Multicast Capacity Scaling Laws for Multihop Cognitive Networks

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Abstract—In this paper, we study multicast capacity for *cognitive networks*. We consider the cognitive network model consisting of two overlapping ad hoc networks, called the primary ad hoc network (PaN) and secondary ad hoc network (SaN), respectively. PaN and SaN operate on the same space and spectrum. For PaN (or SaN, respectively), we assume that primary (or secondary, respectively) nodes are placed according to a Poisson point process of intensity n (or m, respectively) over a unit square region. We randomly choose n_s (or m_s, respectively) nodes as the sources of multicast sessions in PaN (or SaN, respectively), and for each primary source v^p (or secondary source v^s, respectively), we pick uniformly at random n_d primary nodes (or m_d secondary nodes, respectively) as the destinations of v^p (or v^s, respectively). Above all, we assume that PaN can adopt the optimal protocol in terms of the throughput. Our main work is to design the multicast strategy for SaN by which the optimal throughput can be achieved, without any negative impact on the throughput for PaN in order sense. Depending on n_d and n, we choose the optimal one for PaN from two strategies called *percolation strategy* and *connectivity strategy*, respectively. Subsequently, we design the corresponding throughput-optimal strategy for SaN. We derive the regimes in terms of n, n_d, m, and m_d in which the upper bounds on multicast capacities for PaN and SaN can be achieved simultaneously. *Unicast* and *broadcast* capacities for the cognitive network can be derived by our results as the special cases by letting $n_d = 1$ (or $m_d = 1$) and $n_d = n - 1$ (or $m_d = m - 1$), respectively, which enhances the generality of this work.

Index Terms—Cognitive networks, wireless ad hoc networks, multicast capacity, random networks, percolation theory

1 Introduction

The demand for bandwidth is increasing sharply, while a large portion of the assigned spectrum is used sporadically and geographical variations in the utilization of assigned spectrum ranges from 15 to 85 percent with a high variance in time [1], [2]. A solution to the issue that the limited available spectrum coexists with the inefficiency in the spectrum usage is to permit some users to exploit the wireless spectrum opportunistically without having a negative impact on licensed users. Thus, a new communication paradigm, i.e., *cognitive network*, has been proposed. A cognitive network generally consists of two independent overlapping networks, called the *primary network* and *secondary network*, which operate at the same time, space and frequency. The secondary users equipped with cognitive radios are able to sense the idle spectrum and obtain necessary information about primary users [3], [4].

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In this paper, we study the capacity scaling laws of the cognitive network consisting of the primary ad hoc network (PaN) and the secondary ad hoc network (SaN). We directly derive the multicast capacity that unifies results on unicast and broadcast capacity [5], [15]. An important constraint for a cognitive network model is that the primary network does not alter its protocol due to the secondary network anyway. Otherwise, a simple equal time-sharing can get the same order of throughput for both networks as they are standalone, which makes the problem trivial [6], [7]. Under this constraint, for our cognitive network model consisting of the primary ad hoc network and secondary network, a challenging issue is how to design a communication strategy for SaN, corresponding to a determined scheme for PaN, by which SaN can achieve the optimal throughput without any negative impact on the throughput for PaN. In this work, we focus on solving this question for the dense scaling case, [5], [6], [7], [8], [9], [10], [11].

We assume that PaN adopts the optimal scheme in terms of network throughput. To the best of our knowledge, the optimal-throughput multicast schemes for random dense networks were proposed in [5], [10], and [12]. These schemes are based on hierarchical backbones, consisting of *highways* and *connectivity paths*; all senders transmit signals with a constant power P. We prove that the optimal throughput derived in [5] can be achieved by a similar scheme under which senders transmit with a smaller power then the total cost of energy decreases significantly. We assume that PaN adopts such scheme. Then, we aim to design the optimal-throughput scheme for SaN. Corresponding to the scheme for PaN, we propose two types of multicast strategies for SaN. The first one is the *percolation strategy* under which the routing scheme is constructed based on the *highways* system [8], [13]. The second one is the *connectivity strategy* based on the *connectivity paths* (also called second-class highways in [14]) along which the length of each link is of the minimum order to ensure *global connectivity*. The key solution is to set a *preservation region* (P-R) for every primary node in PaN, [6], [7]. The main difference between the strategies for SaN and PaN is that the preservation regions cannot be passed through by any transmission in SaN. Then, two main technical challenges of the specific designing of multicast strategies for SaN arise as follows: 1) How large of P-Rs are optimal with respect to the capacity for both SaN and PaN? 2) How to build routing backbones for SaN in presence of those P-Rs? For both networks, there are some thresholds on the number of destinations for each multicast session, below which the percolation strategy performs better than the connectivity strategy. Based on those thresholds, we use the two types of strategies cooperatively, and derive the corresponding throughput. In our model, nodes in PaN and SaN are distributed independently according to a Poisson point process (p.p.p.) of intensity n and m, respectively, over a unit square. Let n_d and m_d denote the number of destinations of each multicast session in PaN and SaN, respectively, we show that: when $m_d = \omega(\log m)$ and $n = o(\frac{m}{\log m})$, or when $m_d = O(\log m)$ and $n = o(\frac{m}{m_d \cdot \log m})$, for some cases in terms of n_d and m_d, SaN can achieve the upper bound of throughput as it is stand-alone (Theorem 2), without any negative impact on PaN in order sense. Due to the nature of the wireless medium, both the primary and secondary networks inevitably have a negative impact (interference) on each other under the noncooperative communication scenario as long as they share the same spectrum in the same time, although the communications of secondary users are probably nondestructive to the communications of primary users. Thus, the upper bound on the capacity for SaN is no more than that for a stand-alone network isomorphic to SaN. Combining the upper and lower bounds, we obtain the tight capacity bounds for SaN in some regimes.

The rest of the paper is organized as follows: In Section 2, we introduce the system model. The main results are presented in Section 3. In Section 4, we make technical preparations for the following strategy design and capacity analysis. In Section 5, we propose the multicast strategies for PaN and SaN. In Section 6, we derive the achievable multicast throughput and prove the main results. We review the related work in Section 7, and conclude the paper in Section 8.

2 System Model

Throughout the paper, we mainly consider events that happen with high probability (*w.h.p.*) as the scale of network (the number of users in the network) goes to infinity.

2.1 Network Topology

The ad hoc nodes in PaN and SaN are distributed according to Poisson point processes (p.p.p.) of intensity n and m, respectively, over a unit square $\mathcal{A} = [0,1]^2$, i.e., we consider the *dense network model* [5], [6], [7], [8], [9], [10]. Denote the sets of all primary ad hoc nodes and secondary ad hoc nodes by $\mathcal{V}^p(n)$ and $\mathcal{V}^s(m)$, respectively. From Chebychev's inequality, we can easily obtain, *w.h.p.*, the number of primary nodes (or secondary nodes, respectively), i.e., $|\mathcal{V}^p(n)|$ (or $|\mathcal{V}^s(m)|$, respectively), is within $[(1-\varepsilon)n, (1+\varepsilon)n]$ (or $[(1-\varepsilon)m, (1+\varepsilon)m]$, respectively). To simplify the description, we assume that $|\mathcal{V}^p(n)| = n$ and $|\mathcal{V}^s(m)| = m$, respectively, which does not change the results due to the characteristic of the scaling laws issue [8].

2.2 Assurance of Priority for Primary Networks

The most important constraint for a cognitive network model is that the primary network has an overwhelming priority to access the spectrum. To be specific, we introduce constraints in cognitive network model as following.

Primary Constraint of Cognitive Network Model:

1. PaN operates as if SaN were absent. That is, PaN does not alter its protocol due to SaN anyway.
2. The negative impact on PaN imposed by SaN, e.g., throughput decrement, cannot be beyond the tolerance limit of PaN.

If the first primary constraint above does not hold, a simple equal time-sharing can get the same order of throughput for both PaN and SaN as they are stand-alone, which makes the problem trivial [6], [7]. For the second primary constraint above, we assume that PaN permits SaN operating as long as SaN does not change the order of throughput for PaN, since we only care the capacity scaling laws in this work. We state that it is not sufficient to maintain practically the priority of primary networks. For example, consider a case where the throughput of PaN reduces by 30 percent due to the presence of SaN. If PaN can only tolerate at most 20 percent decrement of the throughput, then the priority of primary users is indeed not respected, although the throughput for PaN is not decreased in order sense. In the future work, we should define some parameters determined by primary users to limit the involvement of SaN and guarantee the priority of PaN.

2.3 Cognitive Functions of Secondary Nodes

We assume that secondary users are equipped with cognitive radios, and able to obtain necessary information about PaN. Specifically, we provide the following assumption.

Assumption A. *For PaN and SaN, we assume that*

1. *Secondary nodes know the locations of primary nodes.*
2. *SaN knows what protocol is running in PaN.*

2.4 Communication Model

We assume that both PaN and SaN operate under the TDMA scheme. Let $\mathcal{V}^p_\tau$ (or $\mathcal{V}^s_\tau$, respectively) denote the set of the primary (or secondary, respectively) ad hoc nodes scheduled at time slot τ. Then, during a time slot τ, any link $v_i \to v_j$, $v_i, v_j \in \mathcal{V}^p_\tau \cup \mathcal{V}^s_\tau$, can communicate via a direct link, over a channel with bandwidth B, of rate $\mathrm{R}_\tau(v_i, v_j) = B\log(1 + \frac{\mathrm{S}_\tau(v_i,v_j)}{N_0 + \mathrm{I}_\tau(v_i,v_j)})$, where $N_0 \geq 0$ is the ambient noise, $\mathrm{S}_\tau(v_i, v_j)$ is the strength of the signal initiated by v_i at the receiver v_j, $\mathrm{I}_\tau(v_i, v_j)$ is the sum interference on v_j produced by all nodes belong to $\mathcal{V}^p_\tau \cup \mathcal{V}^s_\tau - \{v_i\}$.

