Minimum Power Assignment in Wireless Ad Hoc Networks with Spanner Property

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Abstract. Power assignment for wireless ad hoc networks is to assign a power for each wireless node such that the induced communication graph has some required properties. Recently research efforts have focused on finding the minimum power assignment to guarantee the connectivity or fault-tolerance of the network. In this paper, we study a *new* problem of finding the power assignment such that the induced communication graph is a spanner for the original communication graph when all nodes have the maximum power. Here, a spanner means that the length of the shortest path in the induced communication graph is at most a constant times of the length of the shortest path in the original communication graph. Polynomial time algorithm is given, for any property that can be tested in polynomial time and is *monotone* [1], to minimize the maximum assigned power. We also give a polynomial time approximation method to minimize the total transmission radius of all nodes. Finally, we propose two heuristics and conduct extensive simulations to study their performance when we aim to minimize the total assigned power of all nodes.

Keywords: Power assignment, spanner, wireless ad hoc networks.

1. Introduction

In this paper, we address the problem of finding minimum power assignment in wireless ad hoc networks such that the induced communication graph is a *spanner* of the communication graph when all nodes transmit at their maximum power. In a wireless network, each wireless node has an omnidirectional antenna and a single transmission of a node can be received by *any* node within its vicinity (called transmission range) which, we assume, is a disk centered at this node. A wireless node can receive the signal from another node if it is within the transmission range of the sender. Otherwise, they communicate through multi-hop wireless links by using intermediate nodes to relay the message. Larger transmission range of a wireless node means more neighbors it can communicate directly, but it costs more energy. Energy conservation is a critical issue in wireless ad hoc network for the node and network life, as the nodes are powered by small batteries only. Thus research efforts have focused on designing minimum-power-assignment (or called minimum-transmission-range-assignment) algorithms for typical

* The author is partially supported by NSF CCR-0311174.

network tasks such as broadcast transmission [2, 3, 4, 5], routing [6], connectivity [7, 8, 9, 10, 11], and fault-tolerance [12, 13, 14].

We consider a set $V = \{v_1, v_2, \cdots, v_n\}$ of n wireless nodes distributed in a two dimensional plane. We assume that the power w_{uv} needed to support the communication between two nodes u and v is a monotone increasing function of the Euclidean distance $\|uv\|$. In other words, $w_{uv} > w_{xy}$ if $\|uv\| > \|xy\|$ and $w_{uv} = w_{xy}$ if $\|uv\| = \|xy\|$. For example, in the literature it is often assumed that $w_{uv} = c + \|uv\|^\beta$, where c is a positive constant real number, and real number $\beta \in [2, 5]$ depends on the transmission environment. We also assume that all nodes have omnidirectional antennas, i.e., if the signal transmitted by a node u can be received by a node v, then it will be received by all nodes x with $\|ux\| \leq \|uv\|$. In addition, all nodes can adjust the transmission power dynamically. Specifically, each node u has a maximum transmission power $E_{\max}$ and it can adjust its power to be exactly w_{uv} to support the communication to another node v. Consequently, if all wireless nodes transmit in their maximum power, they define a network that has a link uv iff $w_{uv} \leq E_{\max}$. This communication graph is also called unit disk graph (UDG). When nodes adjust their power dynamically, we say that a node u can reach a node v in an *asymmetric* communication model if node u transmits at a power at least w_{uv}. Notice that here, in asymmetric communications, node v may transmit at a power less than w_{vu} and thus cannot reach u. We say that a node u can reach a node v in a *symmetric* communication model if both nodes u and v transmit at a power at least w_{uv}. In this paper, we only concern about symmetric communication model.

An observation of this model is that the network topology is entirely dependent on the transmission range of each individual node. Links can be added or removed when a node adjusts its transmission range. A *power assignment* P is an assignment of power setting $P(v_i)$ to wireless node v_i. Given a power assignment P, we can define an induced direct communication graph $\overrightarrow{G}_P$ in which there is a directed edge $\overrightarrow{uv}$ if and only if $w_{uv} \leq P(u)$. We define the induced undirected communication graph G_P in which there is an edge uv if and only if $w_{uv} \leq P(u)$ and $w_{uv} \leq P(v)$. We will hereby refer G_P to as the *induced communication graph*. If all wireless nodes transmit in their maximum power $E_{\max}$, the induced communication graph is called the *original communication graph* (unit disk graph), which provides information about all possible topologies, in accordance with characteristics of the wireless environment and node power constraints. In other words, all possible achievable network topologies are subgraphs of the original communication graph. On the other hand, given a subgraph $G = (V, E)$ of the original communication graph, we can also extract a minimum power assignment P_G, where $P_G(u) = \max_{\{v|uv \in E\}} w_{uv}$, to support the subgraph. We call this P_G an *induced power assignment* from G.

