Complexity of Data Collection, Aggregation, and Selection for Wireless Sensor Networks

Xiang-Yang Li * Yajun Wang† Yu Wang‡

ABSTRACT

Processing the gathered information efficiently is a key functionality for wireless sensor networks. In this paper, we study the time complexity, message complexity, and energy cost complexity of various processing operations for a multi-hop wireless sensor network of n nodes. For most of the operations studied in this paper, we first present a lower-bound on the complexity for the optimal methods, then we provide an (asymptotically matching) upper-bound on the complexity by presenting efficient distributed algorithms to solve these problems. Let ϱ_T, ϱ_M, and ϱ_E be the approximation ratio of an algorithm in terms of time complexity, message complexity, and energy complexity respectively for a certain operation, such as data collection, data aggregation, or data selection. Specifically, we show that, for data collection, there are networks of n nodes and maximum degree Δ, such that $\varrho_M \varrho_E = \Omega(\Delta)$ for *any* algorithm. We then present an efficient algorithm for data collection with $\varrho_T = O(1)$, $\varrho_M = O(1)$, and $\varrho_E = O(\Delta)$. For data aggregation, we show that there are networks of n nodes and maximum degree Δ, such that $\varrho_T \varrho_E = \Omega(\Delta)$ for any algorithm. We then present an efficient algorithm for data aggregation with $\varrho_T = O(1)$, $\varrho_M = O(1)$, and $\varrho_E = O(\Delta)$. For data selection, we show that any deterministic distributed algorithm needs $\Omega(\Delta + D \log_D N)$ time to find the median of all data items, where N is the number of total elements collected by sensors. We then present a randomized algorithm that achieves this lower-bound with high probability. In terms of the message complexity, there is a graph G, such that $\Omega(n \log h)$ messages are required to compute the k^{th} smallest element in G in expectation and with probability at least $1/n^\delta$ for every constant $\delta < 1/2$, where $h = \min(k, N-k)$. We also present a randomized algorithm that achieves this bound with high probability.

Keywords

Time complexity, message complexity, energy, convergecast, selection, aggregation, sensor networks, distributed algorithms.

*Microsoft Research Asia, BeiJing, and Department of Computer Science, Illinois Institute of Technology, 10 W. 31st Street, Chicago, IL 60616, USA. Email: xli@cs.iit.edu,

†Microsoft Research Asia, BeiJing, China, Email: yalding@gmail.com

‡Department of Computer Science, University of North Carolina at Charlotte. Email: yu.wang@uncc.edu

1. INTRODUCTION

Wireless sensor networks (WSNs) have drawn considerable amount of research interests recently because they can collect the data, monitor the environment in a more efficient and convenient way. To design and deploy successful wireless sensor networks, a number of issues must be resolved such as deployment strategies, energy conservation, routing in dynamic environment, localization and so on. For wireless sensor networks, often the ultimate goal is to collect the data (either the raw data or in-network-processed data or both) from a set of targeted wireless sensors to some sink nodes and then perform some further analysis at sink nodes. Convergecast is the common many-to-one communication pattern used for these sensor network applications. In this paper, we study some fundamental complexity problems arising from different types of convergecast (*i.e.*, data collection, data aggregation, and distributed data selection) in wireless sensor networks.

One of the goals of WSNs is to collect the raw data to the sink node(s), or support various queries from the sink node(s), such as those formed in an SQL-like language. It is envisioned that the sink node issues queries regarding the data collected by some target senors, and the senors collaboratively generate an accurate or approximate response. In this paper, we study three different data processing operations, namely, *data collection*, *data aggregation*, and *data selection*. For each problem, we will study its complexity and present efficient algorithms to solve it. The complexity of a problem is defined as the worst case [1] cost (time, message or energy) by the best algorithm. Studying the complexity of a problem is often challenging. We will also design efficient algorithms whose complexity is asymptotically same as (or within a certain factor of) the complexity of that problem. *Data collection* is to collect the set of data items A_i stored in each individual node v_i to the sink node v_0. In *data aggregation*, the sink node wants to know the value $f(A)$ for a certain function f of all data items A, such as minimum, maximum, average, variance and so on. *Data selection* is to find the kth smallest (or largest) value of the set A where k could be any arbitrary value. One typical example of data selection is to find the median. Generally speaking, selection solves aggregation queries about order statistics and percentiles. Data collection and aggregation has been extensively studied in the community of networking and database. Surprisingly, little is known about distributed (network) selection, despite it is a significant part in understanding the data aggregation, especially for wireless networks.