The wireless propagation channel typically includes path loss with distance, shadowing and fading effects, etc. In this paper, as in [6], [7], [8], and [13], we assume that the channel gain depends only on the distance between the transmitter and its receiver, and ignore shadowing and fading. The channel power gain is given by $\ell(v_i, v_j) = (d(v_i, v_j))^{-\alpha}$, where $d(v_i, v_j) = \|v_i v_j\|$ is the euclidean distance between the two nodes v_i and v_j, $\alpha > 2$ is the power attenuation exponent. Then,

$$\mathrm{S}_\tau(v_i, v_j) = P_\tau(v_i) \cdot \|v_i v_j\|^{-\alpha},$$
$$\mathrm{I}_\tau(v_i, v_j) = \sum_{v_k \in \mathcal{V}_\tau^p \cup \mathcal{V}_\tau^s - \{v_i\}} P_\tau(v_k) \cdot \|v_k v_j\|^{-\alpha},$$

where $P_\tau(v_i)$ denotes the transmission power of v_i when it is scheduled. Note that our results hold as long as *near field effects of electromagnetic propagation* can be neglected [8].

2.5 Capacity Definition

We generalize the formal definition of multicast capacity based on that in [15]. Let $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ denote the set of all nodes in the network and the subset $\mathcal{S} \subseteq \mathcal{V}$ denote the set of source nodes of multicast. Denote the number of multicast sessions as $|\mathcal{S}| = n_s$. For each source $v_{\mathcal{S},i} \in \mathcal{S}$, we uniformly choose n_d nodes at random from other nodes to construct the set of destinations, denoted by $\mathcal{D}_{\mathcal{S},i} = \{v_{\mathcal{S},i_1}, v_{\mathcal{S},i_2}, \dots, v_{\mathcal{S},i_{n_d}}\}$, where obviously $n_d \le n-1$. We call $\mathcal{U}_{\mathcal{S},i} = \{v_{\mathcal{S},i}\} \cup \mathcal{D}_{\mathcal{S},i}$ the *spanning set* of multicast session $\mathcal{M}_{\mathcal{S},i}$. Denote $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \dots, \lambda_{\mathcal{S},n_s})$ as a *rate vector* of the multicast data rate of all multicast sessions.

Definition 1 (Feasible Rate Vector). *A multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \dots, \lambda_{\mathcal{S},n_s})$ is called (ρ_s, ρ_d)-*feasible*, where ρ_s and ρ_d are both constants in $[0, 1]$, if for a subset of sources, denoted by $\mathcal{S}'(\rho_s, \rho_d) \subseteq \mathcal{S}$ with the cardinality $|\mathcal{S}'(\rho_s, \rho_d)| = \rho_s(n) \cdot n_s$, there exists a spatial and temporal scheme for scheduling transmissions by which every source $v_{\mathcal{S},i} \in \mathcal{S}'(\rho_s, \rho_d)$ can deliver data to at least $\rho_d(n, i) \cdot n_d$ destinations at a rate of $\lambda_{\mathcal{S},i}$. That is, there is a $T < \infty$ such that in every time interval (with unit seconds) $[(\tau-1)\cdot T, \tau \cdot T]$, every node $v_{\mathcal{S},i} \in \mathcal{S}'(\rho_s, \rho_d)$ can send $T \cdot \lambda_{\mathcal{S},i}$ bits to at least $\rho_d(n, i) \cdot n_d$ destinations, where*

$$\lim_{n\to\infty} \rho_s(n) = \rho_s; \quad \lim_{n\to\infty} \inf_{v_{\mathcal{S},i} \in \mathcal{S}'(\rho_s,\rho_d)} \{\rho_d(n, i)\} = \rho_d.$$

A multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \lambda_{\mathcal{S},2}, \dots, \lambda_{\mathcal{S},n_s})$ is called feasible *if it is $(1, 1)$-*feasible.

Based on a *multicast rate vector*, we can define two representative types of multicast throughput (MT).

- Aggregated multicast throughput (AMT): $\Lambda^{\mathrm{A}}_{\mathcal{S},n_d}(n) = \sum_{v_{\mathcal{S},i} \in \mathcal{S}'(1,1)} \lambda_{\mathcal{S},i}$.
- Per-session multicast throughput (PMT): $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = \min_{v_{\mathcal{S},i} \in \mathcal{S}'(1,1)} \lambda_{\mathcal{S},i}$.

Correspondingly, we define two types of *achievable* multicast throughput based on the *feasible rate vector*.

Definition 2. *AMT $\Lambda^{\mathrm{A}}_{\mathcal{S},n_d}(n)$ (or PMT $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n)$) is* achievable *if the corresponding multicast rate vector $\Lambda_{\mathcal{S},n_d} = (\lambda_{\mathcal{S},1}, \dots, \lambda_{\mathcal{S},n_s})$ is* feasible.

Now, we define the multicast capacity of random networks.

Definition 3 (Multicast Capacity of Random Networks). *The per-session multicast capacity of a class of random networks is of order $\Theta(g(n))$ bits/sec if there are deterministic constants $c > 0$ and $c < c' < +\infty$ such that*

$$\lim_{n\to\infty} \Pr\left(\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = c \cdot g(n) \text{ is achievable}\right) = 1,$$

$$\liminf_{n\to\infty} \Pr\left(\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n) = c' \cdot g(n) \text{ is achievable}\right) < 1.$$

Similarly, we can define the aggregated multicast capacity for random networks. For the relation between AMT and PMT, it holds that AMT $\Lambda^{\mathrm{A}}_{\mathcal{S},n_d}(n) = |\mathcal{S}'(1,1)| \cdot \Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n)$ is achievable if PMT $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n)$ is achievable. According to Definitions 1 and 2, the cardinality of $\mathcal{S}'(1,1)$ follows that $\lim_{n\to\infty} \frac{|\mathcal{S}'(1,1)|}{n_s} = 1$. Therefore, in the following content, AMT $\Lambda^{\mathrm{A}}_{\mathcal{S},n_d}(n)$ is always achievable at the order of $\Theta(n_s \cdot \Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n))$ when PMT $\Lambda^{\mathrm{P}}_{\mathcal{S},n_d}(n)$ is achievable.

As in most related work, we assume that the number of multicast sessions in PaN (SaN, respectively) is $\Theta(n)$ ($\Theta(m)$, respectively).

TABLE 1
Achievable Per-Session Multicast Throughput

	n_d:$[1, \frac{n}{(\log n)^2}]$	n_d:$[\frac{n}{(\log n)^2}, n]$
m_d:$[1, \frac{m}{(\log m)^2}]$	PaN, P-S, $\mathbf{f}_1(n, n_d)$	PaN, C-S, $\mathbf{f}_2(n, n_d)$
	SaN, P-S, $\mathbf{f}_1(m, m_d)$	SaN, C-S, $\mathbf{f}_2(m, m_d)$
m_d:$[\frac{m}{(\log m)^2}, m]$	PaN, P-S, $\mathbf{f}_1(n, n_d)$	PaN, C-S, $\mathbf{f}_2(n, n_d)$
	SaN, C-S, $\mathbf{f}_2(m, m_d)$	SaN, C-S, $\mathbf{f}_2(m, m_d)$

3 Main Results

Before presenting the main results, we define two functions with positive integer domains as follows:

$$\mathbf{f}_1(x, y) = \begin{cases} \Omega\left(\frac{1}{\sqrt{xy}}\right) & \text{when } y\text{:}\left[1, \frac{x}{(\log x)^3}\right], \\ \Omega\left(\frac{1}{y} \cdot (\log x)^{\frac{3}{2}}\right) & \text{when } y\text{:}\left[\frac{x}{(\log x)^3}, x\right], \end{cases}$$

$$\mathbf{f}_2(x, y) = \begin{cases} \Omega\left(\frac{1}{\sqrt{xy\log x}}\right) & \text{when } y\text{:}\left[1, \frac{x}{\log x}\right], \\ \Omega(1/x) & \text{when } y\text{:}\left[\frac{x}{\log x}, x\right], \end{cases}$$

where the expression $y(x)$:$[y_1(x), y_2(x)]$ means that

$$y(x) = \Omega(y_1(x)) \text{ and } y(x) = O(y_2(n));$$

the expression $y(x)$:$(y_1(x), y_2(x)]$ represents that

$$y(x) = \Omega(y_1(n)) \text{ and } y(n) = O(y_2(n)).$$

For PaN, we adopt two alternative schemes similar to those in [5], i.e., *percolation strategy* (P-S) and the *connectivity strategy* (C-S). Correspondingly, we design the specific percolation and connectivity strategies for SaN. We make a decision of choosing the better strategy according to the value of n_d and m_d as described in Table 1.

Assumption B. *For m, m_d, and n,*

1. *when $m_d = \omega(\log m)$, we assume that $n = o(\frac{m}{\log m})$.*
2. *when $m_d = O(\log m)$, we assume that $n = o(\frac{m}{m_d \cdot \log m})$.*

TABLE 2
Regimes Where the Capacity Bounds for SaN Are Not Tight

	Regimes of m_d where there are gaps on capacities
n_d:$[1, \frac{n}{(\log n)^2}]$	m_d:$(\frac{m}{(\log m)^3}, \frac{m}{\log m})$
n_d:$(\frac{n}{(\log n)^2}, n]$	m_d:$[1, \frac{m}{\log m})$

Theorem 1. *Under Assumption B, the multicast throughputs for PaN and SaN can be achieved as described in Table 1.*

From Theorem 1, we ensure that the presence of SaN does not change the throughput for PaN in order sense. Combining with the upper bounds of multicast capacity for random dense networks proposed in [5] and [10] (described by Lemma C in the Appendix, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TMC.2011.212), we get the tight bounds of multicast capacity for SaN for some regimes according to the specific values of n, n_d, m, and m_d.

Theorem 2. *The per-session multicast capacity for SaN is of* $\mathrm{C}_s(m)$.