Due to the importance of energy efficiency in wireless ad hoc networks, minimum power assignment for different network issues have been addressed recently. Research efforts have focused on finding the minimum power assignment so that the induced communication graph has some "good" properties in terms of network tasks such as disjoint paths, connectivity or fault-tolerance. The minimum energy connectivity problem was first studied by Chen and Huang [7], in which the induced communication graph is strongly connected while the total power assignment is minimized. This problem has been shown by them to be NP-hard. Recently, this problem has been heavily studied and many approximation algorithms have been proposed when the network is modelled by using symmetric links or asymmetric links [8, 9, 10, 11, 15]. Along this line, several authors [12, 13, 14] considered the minimum total power assignment while the resulting network is k-strongly connected or k-connected. This problem has been shown to be NP-hard too. Solving this problem can improve the fault tolerance of the network. In [16, 17, 9], Clementi *et. al* also considered the minimum energy connectivity problem while the induced communication graph have a diameter bounded by a constant h. In [1], Lloyd *et. al* proposed one general framework that leads to an approximation algorithm for minimizing total power assignment. Using the framework they proposed a new 2-connected approximation method for power assignment. In [18], Krumke *et. al* also studied the minimum power assignment so that networks satisfy specific properties such as connectivity, bounded diameter and minimum node degree. Other relevant work in the area of power assignment (or called energy-efficiency) includes energy-efficient broadcasting and multicasting in wireless networks. The problem, given a source node s, is to find a minimum power assignment such that the induced communication graph contains a spanning tree rooted at s. This problem was proved to be NP-hard. In [2, 3, 4, 5], they presented some heuristic solutions and gave some theoretical analysis. Recently, Srinivas and Modiano [6] also studied finding k-disjoint paths for a *given* pair of nodes while minimizing the total node power needed by nodes on these k-disjoint paths. An excellent survey of some recent theoretical advances and open problems on energy consumption in ad hoc networks can be found in [19].

In this paper, we consider a new minimum power assignment problem which is *not* studied previously. The problem is to find the optimum transmission power of each individual node such that 1) the induced communication graph is a spanner of the original communication graph; 2) the total (or the maximum) power of all nodes is minimized. Here, a subgraph $H = (V, E')$ is a *t-spanner* of $G = (V, E)$ if for every $u, v \in V$, the length (or weight) of the shortest path between them in H is at most t times of the length of the shortest path between them in G. The value of t is called the *stretch factor* or *spanning ratio*. If it is bounded by a constant, we say H is a spanner of G. Therefore, if the induced communication graph is a spanner of the original communication graph, then we guarantee there is a path between each pair of nodes whose length or power consumption is similar or "not bad" compared with the original possible ones when every node uses its maximum power. This will benefit routing performance on the network topology a lot. Clearly, for this problem, a necessary and sufficient condition that a solution exists is that the unit disk graph is connected when all nodes transmit at the maximum power $E_{\max}$.

The rest of the paper is organized as follows. In Section 2, we present a polynomial time algorithm to find the power assignment whose maximum is minimized such that the induced communication graph is a spanner. In Section 3, we present an $O(1)$-approximation algorithm to find the minimum total radius assignment such that the induced communication graph is a spanner. In Section 4, we show that it is NP-hard to find the minimum total power assignment such that the induced communication graph is a spanner. Then we give two simple power assignment methods for this problem and present the performances comparison of those two min-total power assignment algorithms.We conclude our paper with discussions of possible future research directions in Section 5.

2. Min-Max Power Assignment

The formal definition of minimum maximum power assignment (min-max power assignment) problem is as follows:

Input: A set of n wireless node V, maximum node power $E_{\max}$, and a real constant $t_0 \geq 1$. Notice that given V and $E_{\max}$, it induces the original communication graph UDG.

Output: A power assignment $P = \{P(v_1), P(v_2), \cdots, P(v_n)\}$.