All these three problems have been extensively studied in wired networks. For data collection, it is a folklore result that the total number of packet relays will be the smallest if we collect data using

[1]The worst case is to consider all networks of n nodes (and possibly diameter D, and maximum nodal degree Δ) and all possible distributions of all data items A over all nodes V.

the breadth-first-search tree (BFS). It also has the smallest delay for wired networks. In [19], five distributive aggregations *max*, *min*, *count*, *sum* and *average* are carried out efficiently on a spanning tree. Subsequent work did not quite settle the time complexity, the message complexity and the energy complexity of data collection and aggregation, nor the tradeoffs among these three possibly conflicting objectives. The closest results are [14, 15, 17]. All assume a complete wireless network, which is usually not true in practice. Furthermore, to the best of our knowledge, no fundamental results on the tradeoffs among the time complexity, message complexity, and energy complexity were known before this work.

Finding the median (generally holistic queries) efficiently often requires sophisticated design. In [18], Kuhn *et al.* studied the problem of distributed selection for general *wired* networks with N data items distributed in a network of n nodes and diameter D. In particular, they proved that distributed selection is strictly harder than convergecast with simple aggregation by giving a lower bound of $\Omega(D \log_D N)$ on the time complexity. They then present a novel Las Vegas algorithm which matches this lower bound with high probability[2] (*w.h.p.*). Finally, they de-randomized their algorithm, and proposed a deterministic distributed selection algorithm with a time complexity of $O(D \log_D^2 n)$ which constitutes a substantial improvement over prior art. By assuming that each data item can be represented in $O(\log N)$ bits, Patt-Shamir [24] presented a deterministic algorithm that computes the median value such that each node transmits only $O((\log n)^2)$ bits, and a randomized algorithm that computes an approximate median in which each node transmits $O((\log \log n)^3)$ bits.

To the best of our knowledge, we are the first to study the tradeoffs among the message complexity, time complexity, and energy complexity for data collection, data aggregation and data selection; we are the first to present lower bounds (and matching upperbounds for some cases) on the message complexity, time complexity, and energy complexity for these three operations in wireless networks. The main contributions of this paper are as follows.

Data Collection: We design algorithms whose time complexity and message complexity are within constant factors of the optimum. The minimum energy data collection can be done using minimum cost shortest path tree. We further show that no data collection algorithm can achieve approximation ratio ϱ_M for message complexity and ϱ_E for energy complexity with $\varrho_M \cdot \varrho_E = o(\Delta)$. We then prove that our data collection algorithm has energy cost within a factor $O(\Delta)$ of the optimum while its time and message complexity are within $O(1)$ of the corresponding optimum. Thus, our method achieves the best tradeoffs among the time complexity, message complexity and energy complexity.

Data Aggregation: We also design algorithms for data aggregation whose time complexity and message complexity are within constant factors of the optimum. The minimum energy data aggregation can be done using minimum cost spanning tree (MST). We further show that no data aggregation algorithm can achieve approximation ratio ϱ_T for time complexity and ϱ_E for energy complexity with $\varrho_T \cdot \varrho_E = o(\Delta)$. We then continue to show that our data aggregation algorithm has energy cost within a factor $O(\Delta)$ of the optimum. In other words, our method achieves the best tradeoffs among the time complexity, message complexity and energy complexity with $\varrho_T = O(1)$, $\varrho_M = 1$, $\varrho_E = O(\Delta)$.

Data Selection: We first show that any deterministic distributed algorithm needs at least $\Omega(\Delta + D \log_D N)$ time to find the median of all data items when each node has at least one data item. We then present a randomized algorithm to find the median in time $O(\Delta + D \log_D N)$ when each node has $O(1)$ data item. In terms of the message complexity, we show that, $\Omega(n \log h)$ messages are required to compute the k^{th} smallest element in expectation, and with probability at least $1/n^\delta$ for every constant $\delta < 1/2$. We also present a randomized algorithm that can find the median with $O(N + n_{\mathbf{C}} \log N)$ messages with high probability, where $n_{\mathbf{C}}$ is the size of the minimum connected dominating set. In terms of energy complexity, we present a randomized efficient method that find the median with energy cost at most $O(\omega(MST) \cdot \log N)$ *w.h.p.*, which is at most $\log N$ times of the minimum. Value sensitive methods (whose complexity depending on the found value f_k) are also presented for finding the kth smallest element.

