ performs slightly better than $H1$, I-MST and I-LMST still perform reasonably good and I-LMST, as we anticipated, performs poorer than I-MST.

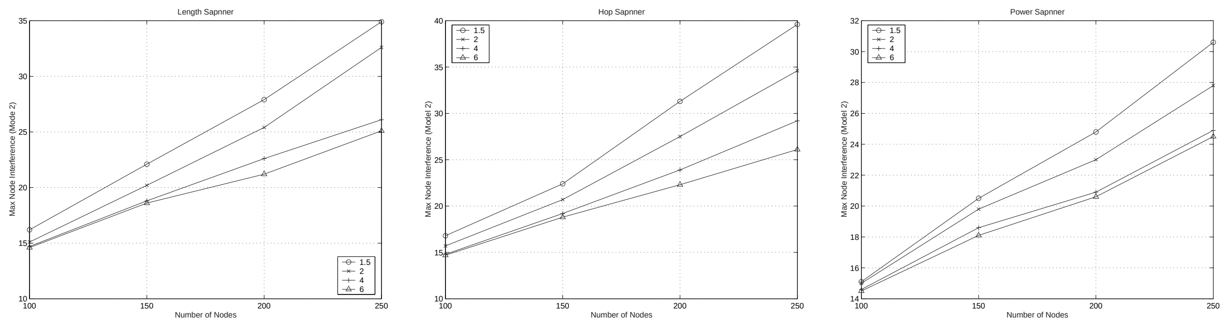
VIII. Conclusion and Future Work

Topology control draw considerable attentions recently in wireless ad hoc networks for energy conservation. In this paper, we studied interference-aware topology control by studying the inherent interference quality of a structure. We optimally solved some problems, gave approximation algorithms for some NP-hard questions, and also gave some efficient heuristics for some questions that seem to be NP-hard. We conducted extensive simulations to see how these new structures perform for random wireless networks. We also theoretically showed that the most commonly used localized structures in the literature have large maximum interference even for random networks. On the other hand, we showed that the Euclidean-LMST, LMST and RNG have a constant bounded average interference ratio for randomly deployed networks. This is just the first step of designing the interference-aware topology. There are many challenging questions left for future researches. In this paper, we proposed several definitions of interference. The ultimate goal of any method will be to increase the throughput, or to decrease the delay and packet loss rate etc. of the network while decreasing the energy consumption. Then what structure is better in practice? And what definition of interference is more proper for maximizing the network throughput? Along this direction, we may need new definitions of interference. One promising definition would be link-based interference: the interference number of a link uv in a topology H is the number of links in H that will be interfered by the communication of uv. A structure with small link-based interference may imply that more simultaneous communications can co-exist in the



(a) MMNISP problem (b) MMNISP Problem

Fig. 6. Performances of various structures for a number of node-interference related problems.



(a) length spanner (b) hop spanner (c) power spanner

Fig. 7. Minimize the maximum node interference with different spanning ratio requirements.

network and thus increase the network throughput. We are currently performing experiments on studying the practical performances of different interference-aware structures and exploring the possible new criteria for measuring the interference of a given structure. To best study these, we need a cross-layer design since the ultimate performance of the network depends on many aspects such as the routing method, the scheduling method, the topology used for routing, the power management techniques and so on.

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