When n_d:$[1, n/(\log n)^2]$,

$$\mathrm{C}_s(m) = \begin{cases} \Theta(1/\sqrt{mm_d}) & \text{when } m_d\text{:}[1, m/(\log m)^3], \\ \Theta(1/m) & \text{when } m_d\text{:}[m/\log m, m]. \end{cases} \quad (1)$$

When n_d:$[n/(\log n)^2, n]$,

$$\mathrm{C}_s(m) = \Theta(1/m) \quad \text{when } m_d\text{:}[m/\log m, m]. \quad (2)$$

Note that in Theorem 2, as in PaN, the regimes of m_d do not cover the whole region $[1, m]$. In those unmentioned regimes, there are still gaps between the upper and lower bounds. Please see the details in Table 2. A challenging issue is to close those gaps by presenting possibly new tighter upper and lower bounds by using some new arguments or designing new multicast strategies.

4 Percolation Model

4.1 Poisson Boolean Percolation Model

In 2D Poisson Boolean model $\mathcal{B}(\lambda, r)$ [16], nodes are distributed according to a p.p.p of intensity λ in 2D euclidean space $\mathbb{R}^2$. Each node is associated to a closed disk of radius $r/2$. Two disks are *directly connected* if they overlap. This setting corresponds to the case where the transmission radius of every node is r. Two disks are *connected* if there exists a sequence of directly connected disks between them. Define a *cluster* as a set of disks in which any two disks are connected. Denote the set of all clusters by $\mathcal{C}(\lambda, r)$. Define the number of disks in the cluster $C_i \in \mathcal{C}(\lambda, r)$ as a random variable $\mathrm{N}(C_i)$. We can associate $\mathcal{B}(\lambda, r)$ to a graph $\mathcal{G}(\lambda, r)$, called *associated graph*, by associating a vertex of $\mathcal{G}(\lambda, r)$ to a node of $\mathcal{B}(\lambda, r)$ and associating an edge of $\mathcal{G}(\lambda, r)$ to a direct connection in $\mathcal{B}(\lambda, r)$. The two models, $\mathcal{B}(\lambda, r)$ and $\mathcal{B}(\lambda_0, r_0)$, lead to a same associated graph, namely $\mathcal{G}(\lambda, r) = \mathcal{G}(\lambda_0, r_0)$ if $\lambda_0 \cdot {r_0}^2 = \lambda \cdot r^2$. Then, the graph properties of $\mathcal{B}(\lambda, r)$ depend only on the parameter $\lambda \cdot r^2$ [9]. The *percolation probability*, denoted by p, is one that a given node belongs to a cluster of an infinite number of nodes. Let C denote the cluster containing the given node, the percolation probability is thus defined as

$$\mathrm{p}(\lambda, r) = \mathrm{p}(\lambda r^2) = \Pr_{\lambda, r}(|C| = \infty) = \Pr_{\mathrm{p}}(|C| = \infty).$$

We call p_c the *critical percolation threshold* of Poisson Boolean model in $\mathbb{R}^2$ when $\mathrm{p}_c = (\lambda r^2)_c = \sup\{\lambda r^2 | \mathrm{p}(\lambda r^2) = 0\}$. The exact value of $(\lambda r^2)_c$ is not yet known. The best analytical results show that it is within $(0.19245, 0.843)$ [16], [17]. In our analysis, we use the following lemma.

Lemma 1 (Meester and Roy [16]). *For a Poisson Boolean model* $\mathcal{B}(\lambda, r)$ *in* $\mathbb{R}^2$, *if* $\lambda r^2 < \mathrm{p}_c$, *it holds that*

$$\Pr(\sup\{\mathrm{N}(C_i) | C_i \in \mathcal{C}(\lambda, r)\} < \infty) = 1,$$

where p_c *is the* critical percolation threshold *of Poisson Boolean model in* $\mathbb{R}^2$.

4.2 Bond Percolation Model

We mainly recall a result proposed in [8] that is to declare the existence of a cluster of nodes forming the *highway system* [8]. The result is derived based on the independent bond percolation model on the square lattice [18], where each edge (bond) of an infinite square grid is *open* with probability p and *closed* otherwise, independently of all other edges.

Let $\mathbb{B}(h, p)$ denote a box of side length h embedded in the square lattice. We call a path consisting of only open edges (bonds) *open path*. For a given $\kappa > 0$, we partition the lattice graph $\mathbb{B}(h, p)$ into horizontal (or vertical) rectangle slabs with the horizontal (or vertical) width of h and the vertical (or horizontal) width of $\kappa \log h - \epsilon(h)$, denoted by R_i^h (or R_i^v). Note that we can choose $\epsilon(h)$ as the smallest value such that the number of rectangle slabs $h/(\kappa \log h - \epsilon(h))$ is an integer. It is obvious that $\epsilon(h) = o(1)$ as $h \to \infty$ [8]. Denote the number of edge-disjoint *open paths*, which pass through the whole box, in slab R_i^h (or R_i^v) as N_i^h (or N_i^v). Let $\mathrm{N}^h = \min_i \mathrm{N}_i^h$, $\mathrm{N}^v = \min_i \mathrm{N}_i^v$. Then, we have the following lemma.

Lemma 2 ([8]). *For any constants* $\kappa > 0$ *and* $p \in (\frac{5}{6}, 1)$ *satisfying* $2 + \kappa \log(6(1-p)) < 0$, *there exists a constant* $\delta(\kappa, p)$ *depending on* κ *and* p *such that*

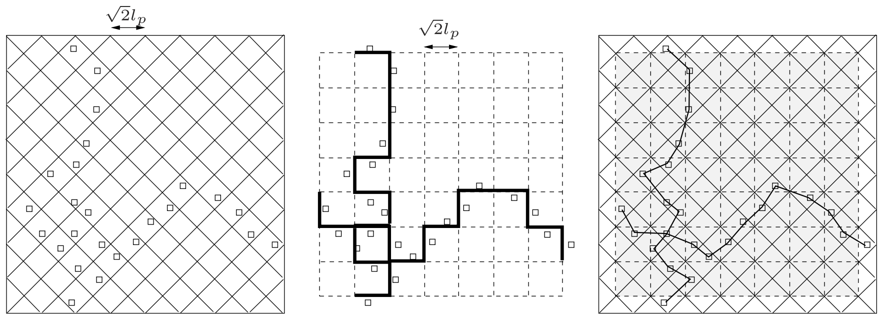
$$\lim_{h \to \infty} \Pr(\mathrm{N}^h \geq \delta \log h) = 1, \lim_{h \to \infty} \Pr(\mathrm{N}^v \geq \delta \log h) = 1.$$

5 Multicast Schemes

Due to the overwhelming priority of accessing the spectrum, PaN can operate as if no SaN were present. Hence, we can design the multicast strategy for PaN as a stand-alone ad hoc network. While, designing the strategy for SaN is a challenging issue. We should maximize the multicast throughput for SaN, ensuring no negative impact on the order of multicast throughput for PaN.

5.1 Schemes for Primary Network

We adopt two multicast strategies similar to those in [5], [10], called the *percolation strategy* and *connectivity strategy*, respectively. The final multicast throughput is derived by using cooperatively two types of strategies according to the specific value of n_d.



(a) Lattice $\mathbb{C}_p(h_p)$ (b) Open Paths (c) Primary Highways

Fig. 1. Construction of highways. (a) Lattice $\mathbb{C}_p(h_p)$. An inclined cell is *open* if it contains at least one node. (b) Open Paths. Two *open paths* are depicted using bold square lines. (c) Primary highways. Two *highways* are depicted by the polygonal chains.

5.1.1 Primary Percolation Strategy $\mathbb{M}_p$

The strategy $\mathbb{M}_p$ is devised based on two types of paths, i.e., *primary-percolation-paths* and *primary-connectivity-paths*.

Primary highway. We concisely introduce the construction of primary-percolation-paths, i.e., *primary highways* in [8], and analyze the density of the primary highways based on Lemma 2. We first partition the region $\mathcal{A}$ into subsquares of side length $l_p = \frac{c}{\sqrt{n}}$ as in Fig. 1a, where c is a constant, and call such subsquares *primary-percolation-cells*. Then there are h_p^2 subsquares, where $h_p = \lceil \sqrt{n}/\sqrt{2}c \rceil$ (we can adjust the value of c such that $\sqrt{n}/\sqrt{2}c$ is an integer). Denote the lattice graph produced by inclined lines by $\mathbb{C}_p(h_p)$ (see Fig. 1a). Let $\mathrm{N}(c_i)$ denote the number of Poisson points inside cell c_i. Thus, for all i, the probability that a square c_i contains at least one Poisson point ($\mathrm{N}(c_i) \geq 1$) is $\mathrm{p}_p \equiv 1 - e^{-c^2}$. We say a square is *open* if it contains at least one point, and *closed* otherwise. Then any square is open with probability p_p, independently from each other. Then, we can map this model into a discrete bond percolation model on the square grid. Draw a horizontal edge across half of the squares, and a vertical edge across the others, as shown in Fig. 1b, by which we obtain the lattice graph $\mathbb{B}(h_p, \mathrm{p}_p)$.

We say a given edge e in $\mathbb{B}(h_p, \mathrm{p}_p)$ is *open* if the inclined subquare in $\mathbb{C}_p(h_p)$, passed through by e, is open. Based on an open crossing path connecting the left side with the right one of $\mathbb{B}(h_p, \mathrm{p}_p)$ (or connecting the top side with the bottom one of $\mathbb{B}(h_p, \mathrm{p}_p)$), depicted in Fig. 1b. By choosing a node from each open cell in $\mathbb{C}_p(h_p)$ corresponding to each open edge of the open path and connect those nodes, we finally obtain a routing crossing path as in Fig. 1c. We call those nodes *primary-highways-stations* and call those crossing paths *primary-percolation-paths* (or *primary highways*). By Lemma 2, we have the following lemma.

Lemma 3 ([8]). *For any* $\kappa > 0$, $c^2 > \log 6 + 2/\kappa$, *there exists a constant* δ_p *such that there are uniform* w.h.p., *at least* $\delta_p \log n$ *horizontal (or vertical) primary highways contained in every horizontal (or vertical) slab with size of* $1 \times \frac{\sqrt{2}c}{\sqrt{n}} \cdot (\kappa \log h_p - \epsilon(h_p))$.