The rest of the paper is organized as follows. In Section 2, we first present our wireless sensor network model, define the problems to be studied in this paper, and then briefly review the connected dominating set. We study the complexity of distributed data collection, data aggregation, and data selection in WSNs in Section 3, Section 4, and Section 5 respectively. We review the related works in Section 6 and conclude the paper in Section 7.

2. PRELIMINARIES

2.1 Network Model

In this paper, we mainly focus on the complexities of various data operations in wireless sensor networks. Thus, for simplicity, we assume a simple and yet general enough model that is widely used in the community. We assume that there are $n + 1$ wireless sensor nodes $V = \{v_0, v_1, v_2, \cdots, v_n\}$ that are deployed in a certain geographic region, where v_0 is the sink node. Each wireless sensor node corresponds to a vertex in a graph G and two vertices are connected iff their corresponding sensor nodes can communicate directly. The graph G is called the communication graph of this sensor network. We assume that links are "reliable": when a node v_i sends some data to a neighboring node v_j, the total message cost is only 1. In some of the results, we further assume that all sensor nodes have a communication range r and a fixed interference range $R = \Theta(r)$. For simplicity, we may assume that $r = 1$, *i.e.*, the communication graph G is a *Unit Disk Graph*(UDG). Let $h(v_i, v_j)$ be the hop number of the minimum hop path connecting v_i and v_j in graph G, and $D(G)$ be the diameter of the graph, *i.e.*, $D(G) = \max_{v_i,v_j} h(v_i, v_j)$. Here, we assume that $D(G) \geq 2$. If $D(G) = 1$, then the graph G is simply a completed graph and all questions studied in this paper can either be trivial or have been solved [14, 15, 17]. For a graph G, we denote its maximum degree as $\Delta(G)$. When each node v_i has n_i data items, we define the weighted degree, denoted as $\tilde{d}_{v_i}(G)$, of a node v_i in graph G as $n_i + \sum_{v_j : v_i v_j \in G} n_j$. The maximum weighted degree of a graph G, denoted as $\tilde{\Delta}(G)$, is defined as $\max_i \tilde{d}_{v_i}(G)$.

Each wireless node has an ability to monitor the environment, and collect some data (such as temperature). Assume that $A = \{a_1, a_2, \cdots, a_N\}$ is a totally ordered multi-set of N elements collected by all n nodes. Here, N is the cardinality of set A. Each node v_i has n_i amount of raw data, denoted as $A_i \subset A$. Since A is a multi-set, we assume $A_i \cap A_j = \emptyset$ and $A = \bigcup_{i=1}^{n} A_i$ Then $\langle A_1, A_2, \cdots, A_n \rangle$ is called a distribution of A at sites of V. We assume that one packet (*i.e.*, message) can contain one data item a_i, the node ID, plus additional constant number of bits, *i.e.*, the packet size is at the order of $\Theta(\log n + \log U)$, where U is the upper-bound on values of a_i. Such a restriction on the message size is realistic and needed, otherwise a single convergecast would suffice to accumulate all data items to the sink which will subsequently solve the problems easily. We consider a TDMA MAC schedule and assume that one time-slot duration allows transmission of exactly one

[2]An event is said to happen with high probability if it happens with probability at least $1 - \frac{1}{n}$.

packet.

If energy consumption is to be optimized, we assume that the *minimum* energy consumption by a node u to send data correctly to a node v, denoted as $E(u, v)$, is $c_1 \cdot \|u - v\|^\alpha + c(v)$, where c_1 (normalized to 1 hereafter) and $\alpha \geq 2$ are constants depending on the environment, and $c(v)$ is the receiving cost of the node v. In some of our results, we assume that $c(v) = 0$. We assume that each wireless sensor node can dynamically adjust its transmission power to the minimum needed. We also assume that when the sensor node is in idle state (not transmitting. not receiving), its energy consumption is negligible. Since we assume that the TDMA MAC is used, the activity cycles for sensor nodes are assumed to be synchronized, and for any time slot, no sensor nodes will be listening for transmissions if it is not scheduled to receive data packets.