Object: Minimize $\max_{v \in V} P(v)$ and guarantee that the induced graph G_P is a t_0-spanner of UDG.

It is obvious that we can solve the min-max power assignment problem in polynomial time by using a binary search scheme. It was proposed in [1] by Lloyd *et. al*. Notice that since the problem only wants to minimize the maximum node power, we only need consider the case when all nodes are assigned the same power, say $P(v)$. Clearly, we can use binary search among all possible power assignments $P(v)$ to find the minimum. We give the classical method in Algorithm 1.

Here spanning ratio could be length or power spanning ratio. The correctness of this algorithm is obvious. The running time of the first step is $O(n^2 + m \log m)$. Recall that the all-pairs shortest paths can be found in $O(n^2 \log n +$

Algorithm 1 MIN-MAX POWER ASSIGNMENT

1. Building UDG:
Using V and $E_{\max}$, we first build the unit disk graph UDG, where there is an edge uv if and only if $w_{uv} \leq E_{\max}$.
Then we sort weights of all edges $uv \in UDG$, and get all possible node powers $w_1, w_2, \cdots, w_m$, where $w_1 < w_2 < \cdots < w_m \leq E_{\max}$ and $m \leq n^2$ is at most the number of links in UDG.
2. Binary search:
Initially $i = 1$, and $k_i = \lceil \frac{m}{2} \rceil$, set the power of all nodes to be $P(v) = w_{k_i}$.
repeat
  a) Building G_P:
  Using V and $P(v)$, build the induced communication graph G_P, where there is an edge uv if and only if $w_{uv} \leq P(v)$.
  b) Computing spanning ratio:
  Call a shortest path algorithm to compute the spanning ratio t for G_P according the UDG.
  c) Select new power $P(v)$:
  if $t \leq t_0$ **then**
    $k_{i+1} = k_i - \lceil \frac{m}{2^{i+1}} \rceil$,
  else
    $k_{i+1} = k_i + \lceil \frac{m}{2^{i+1}} \rceil$.
  end if
  if $k_{i+1} \neq k_i$ **then**
    set the power of all nodes to be $P(v) = w_{k_{i+1}}$ and $i = i + 1$.
  end if
until $k_{i+1} = k_i$

$mn)$, so computing the spanning ratio of given graph G_P costs $O(n^2 \log n + mn)$. The second binary search step will call the all-pairs shortest paths $\log m = O(\log n)$ times, thus, the overall time complexity is $O(\log n \cdot n \cdot (n \log n + m)) = O(n^2 \log^2 n + mn \log n)$. Therefore, the running time of our algorithm is at most $O(n^3 \log n)$.

Notice that here the weight function w_{uv} can be any weight functions, such as Euclidean distance of a link or the power needed to support the communication of the link. In addition, if we change the objective property of the induced graph from spanner to other properties, as long as the property can be tested in polynomial time and is *monotone*[1] [1], we can solve min-max power assignment problem in polynomial time. For example, we can find the min-max power assignment while the induced graph is connected, or has k-

[1] Here a property of the graph is monotone if the property continues to hold even when we increase the powers assigned to some nodes and keep the powers assigned to the other nodes unchanged.

disjoint paths. However, some properties cannot be tested in polynomial time (if $N \neq NP$), e.g., the induced graph is k-connected, and lengths of these k paths are all bounded by some constant factor of the length of shortest path in the original communication graph. In [1], Lloyd *et. al* gave an example property "G IS A TREE", which can be tested in polynomial time and makes the power assignment problem NP-complete even without any minimization objective.

3. Radius Assignment

In this section we consider problem of finding a transmission radius assignment such that the induced graph is a spanner and the total assigned radius of all nodes is minimized. We call it *min-total radius assignment* problem hereafter. There are two differences between min-total radius assignment and min-max power assignment: 1) the weight function now is the Euclidean length of the link, i.e. $w_{uv} = \|uv\|$; 2) we want to minimize the total assigned radius instead of the maximum node power of the network. The formal definition of min-total radius assignment problem is as follows:

Input: A set of n wireless node V, maximum node radius $R_{\max}$, and a real constant $t_0 \geq 1$. Notice that given V and $R_{\max}$, it induces the original communication graph UDG.

Output: A radius assignment $R = \{R(v_1), R(v_2), \cdots, R(v_n)\}$.