Load assignment to primary highways. According to Lemma 3, there are at least $\delta_p \log n$ highways contained in each slab. We can assign the traffic load produced in a slab into these $\delta_p \log n$ highways. An intuitive allocation method is to divide the slab into $\delta_p \log n$ slices and make each highway in charge of the load from exactly one slice. Specifically, we partition each horizontal (or vertical) slab into $\delta_p \log n$ horizontal (or vertical) slices of width

$$w_p = \frac{\frac{\sqrt{2}c}{\sqrt{n}} \cdot (\kappa \log h_p - \epsilon(h_p))}{\delta_p \cdot \log n} = \Theta\left(\frac{1}{\sqrt{n}}\right).$$

By Lemma 3, we can assign one horizontal (or vertical) primary highway to each slice.

Primary connectivity path. Partition the region $\mathcal{A}$ into subsquares of side length $\bar{l}_p = \frac{\sqrt{\log n}}{\sqrt{n}}$ to obtain the lattice graph $\bar{\mathbb{C}}_p(\bar{h}_p)$. Then, there are $\bar{h}_p^2$ subsquares, where $\bar{h}_p = \lceil \frac{n}{\log n} \rceil$, which are called the *primary-connectivity-cells*.

Lemma 4. *All primary-connectivity-cells uniform* w.h.p., *contain at least one primary node.*

Proof. Let N denote the number of nodes in a primary cell, then N follows a Poisson distribution with $\lambda = na_p = \log n$, then $\Pr(\mathrm{N} = 0) = e^{-\lambda} = 1/n$. Thus, the probability that there is at least one cell having no node is upper bounded by $(n/\log n) \Pr(\mathrm{N} = 0) = 1/\log n \to 0$, where union bounds and the fact there are $\Theta(n/\log n)$ cells are used. □

Choose a node from each primary-connectivity-cell and connect them, we finally obtain the *primary-connectivity-paths*. We call those nodes *primary-connectivity-stations*.

Primary multicast routing. Consider the multicast session $\mathcal{M}_{\mathcal{S},k}$ and its *spanning set of nodes* $\mathcal{U}_{\mathcal{S},k}$, where $1 \leq k \leq n_s$. We first construct the euclidean spanning tree (EST) of $\mathcal{U}_{\mathcal{S},k}$ by the method in [15], denoted by $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$. Based on $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$, we propose Algorithm 1 to construct the multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$. The routing scheme is of hierarchical structure composed of the *primary-highways-routing-phase* $\mathbb{M}_p^{r_1}$ (including Steps in Lines 3 and 4 of Algorithm 1) and the *primary-connectivity-paths-routing-phase* $\mathbb{M}_p^{r_2}$ (including Steps in Lines 2 and 5 of Algorithm 1).

Algorithm 1. Primary Percolation Routing $\mathbb{M}_p^r$

Input: A given multicast session $\mathcal{M}_{\mathcal{S},k}$ and its euclidean spanning tree $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$.

Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

1: **for** each link $u_i \to u_j$ of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ **do**
2: Node u_i drains the data into a specific horizontal primary highway along the specific vertical primary-connectivity-path.

3: Data are carried along the specific horizontal primary highway.
4: Data are carried along the specific vertical primary highway.
5: Data are delivered to node u_j from the specific vertical primary highway along a specific horizontal primary-connectivity-path.
6: **end for**
7: Consider the resulted routing graph, we merge the same edges (hops), and remove those circles which have no impact on the connectivity of the communications for $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$. Finally, we obtain the multicast tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

Primary transmission scheduling. We use two independent 9-TDMA schemes to schedule the primary highways and the primary-connectivity-paths based on the lattice graphs $\mathbb{C}_p(h_p)$ and $\bar{\mathbb{C}}_p(\bar{h}_p)$, respectively. To be specific, we divide a scheduling period into two subperiods with the same length, i.e., the *primary-highways-transmission-scheduling-phase* (PH-TSP), denoted by $\mathbb{M}_p^{t_1}$, and the *primary-connectivity-paths-transmission-scheduling-phase* (PCP-TSP), denoted by $\mathbb{M}_p^{t_2}$. The two scheduling phases correspond to the two phases of routing, i.e., $\mathbb{M}_p^{r_1}$ and $\mathbb{M}_p^{r_2}$.

Furthermore, in the PH-TSP, the scheduled sender transmits with power $P \cdot (l_p)^{\alpha}$, and in the PCP-TSP, the scheduled sender transmits with power $P \cdot (\bar{l}_p)^{\alpha}$. Compared with the schemes in [5] that make each scheduled node transmit signals with a constant power P, our schemes have a significant advantage in terms of energy-saving (in Section 6.2, we will prove that our schemes can achieve the throughput at the same order as those of [5]).

5.1.2 Primary Connectivity Strategy $\bar{\mathbb{M}}_p$

Unlike the percolation strategy, the connectivity strategy operates only on the basis of the primary-connectivity-paths.

Primary multicast routing. We adopt a Manhattan routing [15] based on the primary-connectivity-paths. (see the details in Algorithm 2.)

Algorithm 2. Primary Connectivity Routing $\bar{\mathbb{M}}_p^r$
Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.
1: **for** each link $u_i \to u_j$ of $\mathrm{EST}(\mathcal{U}_{\mathcal{S},k})$ **do**
2: Denote the intersection point of the horizontal line through u_i and the vertical line through u_j as $p_{i,j}$.
3: Data are carried along a specific horizontal primary-connectivity-path from u_i to the primary-connectivity-path-station $u_{i,j}$ that is located in the primary-connectivity-cell containing point $p_{i,j}$.
4: Data are carried along the specific vertical primary-connectivity-path passing through $u_{i,j}$ to u_j.
5: **end for**
6: Use a similar method to Line 7 of Algorithm 1 to obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}_{\mathcal{S},k})$.

Primary transmissions scheduling. For this case, since there are only links along the primary-connectivity-paths, we just only use the primary-connectivity-paths-scheduling.

5.2 Schemes for Secondary Network

The main challenge of this work is how to design the multicast strategy for SaN that can achieve the optimal multicast throughput, i.e., the upper bound of the capacity, without changing the order of throughput for PaN. The key technique is to set *preservation regions* [7], [19].

5.2.1 Secondary Percolation Strategy $\mathbb{M}_s$

Like in PaN, there are two types of links in SaN in terms of hop length. The first type of links are those that comprise the *secondary highways* of hop length $O(\frac{1}{\sqrt{m}})$. The second ones are the links that comprise the *secondary connectivity paths* of hop length $O(\sqrt{\log m/m})$. As in PaN, we partition the region $\mathcal{A}$ into subsquares of side length $l_s = \frac{c}{\sqrt{m}}$ to obtain the lattice graph $\mathbb{C}_s(h_s)$ with $h_s = \lceil \frac{\sqrt{m}}{\sqrt{2}c} \rceil$, and we call the subsquares *secondary-percolation-cells*. Similarly, divide $\mathcal{A}$ into subsquares of side length $\bar{l}_s = \frac{\sqrt{\log m}}{\sqrt{m}}$ to obtain the *secondary-connectivity-cells* and lattice graph $\bar{\mathbb{C}}_s(\bar{h}_s)$ with $\bar{h}_s = \lceil \frac{\sqrt{m}}{\sqrt{\log m}} \rceil$.

Unlike in PaN, we must ensure that the secondary transmitters are not too close to the primary nodes operating simultaneously, otherwise, it may produce devastating interference. Hence, we set a *preservation region* [6] for each primary node which the routing of communications in SaN cannot go through.

Preservation region (P-R). Based on lattice graphs $\mathbb{C}_s(h_s)$ and $\bar{\mathbb{C}}_s(\bar{h}_s)$, we define two types of *preservation regions*. The first is the *percolation-preservation-region* (PP-R) that consists of nine secondary-percolation-cells, with a primary node at the center cell. The second is the *connectivity-preservation-region* (CP-R) that consists of nine secondary-connectivity-cells, with a primary node at the center cell.

Secondary highway. We construct the *secondary-percolation-paths*, also called *secondary highways*, based on the lattice graph $\mathbb{C}_s(h_s)$. As in $\mathbb{C}_p(h_p)$, a cell in $\mathbb{C}_s(h_s)$ is *open* if it is nonempty; it is *closed*, otherwise.

To address the fact that the construction of primary highways imposes more constraints on whether a secondary-percolation-cell can be actually used to create the secondary highways, we emphasize a significant *difference* from primary highways. That is, we need to ensure that all secondary nodes on secondary highways, including transmitters and receivers, are at large enough distances to primary nodes; specifically, we ensure that those secondary nodes are not shaded by any percolation-preservation-region. It makes primary and secondary highways not interact with each other in terms of the order of link rate, although they are possibly intersecting geometrically. That will be proven by Lemma 15.

Then, we should modify the definition of availability of cells in $\mathbb{C}_s(h_s)$. We say a secondary-percolation-cell is *cognitive open* if it is open (nonempty) and does not belong to any percolation-preservation-regions; and say it is *cognitive closed*, otherwise. Please see the illustrations in Figs. 2 and 3.

Then, we have the following lemma:

Lemma 5. *When $n = o(m)$, a secondary-percolation-cell in $\mathbb{C}_s(h_s)$ is* cognitive open *with probability p_s, where $\mathrm{p}_s \to \mathrm{p}_p$ as $n \to \infty$.*

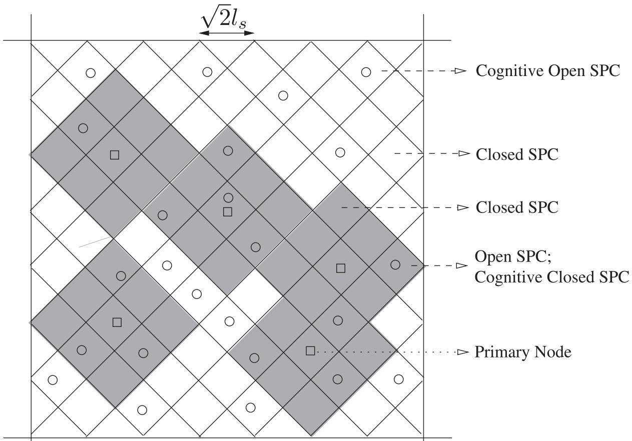


Fig. 2. Cognitive Open *Secondary-Percolation-Cells* (SP-Cs). The shaded regions are the percolation-preservation-regions (PP-Rs). The small square nodes at the center of PP-Rs represent the primary nodes, and the small circle nodes represent the secondary nodes. Those SP-Cs that are not shaded and contain at least one secondary node are *cognitive open*.