For data queries in WSNs, we often need build a spanning tree T of the communication graph G first for pushing down queries and propagating back the intermediate results. Given a tree T, let $H(T)$ denote the height of the tree, *i.e.*, the number of links of the longest path from root to all leave nodes. The depth of a node v_i in T, denoted as $d_T(v_i)$, is the length of the path from the root to v_i. The subtree of T rooted at a node v_i is denoted as $T(v_i)$, the parent node of v_i is denoted as $p_T(v_i)$, and the set of children nodes of a node v_i is denoted as $\mathrm{Child}(v_i)$.

2.2 Problems and Complexities

We will mainly study the time complexity, message complexity, and energy complexity of three different data operations, namely, data collection, data aggregation, and data selection.

The complexity measures we use to evaluate the performance of a given protocol are worst-case measures. The *message complexity* (and the *energy complexity*, respectively), of a protocol is defined as the maximum number of total messages (the total energy used, respectively) by all nodes, over all inputs, *i.e.*, over all possible wireless networks $\mathcal{G}$ of n nodes (and possibly with additional requirement of having diameter D and/or maximum nodal degree Δ) and all possible data distributions of A over V. The *time complexity* is defined as the elapsed time from the time when the first message was sent to the time when the last message was received. The *lower bound* on a complexity measure is the minimum complexity required by *all* protocols that answer the queries correctly. The approximation ratio ϱ_T (resp. ϱ_M and ϱ_E) for an algorithm denotes the worse ratio of the time complexity (resp. message complexity and energy consumption) used by this algorithm compared to an optimal solution over all possible problem instances. Here we assume that TDMA MAC is used for channel usage. Obviously, the complexity depends on the TDMA schedule policy $\mathcal{S}$. Let $X(v_i, t)$ denote whether node v_i will transmit at time slot t or not. Then a TDMA schedule policy $\mathcal{S}$ is to assign 0 or 1 to each variable $X(v_i, t)$. A TDMA schedule should be *interference free*: no receiving node is within the interference range of the other transmitting node. In other words, if the schedule is defined for tree T, for any time slot t, if $X(v_i, t) = 1$, then $X(v_j, t) \neq 1$ for any node v_j such that $p_T(v_i)$ is within the interference range of v_j.

We now define the three data operation problems in a more formal way.

Data collection is an operation to collect the set of *raw* data items A from all sensor nodes to the sink node. It can be done by building a spanning tree T rooted at the sink v_0, and sending the data from every node v_i to the root node along the unique path in the tree. Clearly, the **message complexity** of data collection along T is $\sum_{i=1}^{n} n_i \cdot d_T(v_i)$. The **energy complexity**, defined as the total energy needed by all nodes for completing an operation, of data collection using T is $\sum_{i=1}^{n}[E(v_i, p_T(v_i)) \cdot \sum_{v_j \in T(v_i)} n_j]$.

The TDMA schedule should also be *valid* in the sense that every datum in the network will be relayed to the root. In other words, in tree T, when node v_i sends a datum to its parent $p_T(v_i)$ at a time slot t, node $p_T(v_i)$ should relay this datum at some time-slot $t' > t$. The largest time $\mathcal{D}$ such that there exists a node v_i with $X(v_i, \mathcal{D}) = 1$ is called the **time complexity** of this valid schedule. Time $\mathcal{D}$ is also called the *mark-span* of the schedule $\mathcal{S}$. Clearly, a schedule $\mathcal{S}$ is *valid* for data collection of A using tree T, iff for every node v_i and time slot t such that $X(v_i, t) = 1$, then

$$\sum_{v_j \in \mathrm{Child}(v_i)} \sum_{b=1}^{t-1} X(v_j, b) + n_i \geq \sum_{b=1}^{t} X(v_i, b).$$

Here $\sum_{v_j \in \mathrm{Child}(v_i)} \sum_{b=1}^{t-1} X(v_j, b) + n_i$ is the total number of data items node v_i has seen so far till time slot t and $\sum_{b=1}^{t-1} X(v_i, b)$ is the total number of data items that have been relayed by node v_i so far till time slot t. Then the time-complexity optimizing data collection problem is to find a spanning tree T and a *valid*, *interference-free* schedule $\mathcal{S}$ such that the mark-span is minimized.