Object: Minimize $\sum_{v \in V} R(v)$ and guarantee that the induced graph G_R is a t_0-spanner of UDG.

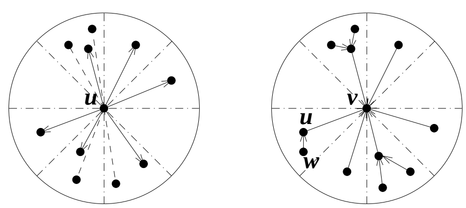
This problem seems much harder than min-max power/radius assignment, although it is still open whether it is a NP-hard problem. In this section, we now present an $O(1)$-approximation algorithm for this problem, in which we first construct a spanner using a method presented in [20, 21] and then bound the total edge length of the structure using a greedy method in [22]. Our algorithm is given in Algorithm 2.

For completeness of presentation, we review the methods of constructing a bounded degree spanner with spanning ratio t_1. We first divide the unit disk centered at each node u into k-equal sized cones, where $k \geq \pi / \arcsin \frac{1-1/\sqrt{t_1}}{2}$. For each cone apexed at node u, we select the shortest link uv (the link $\overrightarrow{uv}$ is directed actually). After processing all nodes, we have a directed graph called *Yao* structure [25]. See Figure 1 (a) for an illustration. For each node v, for each cone, we select the shortest incoming link $\overrightarrow{uv}$, and then partition the incoming neighbors locating inside this cone using the cone partition centered at node u. Then select the closest such neighbor (say w) at each cone apexed

Algorithm 2 Min-Total Radius Assignment

1. Building UDG:
Using V and $R_{\max}$, we build the unit disk graph, where there is an edge uv if and only if $w_{uv} \leq R_{\max}$.
2. Building spanner:
Use the method by [20, 21] to build a $\sqrt{t_0/t}$-spanner H of UDG where t is a positive real constant smaller than t_0.
3. Bounding weight:
Run the method in [22] to bound the total edge length of H while the spanning ratio of the final structure is t_0. The parameter of the greedy method is $\alpha = \sqrt{t_0 \cdot t}$. Clearly, the final structure (denoted by G) has spanning ratio t_0.
4. Radius assignment:
Extract the induced radius assignment R_G, where $R_G(u) = \max_{\{v|uv \in G\}} w_{uv}$, to support the subgraph.

at u and add link $\overrightarrow{wu}$. Repeat the above procedure until all neighbors are processed. See Figure 1 (b) for an illustration. The final structure by ignoring the link direction is called *YaoSink*[21], which is a t_1 spanner, and the node degree is bounded by $(k+1)^2 - 1$. Notice that the length spanning ratio of YaoSink is at most $\frac{1}{(1-(2\sin\frac{\pi}{k}))^2}$[21] and $t_1 \geq \frac{1}{(1-(2\sin\frac{\pi}{k}))^2}$ due to the selection of k.



(a) (b)

Figure 1. The structures of Yao and YaoSink, when $k = 8$. (a): The shortest edge in each cone is added as a neighbor of u for Yao. (b): The sink structure is built recursively by the center v.

We then review the greedy method with parameter α to bound the total edge length of a t_1-spanner. Consider any sparse spanner G with spanning ratio t_1 on a point set. Initialize the final structure H to be empty. We first add all edges in G with length at most D/n to H, where D is the diameter of the point set. Then we process the remaining edges of G in the increasing order of their lengths. An edge $uv \in G$ is added to H if there is no path in H connecting u and v with length $\leq \alpha\|uv\|$. Gudmundsson *et al.* [22] gave a method to perform such query efficiently by bucketing the remaining edges of G into $\log n$ groups. It is proven that the final structure H has spanning ratio

$\alpha \cdot t_1$ and its total edge length is at most $O(w(EMST))$, where $w(EMST)$ is the total edge length of Euclidean MST. Generally, for a general weighted graph $G = (V, E, w)$, let $w(G) = \sum_{uv \in G} w_{uv}$, where w_{uv} is the weight of link uv. When the weight is the Euclidean distance, the weight function is omitted hereafter. The weight of a node u in the weighted graph $G = (V, E, w)$ is $P(u) = \max_{uv \in E} w_{uv}$, and the total node weight of the graph is $P(G) = \sum_{u \in V} P(u)$.