Proof. For any secondary-percolation-cell (SP-C) G_s, define an event $\mathbf{E}_1(G_s)$: the cell G_s is open (nonempty). Then,

$$\Pr(\mathbf{E}_1(G_s)) = 1 - e^{m\cdot(\frac{c}{\sqrt{m}})^2} = 1 - e^{-c^2}.$$

Next, we define an event $\mathbf{E}_2(G_s)$: the cell G_s does not belong to any percolation-preservation-regions. Denote a set of 3×3 SP-Cs centered at G_s by $\mathcal{C}(G_s)$. Then, the event $\mathbf{E}_2(G_s)$ happens if and only if there is no primary node falling into the region $\mathcal{C}(G_s)$. Since primary nodes are distributed according to a Poisson point process (p.p.p.) of intensity n, the number of primary nodes falling into $\mathcal{C}(G_s)$ follows a Poisson distribution of mean $\lambda(\mathcal{C}(G_s)) = n \cdot 9 \cdot (\frac{c}{\sqrt{m}})^2 = \frac{9c^2 n}{m}$. Thus, $\Pr(\mathbf{E}_2(G_s)) = e^{-\lambda(\mathcal{C}(G_s))} = e^{-\frac{9c^2 \cdot n}{m}}$.

Since the processes of distributing primary and secondary nodes are independent, the events $\mathbf{E}_1(G_s)$ and $\mathbf{E}_2(G_s)$ are independent, too. Therefore, according to the definition of *cognitive open*, we get,

$$\mathrm{p}_s = \Pr(\mathbf{E}_1(G_s)) \times \Pr(\mathbf{E}_2(G_s)) = (1 - e^{-c^2}) \cdot e^{-\frac{9c^2 \cdot n}{m}}.$$

Combining with the condition $\lim_{n\to\infty} \frac{n}{m} = 0$, we can prove that $\lim_{n\to\infty} \mathrm{p}_s = \mathrm{p}_p$. □

By a similar procedure of the mapping from $\mathbb{C}_p(h_p)$ to $\mathbb{B}(h_p, \mathrm{p}_p)$, we can construct the lattice graph $\mathbb{B}(h_s, \mathrm{p}_s)$ based on $\mathbb{C}_s(h_s)$, which serves as the basic frame of the bond percolation model. By Lemmas 2 and 3, we have the following lemma:

Lemma 6. *When $n = o(m)$, for any $\kappa > 0$ and $c^2 > \log 6 + 2/\kappa$, there exists a constant δ_s such that there are uniform* w.h.p., *at least $\delta_s \log m$ horizontal (vertical)* secondary highways *in each horizontal (vertical) slab with size of $1 \times \frac{\sqrt{2}c}{\sqrt{m}}(\kappa \log h_s - \epsilon(h_s))$.*

Load assignment to secondary highways. As in PaN, we can partition each horizontal (or vertical) slab into $\delta_s \log m$ horizontal (or vertical) slices of width $w_s = \Theta(\frac{1}{\sqrt{m}})$.

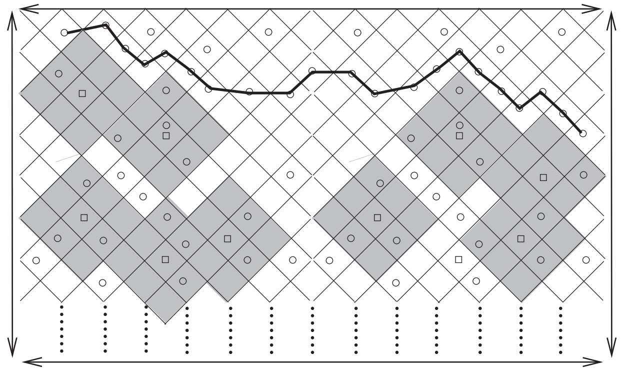


Fig. 3. An illustration of a secondary highway. The cells are of side length $l_s = \frac{c}{\sqrt{m}}$. The slab is of side length $\frac{\sqrt{2}c}{\sqrt{m}}(\kappa \log h_s - \epsilon(h_s))$. The shaded regions are percolation-preservation-regions (PPRs). The small square nodes at the center of PPRs represent primary nodes, and the small circle nodes represent secondary nodes.

According to Lemma 3, we can assign at least one horizontal (or vertical) secondary highway to each horizontal (or vertical) slice.

Secondary connectivity path. We build the secondary-connectivity-paths (SC-Ps) based on the lattice graph $\bar{\mathbb{C}}_s(\bar{h}_s)$. Similar to Lemma 4, we can obtain following lemma:

Lemma 7. *All secondary-connectivity-cells uniform* w.h.p., *contain at least one secondary node.*

Thus, we can choose a secondary node from each secondary-connectivity-cell and connect them to obtain secondary-connectivity-like-paths (secondary CP-like paths). We construct the *secondary-connectivity-paths* based on those secondary CP-like paths. Note that the difference between the primary-connectivity-paths and the secondary-connectivity-paths is that the latter do *not* pass through any connectivity-preservation-regions. Thus, we construct the secondary-connectivity-paths by modifying the CP-like paths by a similar method in [7]: when a secondary CP-like path collides with the connectivity-preservation-regions (CPR), the path detours the CPR along its boundary secondary-connectivity-cells, see Fig. 4. We call all joint nodes on the secondary-connectivity-paths the *secondary-connectivity-path-stations*.

Served set. Unlike in PaN, there are possibly some secondary cells (secondary-percolation-cells or secondary-connectivity-cells) that are not served in SaN, because they are covered by *preservation regions* (percolation-preservation-regions or connectivity-preservation-regions) or by the closed regions encompassed with the cluster of preservation regions. We call those cells *nonserved cells*, and define the set of all secondary nodes contained in the *nonserved cells* as $\bar{\mathcal{V}}^s(m)$. Denote the set of all secondary sources for multicast sessions in SaN as $\mathcal{S}$. (For succinctness, we denote both sets of sources of multicast sessions in PaN and SaN by $\mathcal{S}$ without confusion, but we should learn that they are really different sets.)

Algorithm 3. Secondary Percolation Routing $\mathbb{M}_s^r$
Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}'_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.
1: **for** each link $u_i \to u_j$ of $\mathrm{EST}(\mathcal{U}'_{\mathcal{S},k})$ **do**

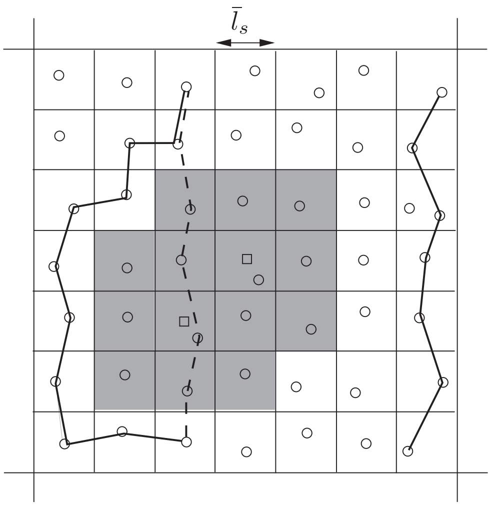


Fig. 4. CP-Rs and SC-Ps. The shaded region denotes the union of connectivity-preservation-region (CP-Rs). The chains denote secondary-connectivity-paths (SC-Ps).

Node u_i drains the data into a specific horizontal secondary highway along a specific vertical secondary-connectivity-path.
Data are carried along the specific horizontal secondary highway.
Data are carried along the specific vertical secondary highway.
Data are delivered to node u_j from the specific vertical secondary highway along a specific horizontal secondary-connectivity-path.
end for
Use a similar method to Line 7 of Algorithm 1 to obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.

Based on the sets $\bar{\mathcal{V}}^s(m)$ and $\mathcal{S}$, we define a new notion called *served set*. The definition of *served set* can be divided into two regions depending on m_d and m_s.

Definition 4 (Served Set). *The* served set, *denoted by* $\mathcal{S}'$, *is a subset of* $\mathcal{S}$, *and*

1. *when* $m_d = \omega(\log m_s)$, *define* $\mathcal{S}' := \mathcal{S} - \mathcal{S} \cap \bar{\mathcal{V}}^s(m)$;
2. *when* $m_d = O(\log m_s)$, *define* $\mathcal{S}' := \{v_{\mathcal{S},i} | \mathcal{U}_{\mathcal{S},i} \cap \bar{\mathcal{V}}^s(m) = \emptyset\}$.

For each multicast session $\mathcal{M}_{\mathcal{S},i}$ with source $v_{\mathcal{S},i} \in \mathcal{S}'$ in SaN, define a set as

$$\mathcal{U}'_{\mathcal{S},i} = \{v_{\mathcal{S},i}\} \cup \mathcal{D}'_{\mathcal{S},i}, \text{ where } \mathcal{D}'_{\mathcal{S},i} = \mathcal{D}_{\mathcal{S},i} - \mathcal{D}_{\mathcal{S},i} \cap \bar{\mathcal{V}}^s(m).$$

In Section 6.1, we will prove that under Assumption B, $|\mathcal{S}'| \to |\mathcal{S}| = m_s$, and for all $v_{\mathcal{S},i} \in \mathcal{S}'$, uniform *w.h.p.*, $|\mathcal{D}'_{\mathcal{S},i}| \to |\mathcal{D}_{\mathcal{S},i}| = m_d$, as $n, m \to \infty$.

Secondary multicast routing. Based on every set $\mathcal{U}'_{\mathcal{S},i}$ for $v_{\mathcal{S},i} \in \mathcal{S}'$, we can build $\mathrm{EST}(\mathcal{U}'_{\mathcal{S},i})$ by the method in [15]. The detailed routing scheme is presented in Algorithm 3.