Data Aggregation: The database community classifies aggregation functions into three categories: be distributive (*e.g.*, max, min, sum, and $count$), algebraic (*e.g.*, $plus$, $minus$, $average$, $variance$) and holistic (*e.g.*, $median$, k^{th} smallest or largest). In this paper, we call the distributive or algebraic aggregation *data aggregation* and the holistic aggregation *data selection*. A function f is said to *distributive* if for every disjoint pair of data set X_1, X_2, we have $f(X_1 \cup X_2) = h(f(X_1), f(X_2))$ for some function h. Actually, except for $count$, we have $h = f$. For example, when f is sum, then h can be set as sum. For wired networks, it has been well-known that the distributive and algebraic functions can easily be computed using *convergecast* operations, which is a straightforward applications of flooding-echo on a spanning tree.

Given an algebraic function f and a wireless network G, it is easy to show that each node only needs to send out information once. Hence, the connectivity of the communication graph of the data aggregation implies that it should be a tree to be optimal. Our task is to construct a data aggregation tree T and nodes' transmission schedule to optimize the time-complexity, or the message complexity, or the energy-cost complexity. Generally, we assume that the algebraic aggregation function F can be expressed as the combination of k distributive functions for some constant integer k, *i.e.*, $f(X) = h(g_1(X), g_2(X), \cdots, g_k(X))$. For example, when f is $average$, then $k = 2$ and g_1 can be set as sum and g_2 can be set as $count$ (obviously both g_1 and g_2 are distributive) and h can be set as $h(y_1, y_2) = y_1/y_2$. Hereafter, we assume that an algebraic function f is given in formula $h(g_1(X), g_2(X), \cdots, g_k(X))$. Thus, instead of computing f, we will just compute $y_i = g_i(X)$ distributively for $i \in [1, k]$ and $h(y_1, y_2, \cdots, y_k)$ at the sink node.

Given a distributive function g_i (and their corresponding function h_i) and a data aggregation tree T, the **message complexity** clearly is the number of edges in T, which is fixed as n (recall that the root is node v_0). The **energy-cost complexity** clearly is the total energy-cost used by all n links, *i.e.*, $\sum_{i=1}^{n} E(v_i, p_T(v_i))$. This can be easily found using minimum spanning tree algorithm where the link cost of uv is the energy-cost for supporting the communication of a link uv. The **time complexity** of data aggregation depends on the schedule $\mathcal{S}$. Clearly, a schedule $\mathcal{S}$ is *valid* for data aggregation of A using tree T, if for every node v_i it is scheduled to transmit at a time slot t only if it has received data from *all* of its children nodes. Consequently, the time-complexity of *any* data aggregation scheme for a wireless network G is at least the height of the BFS tree, $H(\mathrm{BFS}(G))$, rooted at sink v_0.

Data Selection is to find the kth ranked number from a given N

numbers (possibly stored in a network). It is well-known that data selection can be done in linear time in a centralized manner [10]. Data selection is a holistic operation. Aggregate function f is *holistic* if there is no constant bound on the size of the storage needed to describe a sub-aggregate. All proposed algorithms for data selection are *iterative*, in the sense that they continuously reduce the set of possible solutions. The search space is iteratively reduced until the correct answer is located.

In this paper, we will mainly study the complexity and efficient algorithm for these operations in wireless sensor networks. To address each of these problems, we usually first build a spanning tree T and then decide a interference-free and valid schedule of nodes activities such that certain complexity measure is optimized. However, our lower bound and approximation argument do not depend on the communication graph used, which may not be a tree.

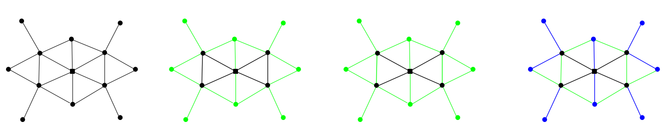
2.3 Connected Dominating Set

A number of our methods will be based on a "good"[3] connected dominating set (CDS) that has bounded degree **d** and bounded hop spanning ratio. Here a subgraph H of G is a connected dominating set if (1) graph H is connected, and (2) the set of vertices of H is a *dominating set*, *i.e.*, for every node $v \in G \setminus H$, there is a neighboring node $u \in H$, *i.e.*, $uv \in G$. A subgraph H of G has a bounded spanning ratio if for every pair of nodes u and v in H, the distance (hop or weighted distance) of the shortest path connecting u and v in H is at most a constant times of the distance of the shortest path connecting them in the original graph G.