Algorithm 2 has running time $O(n \log n)$ (after UDG is built) since remaining steps have running time at most $O(n \log n)$ [20, 21, 22]. Obviously, the summation of radii assigned to all nodes is at most $2w(G)$, which is still at most $O(w(EMST))$.

We then show that the lower bound of min-total radius assignment is $w(EMST)$. Generally, the total power assignment $P(G)$ based on any weighted graph G, to guarantee the connectivity, satisfying the following condition

$$w(EMST(G)) \leq P(G).$$

Notice that the communication graph induced by the power assignment P_G is connected. We root the tree $EMST(G)$ at an arbitrary node. For any link $uv \in EMST(G)$ where u is the parent of v, we associate link uv to node v, and call uv as $A(v)$. The definition is valid since each node can only have one parent. Clearly, $w(EMST(G)) = \sum_u w(A(u))$. On the other hand, $P(u)$ is at least the weight of the link $A(u)$. Consequently,

$$w(EMST(G)) = \sum_u w(A(u)) \leq \sum_u P(u).$$

Since the min-total radius assignment produces a communication graph with bounded spanning ratio, it clearly guarantees the connectivity of the induced communication graph. Thus, we have the following lemma and theorem.

LEMMA 1. *The optimum radius assignment for min-total radius assignment problem has total radius at least* $w(EMST)$.

THEOREM 2. *Algorithm 2 gives a solution that is within a constant factor of the optimum.*

Obviously, we can find a bounded degree subgraph with the same spanning ratio of the communication graph induced by the radius assignment calculated by Algorithm 2. If we want to find a subgraph of the induced communication graph with some additional properties such as *planar*, *fault-tolerance*, we have to replace the second step of Algorithm 2 by some other spanners. For example, Li and Wang [23] gave a method to construct a planar spanner with bounded degree. Recently, Czumaj and Zhao [24] also proposed a k-vertex fault-tolerant spanner whose total cost is $O(k^2 \cdot w(EMST))$.

4. Min-Total Power Assignment

Finally, we consider the minimum total power assignment (min-total power assignment) problem which is defined as follows.

Input: A set of n wireless node V, maximum node power $E_{\max}$, and a real constant $t_0 \geq 1$. Given V and $E_{\max}$, it induces the original communication graph UDG. Here, the weight function of a link uv becomes $w_{uv} = \|uv\|^2$ ($\beta = 2$).

Output: A power assignment $P = \{P(v_1), P(v_2), \cdots, P(v_n)\}$.

Object: Minimize $\sum_{v \in V} P(v)$ and guarantee that the induced graph G_P is a t_0-spanner of UDG.

Clearly, this problem is a NP-hard problem since the minimum energy connectivity problem is the special case of the minimum total power assignment problem in which t_0 is chosen sufficiently large. Remember the minimum total power assignment problem for connectivity is NP-hard [7]. Although there are several constant approximation methods for the minimum total power assignment problem for connectivity, it is still an open problem whether we can find a constant approximation algorithm for the minimum total power assignment problem with bounded spanning ratio. In this paper, we give two simple heuristic algorithms.

Our first approach is a simple greedy heuristic algorithm.

Algorithm 3 GREEDY MIN-TOTAL POWER ASSIGNMENT

1. Building UDG:
Using V and $E_{\max}$, we first build the unit disk graph UDG.
2. Sorting UDG edges:
Sorting edges in UDG according their weights, get $e_1, e_2, \cdots, e_m$, where $w_{e_1} \leq w_{e_2} \leq \cdots \leq w_{e_m} \leq E_{\max}$.
3. Greedy method:
Initialize G to be an empty graph. Following the increasing order, add an edge $e_i = uv$ to G if and only if no path in G (already added edges) with total power no more than $t_0 \cdot \|uv\|^2$.
4. Power assignment:
Extract the induced power assignment P_G, where $P_G(u) = \max_{\{v | uv \in G\}} w_{uv}$.

The running time of the first step is $O(n^2)$. Sorting the edges takes $O(m \log m)$. Recall that the single source shortest path algorithm can be done in $O(n \log n + m)$. The greedy step calls at most m times shortest path algorithm, so the cost is $O(n^2 \log n + mn)$. The last step takes at most $O(m)$, thus, the total costs is $O(n^2 + m \log m + n^2 \log n + mn + m)$ which is $O(n^3)$ when $m = O(n^2)$.