Secondary transmissions scheduling. To be synchronous with PaN, in SaN, the secondary highways in Phase $\mathbb{M}_s^{t_1}$ and secondary-connectivity-paths in Phase $\mathbb{M}_s^{t_2}$ should be independently scheduled, where the length of each time slot in $\mathbb{M}_s^{t_i}$ equals that of $\mathbb{M}_p^{t_i}$ (for $i = 1, 2$). While, similar to [7], the scheduling periods of $\mathbb{M}_s^{t_i}$ are three times those of $\mathbb{M}_p^{t_i}$. That is, we adopt two independent 27-TDMA schemes for secondary highways and secondary-connectivity-paths, in which each secondary cell is scheduled for three continuous time slots in a scheduling period (27 time slots).

Furthermore, for each transmission in Phase $\mathbb{M}_s^{t_1}$, the transmitter transmits with power $P \cdot (l_s)^\alpha$, and in Phase $\mathbb{M}_s^{t_2}$, the transmitter transmits with power $P \cdot (\bar{l}_s)^\alpha$.

5.2.2 Secondary Connectivity Strategy $\bar{\mathbb{M}}_s$

We use a Manhattan Routing [15] based on the secondary-connectivity-paths system (please see the details in Algorithm 4), and for this case, we only use the scheduling $\bar{\mathbb{M}}_s^{t_2}$ since there are only links along the secondary-connectivity-paths to be scheduled.

Algorithm 4. Secondary Connectivity Routing $\bar{\mathbb{M}}_s^r$
Input: The multicast session $\mathcal{M}_{\mathcal{S},k}$ and $\mathrm{EST}(\mathcal{U}'_{\mathcal{S},k})$.
Output: A multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.
for each link $u_i \to u_j$ of $\mathrm{EST}(\mathcal{U}'_{\mathcal{S},k})$ **do**
Denote the intersection point of the horizontal line through u_i and the vertical line through u_j by $p_{i,j}$.
Data are carried along a specific horizontal secondary-connectivity-path from u_i to the *connectivity station* $u_{i,j}$ that is located in the secondary-connectivity-cell containing point $p_{i,j}$.
Data are carried along the specific vertical secondary-connectivity-path passing through $u_{i,j}$ to u_j.
end for
Use a similar method to Line 7 of Algorithm 1 to obtain the final multicast routing tree $\mathcal{T}(\mathcal{U}'_{\mathcal{S},k})$.

5.3 Decision of Optimal Strategy

According to the relations among n, n_d, m, and m_d, we choose better strategies in terms of multicast throughput for PaN and SaN. Specifically, we select the better strategy from $\mathbb{M}_p$ and $\bar{\mathbb{M}}_p$ for PaN, and determine the secondary multicast strategy from $\mathbb{M}_s$ and $\bar{\mathbb{M}}_s$ to match the selected primary multicast strategy.

Strategy Alternatives:

- When $n_d \in [1, \frac{n}{(\log n)^2}]$, we adopt $\mathbb{M}_p$ for PaN.
  - When $m_d \in [1, \frac{m}{(\log m)^2}]$, we adopt $\mathbb{M}_s$ for SaN.
  - When $m_d \in [\frac{m}{(\log m)^2}, m]$, we adopt $\bar{\mathbb{M}}_s$ for SaN.
- When $n_d \in [\frac{n}{(\log n)^2}, n]$, we adopt $\bar{\mathbb{M}}_p$ for PaN.
  - We always adopt $\mathbb{M}_s$ for SaN.

6 Analysis of Multicast Throughput

Since the definition of multicast capacity in [15] can be regarded as a special case of Definition 3 (let $\rho_s \equiv 1$ and $\rho_d \equiv 1$), we can derive the achievable multicast throughput for PaN under the formal definition in [15], as well as under Definition 3. While, for SaN, we consider the achievable multicast throughput based on Definition 3, and we only focus on the multicast sessions whose sources are included in the *served set* (Definition 4).

6.1 Analysis of Served Set

Now, we start to analyze the *served set* and mainly prove Lemma 8. We discuss the issue separately for two cases when $m_d = \omega(\log m_s)$ and when $m_d = O(\log m_s)$.

Lemma 8. *The cardinality of* served set *for SaN defined in Definition 4 goes to m_s, i.e., $\frac{|\mathcal{S}'|}{|\mathcal{S}|} \to 1$, and for all $v_{\mathcal{S},i} \in \mathcal{S}'$, uniform* w.h.p., $\frac{|\mathcal{D}'_{\mathcal{S},i}|}{|\mathcal{D}_{\mathcal{S},i}|} \to 1$, *as $n, m \to \infty$.*

6.1.1 Total Area of Nonserved Cells

Based on Lemma 1, we propose a lemma to show that the sizes of all clusters of preservation regions are bounded.

Lemma 9. *When $n < \frac{\mathrm{p}_c}{8} \cdot \frac{m}{\log m}$, any cluster of* preservation regions *(percolation-preservation-regions or connectivity-preservation-regions) includes at most a constant μ preservation regions* w.h.p., *where p_c is the* critical percolation threshold *of Poisson Boolean model in $\mathbb{R}^2$, m and n are the density of primary and secondary networks, respectively.*

Proof. Consider the Poisson Boolean model $\mathcal{B}(\lambda, r)$, where

$$r = 2\sqrt{2}\max\{l_s, \bar{l}_s\} = 2\sqrt{2} \cdot \frac{\sqrt{\log m}}{\sqrt{m}}$$

and $\lambda = n$. Since the *associated graphs* $\mathcal{G}(\lambda_0, r_0) = \mathcal{G}(\lambda, r)$ when $\lambda_0 \cdot {r_0}^2 = \lambda \cdot r^2$. Hence, $\mathcal{B}(\lambda, r)$ is equivalent to $\mathcal{B}(\lambda_0, r_0)$ in terms of the connectivity, where $r_0 = 1$ and $\lambda_0 = 8n \cdot \frac{\log m}{m}$. Since $n < \frac{\mathrm{p}_c}{8} \cdot \frac{m}{\log m}$, we have $\lambda \cdot r^2 < \mathrm{p}_c$. By Lemma 1, the size of any cluster is at most a constant μ. Since a disk of radius $r/2$ contains a square preservation region, it is also true for all clusters of preservation regions. □

Lemma 10. *The sum area of the* nonserved cells, *denoted by $S(m)$, is at most $9 \cdot \mu \cdot n \cdot \frac{\log m}{m}$, where the constant μ is the maximum size of the clusters of* preservation regions.

Proof. For any cluster with size μ_i, it is true that there exists a square of side length $3\mu_i \bar{l}_s$ containing completely all μ_i *preservation regions* and the *nonserved cells* encompassed by them. So, the sum area of the *nonserved cells* produced by μ_i *preservation regions*, say $S(m, \mu_i)$, holds that $S(m, \mu_i) \le 9 \cdot \mu_i^2 \cdot \frac{\log m}{m}$. Then, the sum area of the *nonserved cells* $S(m) \le S'_{max}$, where S_{max} is the optimum solution of the optimization problem:

$$\begin{cases} \max & S = 9 \cdot \frac{\log m}{m} \cdot \sum_{i=1}^{n} \mu_i^2 \\ \text{s.t.} & \sum_{i=1}^{n} \mu_i = n, 1 \le \mu_i \le \mu, \quad i = 1, 2, \ldots, n. \end{cases}$$

It is easy to derive that $S_{max} = \frac{n}{\mu} \cdot \mu^2 \cdot 9 \cdot \frac{\log m}{m} = 9 \cdot \mu \cdot n \cdot \frac{\log m}{m}$, which completes the proof. □

6.1.2 When $m_d = \omega(\log m_s)$

For this case, according to Definition 4, we define the *served set* as $\mathcal{S}' = \mathcal{S} - \mathcal{S} \cap \bar{\mathcal{V}}^s(m)$. Then, we get that $|\mathcal{S}'| = |\mathcal{S}| - |\mathcal{S} \cap \bar{\mathcal{V}}^s(m)|$. Note that we need the condition that $n = o(m/\log m)$ made in Assumption B.

Lemma 11. *With high probability, it holds that $|\mathcal{S} \cap \bar{\mathcal{V}}^s(m)| \le \bar{\rho}_s(m) \cdot m_s$, where $\bar{\rho}_s(m) \to 0$, as $m \to \infty$.*

Proof. Define a random variable $\bar{\xi}^s = |\mathcal{S} \cap \bar{\mathcal{V}}^s(m)|$. Then, by Lemma 10, it follows a poisson distribution of mean

$$\bar{\lambda}^s \le m_s \cdot S_{max} = 9 \cdot \mu \cdot m_s \cdot n \cdot \log m/m.$$

According to Lemma A in the Appendix, available in the online supplemental material, we get that

$$\Pr(\bar{\xi}^s \ge 18\mu \cdot m_s \cdot n \cdot \log m/m) \le (e/4)^{9\mu \cdot m_s n \cdot \frac{\log m}{m}} \to 0.$$

By $n = o(\frac{m}{\log m})$, we get that $\bar{\rho}_s(m) = o(1)$. □

According to Lemma 11, it can be easily obtained that $\frac{|\mathcal{S}'|}{|\mathcal{S}|} \to 1$, as $n, m \to \infty$.

Next, we derive the uniform lower bound of $\frac{|\mathcal{D}_{\mathcal{S},i}|}{|\mathcal{D}'_{\mathcal{S},i}|}$ for all $v_{\mathcal{S},i} \in \mathcal{S}'$. We first consider $|\mathcal{D}_{\mathcal{S},i} - \mathcal{D}'_{\mathcal{S},i}|$.