The second method is based on *Yao graph*. The *Yao graph* [25] with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k(G)$, is defined as follows. At each node u, any k equally-separated rays originated at u define k cones. In each cone, choose the shortest edge uv among all edges from u, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily. The resulting directed graph is called the Yao graph. See Figure 1 (a) for an illustration. Let $YG_k(G)$ be the undirected graph by ignoring the direction of each link in $\overrightarrow{YG}_k(G)$. Li *et al.* [21] proved the power stretch factor of the Yao graph $YG_k(V)$ is at most $\frac{1}{1-(2\sin\frac{\pi}{k})^2}$. The idea of our second method is to construct the t_0-spanner based on Yao structure. Consider UDG, for each node, we partition the disk into cones, and select the shortest edge of UDG in each cone. The number of cones k is chosen so that the power spanning ratio is t_0, i.e. $\frac{1}{1-(2\sin\frac{\pi}{k})^2} \leq t_0$. Thus, $k \geq \pi/\arcsin\frac{\sqrt{1-1/t_0}}{2}$. Notice, in Yao graph the cone partition does not need to be aligned. Therefore, we can choose a rotation for each node such that the maximum chosen incident link is the smallest. Obviously, there are only d_u different rotations that may produce different power assignments at node u, d_u is the degree of the node u in UDG.

Algorithm 4 YAO-BASED MIN-TOTAL POWER ASSIGNMENT

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1. Building UDG:

Using V and $E_{\max}$, we first build the unit disk graph UDG.

2. Building Yao graph:

Set $k \geq \pi/\arcsin\frac{\sqrt{1-1/t_0}}{2}$, apply YG_k on UDG. For each node u, assume that it has d_u edges $uv_1, uv_2, \cdots, uv_{d_u}$ in UDG. Then for each edge uv_i, we can assign a cone partition C_i (one of the cones started at link uv_i). We test Yao structure of u for all the d_u cone partitions C_i, and select the one whose maximum chosen link incident is the smallest. Then the union of the Yao structures of all nodes forms a graph G.

3. Power assignment:

Extract the induced power assignment P_G, where $P_G(u) = \max_{\{v|uv\in G\}} w_{uv}$.

The running time of the first step and last are the same with those of the previous algorithm. The total time of building one Yao graph takes $O(m)$. In our algorithm, we build at most d_u Yao structures at node u, so totally at most $\max_u(d_u)$ Yao graph. Therefore, the cost is at most $O(mn)$. Then, the total costs of Yao-based algorithm is $O(mn)$, which is $O(n^3)$ when $m = O(n^2)$. It seems that running time of this second algorithm is similar with the first one. However, this algorithm is much faster than the first one practically , and more importantly it can be performed in a localized way. Remember for each node to building one Yao structure, it only takes at most $O(d_u)$. So at each node, building d_u Yao structures takes at most $O(d_u^2)$. And since this algorithm can be done locally, it is quite suitable for wireless ad hoc networks.

Originally, we was planning using a subgraph of UDG called Gabriel graph [26] (GG) to save some computation in our algorithms. Let $disk(u, v)$ be the disk with diameter uv. The *Gabriel graph* contains an edge uv from UDG if and only if $disk(u, v)$ contains no other nodes $w \in V$. In [21], Li *et. al* proved Gabriel graph is a power spanner and its power stretch factor is one. Therefore, we first conjectured that it is enough to only consider the power assignment induced from subgraphs of the Gabriel graph instead of considering all possible subgraphs of UDG. However, we construct a counter example to disprove the following conjecture.

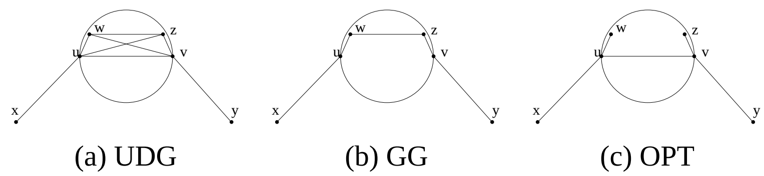


Figure 2. A counter-example for Conjecture 3. (a): the unit disk graph. (b): the Gariel graph. (c): the induced communication graph from the optimum power assignment

CONJECTURE 3. *The optimum power assignment is induced from some connected subgraph H of GG.*

DISPROOF. Assume that we have six wireless nodes and they are distributed as in Figure 2 (a). And when all nodes transmit at their maximum power, the communication graph (the unit disk graph) is shown in Figure 2 (a). Notice that $||xu|| = |yv|| > ||uv|| > ||wz|| > ||uw|| = ||vz||$. Since node w and z are inside the $disk(u, v)$, from the definition of GG, we know uv are removed in GG. Figure 2 (b) shows the Gabriel graph. The power assignment induced from GG will be $P(u) = P(v) = P(x) = P(y) = ||xu||^2$ and $P(w) = P(z) = ||wz||^2$. Therefore, the total power assignment is $P_{GG} = 4||xu||^2 + 2||wz||^2$. However, in the optimum power assignment shown in Figure 2 (c), since the power at node u needs to cover x, it is strong enough to connect u to v. Thus, link wz is removed in the optimum power assignment OPT. The power assignment induced from OPT will be $P(u) = P(v) = P(x) = P(y) = ||xu||^2$ and $P(w) = P(z) = ||uw||^2$. Clearly, the total power assignment $P_{OPT} = 4||xu||^2 + 2||uw||^2$ is less than the one induced from GG. Also it is easy to see there are no connected subgraphs H of GG that can induce the optimum power assignment, since for this special case we cannot remove any edge in GG while still keep it connected. □

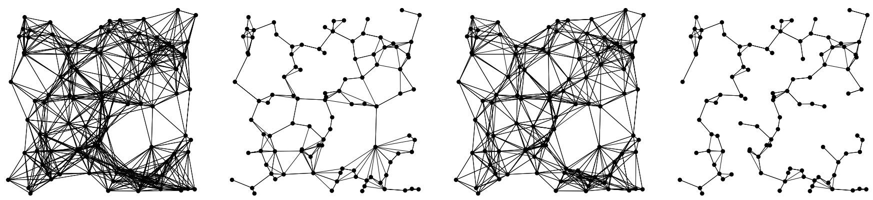


Figure 3. Different induced communication graphs under the different power assignments from the same original communication graph (UDG).

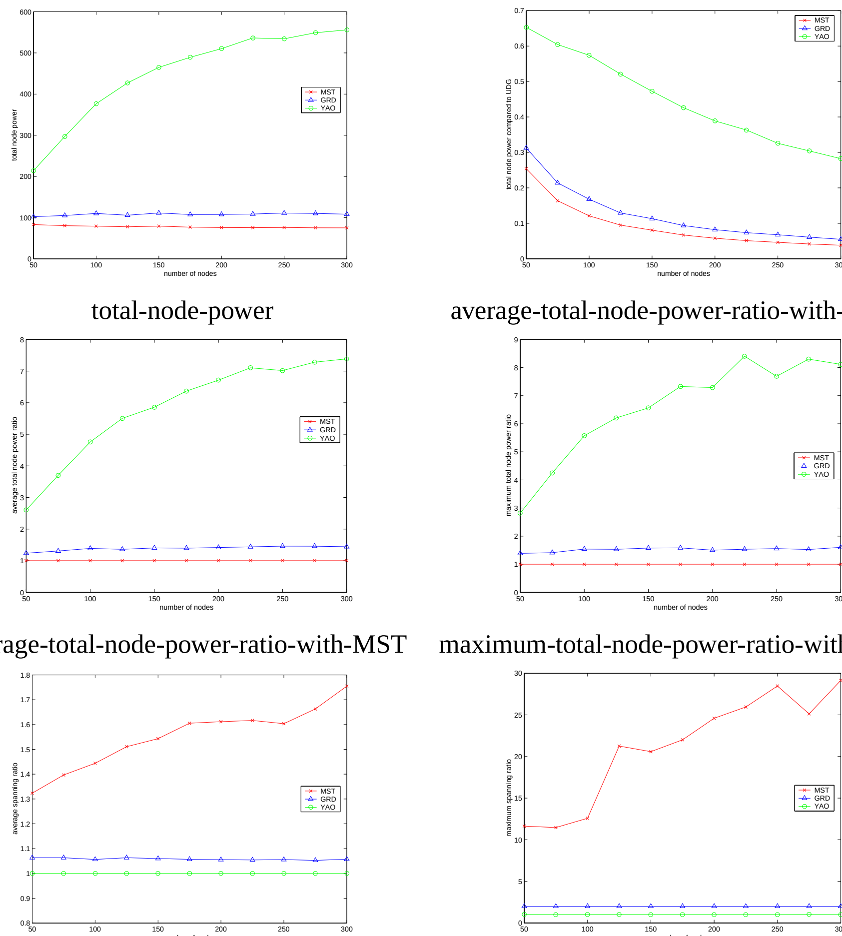
Table I. Total assigned power and spanning ratios of graphs induced by power assignment methods.