Lemma 12. *For all $v_{\mathcal{S},i} \in \mathcal{S}'$, it holds uniform* w.h.p., *that $|\mathcal{D}_{\mathcal{S},i} - \mathcal{D}'_{\mathcal{S},i}| \le \bar{\rho}_d(m) \cdot m_d$, where $\bar{\rho}_d(m) = o(1)$.*

Proof. For each $v_{\mathcal{S},i} \in \mathcal{S}'$, define a random variable $\bar{\xi}^d_{\mathcal{S},i} = |\mathcal{D}_{\mathcal{S},i} - \mathcal{D}'_{\mathcal{S},i}|$. Then, according to Lemma 10, $\bar{\xi}^d_{\mathcal{S},i}$ follows a poisson distribution of at most $9 \cdot \mu \cdot m_d \cdot n \cdot \frac{\log m}{m}$. We consider separately two cases of $m_d \cdot n \cdot \frac{\log m}{m} = \Omega(\log m_s)$ and $m_d \cdot n \cdot \frac{\log m}{m} = O(\log m_s)$. Then, according to Lemma A in the Appendix, available in the online supplemental material, (Tails of the Chernoff bounds) and union bounds, we can obtain

$$\bar{\rho}_d(m) = \begin{cases} O\left(\frac{n \cdot \log m}{m}\right) & \text{when} \quad m_d \cdot n \cdot \frac{\log m}{m} = \Omega(\log m_s), \\ O\left(\frac{\log m_s}{m_d}\right) & \text{when} \quad m_d \cdot n \cdot \frac{\log m}{m} = O(\log m_s). \end{cases}$$

Thus, we get that $\bar{\rho}_d(m) = o(1)$ when $m_d = \omega(\log m_s)$. □

According to Lemma 12, we can easily get that $\frac{|\mathcal{D}_{\mathcal{S},i}|}{|\mathcal{D}'_{\mathcal{S},i}|} \to 1$, as $n, m \to \infty$.

Combining Lemmas 11 and 12, we can obtain Lemma 8 for the case when $m_d = \omega(\log m_s)$.

6.1.3 When $m_d = O(\log m_s)$

For this case, based on Definition 4, we obtain a *served set* as

$$\mathcal{S}' = \{v_{\mathcal{S},i} | (v_{\mathcal{S},i} \in \mathcal{S}) \wedge (\mathcal{U}_{\mathcal{S},i} \cap \bar{\mathcal{V}}^s(m) = \emptyset)\}.$$

Unlike in the case when $m_d = \omega(\log m_s)$, we need a new condition that $n = o(\frac{m}{m_d \cdot \log m})$ as in Assumption B. We first propose following lemma.

Lemma 13. *For all $v_{\mathcal{S},i} \in \mathcal{S}'$, it holds that $\mathcal{D}'_{\mathcal{S},i} = \mathcal{D}_{\mathcal{S},i}$.*

Proof. According to the definition of $\mathcal{S}'$, for all $v_{\mathcal{S},i} \in \mathcal{S}'$, $\mathcal{U}_{\mathcal{S},i} \cap \bar{\mathcal{V}}^s(m) = \emptyset$. Since $\mathcal{D}_{\mathcal{S},i} \subseteq \mathcal{U}_{\mathcal{S},i}$, then $\mathcal{D}_{\mathcal{S},i} \cap \bar{\mathcal{V}}^s(m) = \emptyset$. Hence, $\mathcal{D}'_{\mathcal{S},i} = \mathcal{D}_{\mathcal{S},i} - \mathcal{D}_{\mathcal{S},i} \cap \bar{\mathcal{V}}^s(m) = \mathcal{D}_{\mathcal{S},i}$. □

By Lemma 13, it obviously holds that $\frac{|\mathcal{D}_{\mathcal{S},i}|}{|\mathcal{D}'_{\mathcal{S},i}|} \to 1$, as $n, m \to \infty$.

Next, we consider the cardinality of $\mathcal{S}'$. Above all, we notice that $|\mathcal{S}'| \ge |\mathcal{S}| - |\mathcal{S} - \mathcal{S}'|$.

Lemma 14. *With high probability, $|\mathcal{S} - \mathcal{S}'| \le \bar{\bar{\rho}}_s(m) \cdot m_s$, where $\bar{\bar{\rho}}_s(m) = o(1)$.*

Proof. Define a random variable $\bar{\bar{\xi}}^s = |\mathcal{S} - \mathcal{S}'|$. Then by Lemma 10, $\bar{\bar{\xi}}^s$ follows a Poisson distribution of the mean $\bar{\bar{\lambda}}^s \le m_s \cdot (m_d + 1) \cdot 9 \cdot \mu \cdot n \cdot \log m/m$. By the Chernoff bounds in Lemma A in the Appendix, available in the online supplemental material, we get that

$$\Pr\left(\bar{\bar{\lambda}}^s \geq \frac{18m_s \cdot (m_d+1) \cdot \mu \cdot n \cdot \log m}{m}\right) \leq \left(\frac{e}{4}\right)^{\frac{9m_s \cdot (m_d+1) \cdot \mu \cdot n \cdot \log m}{m}} \rightarrow 0.$$

By $n = o(\frac{m}{m_d \log m})$, we have that $\bar{\bar{\rho}}_s(m) \leq 18m_s \cdot (m_d+1) \cdot \mu \cdot n \cdot \log m/m \rightarrow 0$, which completes the proof. □

Thus, by Lemma 14, it holds that $\frac{|\mathcal{S}'|}{|\mathcal{S}|} \rightarrow 1$, as $n, m \rightarrow \infty$.

Combining Lemmas 13 and 14, we can obtain Lemma 8 for the case when $m_d = O(\log m_s)$.

6.1.4 Role of Served Set

Now, we discuss what role the *served set*, i.e., $\mathcal{S}'$, will play. According to the routing schemes presented in Section 5.2, only the multicast sessions whose sources belong to the *served set* $\mathcal{S}'$ are considered, and for each considered session $\mathcal{M}_{\mathcal{S},i}$, only the destinations belong to $\mathcal{D}'_{\mathcal{S},i}$ are considered. Thus, by Lemma 8, we can state that the per-session throughput for SaN is achieved of λ if we can prove that, in SaN, for each multicast session $\mathcal{M}_{\mathcal{S},i}$ with source $v_{\mathcal{S},i} \in \mathcal{S}'$, uniform *w.h.p.*, data can be delivered to all destinations in $\mathcal{D}'_{\mathcal{S},i}$ at a rate of λ. Here, obviously, $\mathcal{S}'$ acts as the $\mathcal{S}'(1,1)$ introduced in Definition 1.

6.2 Multicast Throughput Analysis

6.2.1 When $n_d \in [1, \frac{n}{(\log n)^2}]$ and $m_d \in [1, \frac{m}{(\log m)^2}]$

For this case, we implement the scheme $\mathbb{M}_p$ for PaN and $\mathbb{M}_s$ for SaN, and we analyze the multicast throughput achieved by $\mathbb{M}_p^r$ with $\mathbb{M}_s^r$ according to Lemma D in the Appendix, available in the online supplemental material. First, we consider Phase $\mathbb{M}_p^{r_1}$ (or $\mathbb{M}_s^{t_1}$).

Lemma 15. *During Phase $\mathbb{M}_p^{t_1}$ (or Phase $\mathbb{M}_s^{t_1}$), the achievable total rate along the highways (including primary highways in PaN and secondary highways in SaN) is of order $\Omega(1)$.*

For the multicast throughput during highways phases, by using Lemma D in the Appendix, available in the online supplemental material, we can derive the following result.

Lemma 16. *During Phase $\mathbb{M}_p^{r_1}$ (or Phase $\mathbb{M}_s^{r_1}$), the achievable per-session multicast throughput for PaN (or for SaN) is of order $\Omega(\frac{1}{\sqrt{nn_d}})$ (or $\Omega(\frac{1}{\sqrt{mm_d}})$).*

Next, we analyze the throughput in Phase $\mathbb{M}_p^{r_2}$ (or $\mathbb{M}_s^{r_2}$).

Lemma 17. *During Phase $\mathbb{M}_p^{r_2}$ (or $\mathbb{M}_s^{r_2}$), the achievable per-session multicast throughput for PaN (or for SaN) is of order $\Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$ (or $\Omega(\frac{1}{m_d} \cdot (\log m)^{-\frac{3}{2}})$).*

Combining Lemmas 16 and 17, we get the following theorem:

Theorem 3. *When n_d:$[1, \frac{n}{(\log n)^2}]$ and m_d:$[1, \frac{m}{(\log m)^2}]$, the achievable per-session multicast throughputs for PaN and SaN are of $\Omega(\mathbf{f}_1(n, n_d))$ and $\Omega(\mathbf{f}_1(m, m_d))$, respectively.*

6.2.2 When $n_d \in [1, \frac{n}{(\log n)^2}]$ and $m_d \in [\frac{m}{(\log m)^2}, m]$

For this case, we implement the scheme $\mathbb{M}_p$ for PaN and the scheme $\bar{\mathbb{M}}_s$ for SaN. In Phase $\mathbb{M}_p^{r_1}$, we can set SaN to be idle, which has no impact on the throughput in order sense. Then, it is obviously true that the throughput for PaN during phase $\mathbb{M}_p^{r_1}$ for this case is no less than that for the previous case. Furthermore, during Phase $\mathbb{M}_p^{r_2}$, we implement strategy $\bar{\mathbb{M}}_s$ for SaN. Using a similar method to Lemma 17, we can get that the interference produced by $\bar{\mathbb{M}}_s$ to transmissions of PaN is no more than our estimation (in Lemma 17) of that produced by $\mathbb{M}_s^{t_1}$. Based on the analysis above, we easily obtain the following.

Lemma 18. *During Phase $\mathbb{M}_p^{r_1}$ (or $\mathbb{M}_p^{r_2}$), the achievable per-session multicast throughput for PaN is of order $\Omega(\frac{1}{\sqrt{nn_d}})$ (or $\Omega(\frac{1}{n_d} \cdot (\log n)^{-\frac{3}{2}})$).*

According to Lemma 18, it is obviously true that the per-session multicast throughput for PaN is achieved of order $\mathbf{f}_1(n, n_d)$ when $n_d \in [1, \frac{n}{(\log n)^2}]$ and $m_d \in [\frac{m}{(\log m)^2}, m]$. Next, we consider the multicast throughput for SaN.