	MST	GREEDY	YAO
Avg Total-Power ($P(G)$)	78.92	106.72	366.21
Avg $P(G)/P(UDG)$	0.126	0.170	0.585
Avg $P(G)/P(MST)$	1.00	1.352	4.65
Max $P(G)/P(MST)$	1.00	1.650	5.53
Avg Spanning Ratio	1.424	1.060	1.000
Max Spanning Ratio	14.84	1.999	1.097

Since we do not give the theoretical performance analysis for our min-total power assignment heuristics, we conducted extensive simulations of both min-total power assignment methods. In experiments, we randomly generate a set V of n wireless nodes and its $UDG(V)$, and test the connectivity of $UDG(V)$. If it is connected, we apply these two min-total power assignment methods and also the MST-based method to assign power for each node. Then we compare the total power of the final power assignments.

In the first simulation, we generate 100 random wireless nodes in a 10×10 square; the spanner parameter $t_0 = 2$; and the maximum power is 2.5. We generate 100 vertex sets V (each with 100 nodes) and then apply the min-total power assignment methods for each of these 100 vertex sets. The average and the maximum are computed over all these 100 vertex sets. Figure 3 gives an example of the original communication graph and different induced communication graphs by different min-total power assignment methods. It is clear that Yao-based method keeps more links than others. Table I compares the performances of our methods with the performance of the power assignment based on MST. Remember that, it is already known [7, 8, 9] that the power assignment based on MST is within twice of the optimum power assignment for connectivity only. In this paper, we are interested in power assignment such that the induced communication graph is a spanner and we also proved in Section 3 that the optimum min-total power assignment has a lower bound

$w(MST(UDG))$. From Table I, we found that the total power assignment by greedy-based and Yao-based methods are within small constant factor of $w(MST(UDG))$. Also both the power assignment methods save many energy compared with UDG (i.e. every node uses the maximum transmission power). Notice that the spanning ratio of the communication graph induced from the power assignment induced from MST is large (almost 15 in the worst case) while the communication graph induced by our power assignment methods has spanning ratios less than 2.



total-node-power — average-total-node-power-ratio-with-UDG

average-total-node-power-ratio-with-MST — maximum-total-node-power-ratio-with-MST

average-spanning-ratio — maximum-spanning-ratio

Figure 4. Results when the number of nodes in the networks are different (from 50 to 300). Here the maximum transmission range is set as 2.5.

We then vary the number of nodes in the region from 50 to 300. The maximum transmission range is still set as 2.5. We plot the performances (with the same six metrics in Table I) of all structures in Figure 4. We also conduct experiments where we fix the number of nodes and vary the maximum transmission range. The results are similar. Due to space limit, we ignore those results here. All the results show that the spanning ratios of communication graphs induced by our greedy-based and Yao-based power assignment methods are satisfied with the input requirement while the one by MST-based method maybe large. Moreover, the total power assignments by our new methods are within small constant factor of $w(MST(UDG))$, even though we do not have theoretical results for its approximation ratios. Yao-based method keeps more links and spends more power, however it is easy to perform and can be run locally. In practice, both of our min-total power heuristics are suitable for power assignment tasks in ad hoc networks.

5. Conclusion

In this paper, we studied the power assignment for wireless ad hoc networks such that the induced communication graph is a spanner for the original communication graph when all nodes have the maximum power. Polynomial time algorithm was given, for any property that can be tested in polynomial time and is monotone, to minimize the maximum assigned power. We also proposed a polynomial time approximation method to minimize the total transmission radius of all nodes. We gave two heuristics and conducted extensive simulations to study their performance when we want to minimize the total assigned power of all nodes. For future work, we would like to know if the min-total radius assignment is NP-hard and to design approximation algorithms for min-total power assignment problem.

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