Lemma 19. *The achievable per-session multicast throughput for SaN is of order $\Omega(\mathbf{f}_2(m, m_d))$.*

Combining Lemmas 18 and 19, we get the following theorem:

Theorem 4. *When n_d:$[1, \frac{n}{(\log n)^2}]$ and m_d:$[\frac{m}{(\log m)^2}, m]$, the achievable per-session multicast throughputs for PaN and SaN are of $\Omega(\mathbf{f}_1(n, n_d))$ and $\Omega(\mathbf{f}_2(m, m_d))$, respectively.*

6.2.3 When $n_d \in [\frac{n}{(\log n)^2}, n]$

For this case, we adopt the scheme $\bar{\mathbb{M}}_p$ for PaN and adopt the scheme $\bar{\mathbb{M}}_s$ for SaN. By a similar proof to that of Lemma 19, we can obtain Theorem 5 due to the nonhierarchical structure of strategies $\bar{\mathbb{M}}_p$ and $\bar{\mathbb{M}}_s$.

Theorem 5. *When n_d:$[\frac{n}{(\log n)^2}, n]$, for all m_d:$[1, m]$, the achievable per-session multicast throughputs for PaN and SaN are of $\Omega(\mathbf{f}_2(n, n_d))$ and $\Omega(\mathbf{f}_2(m, m_d))$, respectively.*

7 Literature Review

In the issue of capacity scaling laws, there are generally two levels of asymptotic bounds. The first level is *information-theoretic* bound that is obtained by allowing arbitrary (physical layer) cooperative relay strategies, [20], [21], [22], [23], [24]. This issue was first addressed by Xie and Kumar [25]. The second level is *networking-theoretic* bound that assumes the signals received from nodes other than the particular transmitter are simply regarded as noise degrading the communication link. The pioneer work [26] was just done for this issue. Since we limit our scope into the networking-theoretic capacity bounds in this paper, we subsequently review the literature about the networking-theoretic capacity for stand-alone wireless networks and cognitive networks.

7.1 Capacity Scaling Laws for Stand-Alone Wireless Networks

7.1.1 Capacity Scaling Laws for Stand-Alone Ad Hoc Networks

From the aspect of state of ad hoc nodes, ad hoc networks can be separated into two classes: random *static ad hoc network* and random *mobile ad hoc network*.

Random Static Ad Hoc Networks (SANETs). For random dense SANETs, Gupta and Kumar [26] showed that the per-session unicast capacity under the *protocol model* and *physical model* is of order $\Theta(1/\sqrt{n \log n})$. Li et al. [27] showed that for total capacity to scale up with network size the average distance between source and destination nodes must remain small as the network grows. Tavli [28] proved that the per-session broadcast capacity is only of order $\Theta(1/n)$. Similar result on broadcast capacity was proposed in [29]. For general scaling random SANETs, including random dense SANETs and random extended SANETs, Li [15] proposed that the per-session multicast capacity under the protocol model [26] is of order $\Theta(1/\sqrt{n_d n \log n})$ when $n_d = O(n/\log n)$, and is of order $\Theta(1/n)$ when $n_d = \Omega(n/\log n)$ under the assumption that $n_s = n$. At the same time as [15], Shakkottai et al. [30] studied the multicast capacity of random dense SANETs when the number of multicast sessions is n^{ϵ} for some $\epsilon > 0$, and the number of receivers of each multicast session is $n^{1-\epsilon}$. Under the protocol model, they designed a novel multicast routing called *comb scheme*. Under the *Gaussian Channel* model [8], [13] that captures better the property of physical layer in wireless networks, Franceschetti et al. [8] showed that the unicast throughput for random SANETs can be achieved of order $\Omega(1/\sqrt{n})$; Zheng [21] pointed out that using multihop relay, the per-session broadcast capacity for random SANETs is of order $\Theta(\frac{1}{n} \cdot (\log n)^{-\frac{\alpha}{2}})$; Li et al. [13] proposed that when $n_d = O(\frac{n}{(\log n)^{2\alpha+6}})$, the per-session multicast throughput for random extended SANETs is achieved of order $\Omega(1/\sqrt{nn_d})$ by using the percolation-based routing [8], [16]; Wang et al. [14] improved the threshold value of n_d mentioned in [13] to $n_d = O(\frac{n}{(\log n)^{\alpha+1}})$. By exploiting a novel technique called *arena*, Keshavarz-Haddad and Riedi [5], [10] proposed the upper bounds of the multicast capacity for random dense SANETs. In [31], Hu et al. further improved the multicast capacity by the technique of *hierarchical cooperation*.

Random Mobile Ad Hoc Networks (MANETs). The mobility of ad hoc nodes brings the change in the capacity issue due to the frequent change of network topology. The original work done by Grossglauser and Tse [32] showed that one can achieve the *linear scaling* unicast capacity in assistance of the mobility. This is an exciting result: a linear scaling means that there is essentially no interference limitation; the rate for each source-destination pair does not degrade significantly even as the size of network goes to infinity. To be specific, in [32], a classic *2-hop scheme* was proposed, and the I.I.D mobility model was adopted. Many researchers have designed the protocols based on this *2-hop scheme* [32] to derive the capacity or examine the tradeoffs between the capacity and delay for more general session patterns, such as multicast session [33], broadcast session [5], or under more realistic mobility models, such as random walk mobility model [34], Brownian mobility model [34], [35], [36], random way-point mobility model [35], [37], [38]. In [39], Thrasyvoulos et al. derived accurate closed form expressions for the expected encounter time between different nodes under commonly used mobility models. The work helps understanding the performance of various approaches in different settings and designing new protocols.

7.1.2 Capacity Scaling Laws for Stand-Alone Ad Hoc Network with Infrastructure Support

Based on a pure ad hoc network, one can build the *ad hoc network with infrastructure* [40], [41], in which some amount of base stations (BSs) that neither produce data nor consume data are erected, and they are connected by high bandwidth links. These BSs support the underlying ad hoc networks by relaying data through the infrastructure. In such network, data can be transported in a multihop fashion as in an ad hoc network or by the relay of the BSs as in a cellular network. The SANET with infrastructure and MANET with infrastructure are often referred to as *hybrid wireless network* [41], [42], [43], [44], and *multihop cellular network* [40], [45], [46], respectively.

Hybrid Wireless Networks. Under threshold-based communication model [14], Liu et al. [47] introduced the model based on the *dense network* in which the base stations are regularly placed and the ad hoc nodes are randomly distributed; Kozat and Tassiulas [48] studied the case where both base stations and ad hoc nodes are randomly placed in the *dense network*; Agarwal and Kumar [49] considered the unicast capacity for hybrid networks under physical model; recently, Mao et al. [43] studied the *multicast capacity* for hybrid networks by assuming that $m = O(n/\log n)$. Under the Gaussian Channel model, Agarwal and Kumar [49] studied the unicast capacity for hybrid dense networks, and they derived the same bounds as those under the threshold-based communication model; Liu et al. [41] studied the achievable unicast throughput for hybrid extended networks, they showed that in a 2D square hybrid wireless network with n ordinary ad hoc nodes and m base stations, it is necessary that $m = \Omega(\sqrt{n})$ in order to obtain a linear gain of capacity; recently, Wang et al. [44] derived the achievable multicast throughput for *hybrid extended networks* by cooperatively using three types of multicast strategies called *ordinary ad hoc strategy*, *BS-based strategy*, and *hybrid strategy*.

Multihop Cellular Networks. To the best of our knowledge, capacity scaling laws of multihop cellular networks were not fully studied in the literature; and the latest result is proposed in the work [50] done by Huang et al. We notice that a study of capacity scaling laws for the cognitive network model consisting of a multihop cellular primary network and an ad hoc secondary network is of both theoretical and practical significance, because of the popularity and reality of such model.

7.2 Capacity Scaling Laws for Cognitive Networks

The issue of capacity scaling laws for cognitive networks is a relatively new topic. In [3], the primary source-destination (S-D) and cognitive S-D pairs are modeled as an interference channel with asymmetric side information. In [51], the communication opportunities are modeled as a two-switch channel. The works in [3] and [51] both considered the *single-user* case in which a single primary and a single cognitive S-D pairs share the spectrum. Recently, a *single-hop* cognitive network was considered in [6], where multiple secondary S-D pairs transmit in the presence of a single primary S-D pair. The authors showed that a linear scaling law of the *single-hop* secondary network is obtained when its operation is constrained to guarantee a particular outage constraint for the primary S-D pair.

For multihop and multiple users case, Jeon et al. [7] first studied the achievable unicast throughput for cognitive networks. In their cognitive model, the primary network is a random dense SANET or a dense BS-based network [41], and the secondary network is always a random dense SANET; two networks operate on the same space and spectrum. Following the model of [7], Wang et al. [52] studied the multicast throughput for the primary and secondary networks. In order to ensure the priority of primary users in meanings of throughput, they defined a new metric called *throughput decrement ratio* (TDR) to measure the ratio of the throughput of PaN in presence of SaN to that of PaN in absence of SaN. Endowing PaN with the right to determine the threshold of the TDR, they [52] devised the multicast strategies for SaN. Both the unicast routing in [7] and multicast routing in [52] are built based on the backbones similar to the *second-class highways* in [14], which suggests that the derived throughputs are not optimal for most cases, [5], [13], [14], [53].

8 Conclusion and Future Work

We study the multicast capacity of cognitive networks. For two classical multicast strategies adopted in the primary ad hoc network (PaN), we design the corresponding strategies for the secondary ad hoc network (SaN) to achieve asymptotically the optimal throughput, ensuring the achieving throughput for PaN not decrease in order sense.

It is an interesting work to extend our results to the case where the *primary network* is an infrastructure-supported *hybrid network* or *multihop cellular network*. Another significant issue is to extend our results only applicable to *dense networks* into ones applicable to *extended networks*. Furthermore, as in the stand-alone ad hoc network, the most challenging question is to close the remaining gaps between the lower and upper bounds of multicast capacity in some regimes by presenting possibly new tight upper bounds or designing strategies to improve the asymptotic multicast throughput.

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