A number of methods have been proposed in the literature to construct such a good CDS. See [1, 2] for more details. A simple method is to partition the deployment region into grid of size $r/\sqrt{2}$, select a node (called *dominator*) from each cell if there is any, and then find nodes (called *connectors*) to connect every pair of dominators that are at most 3-hops apart. Then the diameter of the CDS is at most a constant times of the original diameter of graph G. We assume the availability of a good CDS hereafter.

THEOREM 1 ([1]). *Consider any graph $G = (V, E)$ and a dominating set V' of G. Let $G' = (V, E')$ be the graph obtained by connecting all pairs of dominating nodes in V' that are at most 3-hops apart by connectors, and each dominatee node to a unique neighboring dominator. Then $D(G') \le 5D(G)$ and $\Delta(G') \le$ **d**, for a constant **d**.*

Given a graph $G = (V, E)$, let $\mathbf{C} = (V_{\mathbf{C}}, E_{\mathbf{C}})$ be a connected dominating set of G. See Figure 1 for illustration. For a node $v \in V_{\mathbf{C}}$, let $T_{\mathbf{C}}$ be a BFS tree of $\mathbf{C}$. For a node $v \in V \setminus V_{\mathbf{C}}$, we define a unique dominator $d(v)$ which is the one having the shortest hop distance to the sink v_0.



(a) Graph G (b) CDS $\mathbf{C}$ (c) BFS Tree $T_{\mathbf{C}}$ (d) DCT Tree T

Figure 1: Illustrations of a graph G, a CDS $\mathbf{C}$ of G, the BFS tree $T_{\mathbf{C}}$, and the data communication tree (DCT) T.

DEFINITION 1 (DATA COMMUNICATION TREE (DCT)). *For a graph G and its CDS, the data communication tree T is defined as follows: $T = (V, T_C \cup \{\overline{vd(v)} \mid v \in V \setminus V_C\}$.*

[3] A CDS for a UDG graph is said to be "good" here if each node has a constant degree in CDS, and the for each pair of nodes, the shortest path connecting them via CDS is at most a constant factor of the shortest path connecting them in the original UDG graph.

Given data communication tree, an aggregate operation consists of (possibly repeated) two phases: a *propagation* phase where the query demands are pushed down into the sensor network along the tree; and an *aggregation* phase where the aggregated values are propagated up from the children to their parents. We discuss some properties of the data communication tree.

THEOREM 2. *Let G and **C** be a graph and its CDS respectively. The data communication tree T has following properties:*

1. *$\Delta(T_C) \le$ **d**.*
2. *For any edge $e \in E_T$, let $I(e)$ be the set of edges in T_C that have interferences with e, then $|I(e)| \le c \cdot \mathbf{d} \cdot \Delta(G)$ for some constant c depending on R/r.*

PROOF. The first property directly comes from the property of the CDS **C**. For any edge $e = \overline{uv} \in E_T$, either u or v will be in **C** based on our construction. Assume $u \in V(\mathbf{C})$. For all edges having interferences with e, both end nodes should be within distance $2r + R$ from u. Since $R = \Theta(r)$ and the CDS has a constant-bounded degree, there are at most a constant number of nodes of **C** within this range. On the other hand, all edges of T have at least one node in **C**. Then it is easy to show that $|I(e)| \le (\frac{R+2r}{r})^2 \mathbf{d} \cdot \Delta(G)$ by an area argument. This finishes the proof. □

All our methods will be based on a good CDS and using **data clustering**: given a good CDS, for a node $v \in V \setminus V_{\mathbf{C}}$, it sends the data items to its dominator $d(v)$ in a TDMA manner.

LEMMA 3. *Given a good CDS of the graph G, data clustering can be done in time $O(\tilde{\Delta}(G))$.*

PROOF. We use the communication tree T to do data clustering. For a node $v \in V \setminus V_{\mathbf{C}}$, assume that the edge $\overline{vd(v)}$ interferes with an edge $\overline{ud(u)}$. Then dominator nodes $d(u)$ and $d(v)$ are within distance at most $R + 2r$. Thus, there are at most $\frac{(R+2r)^2}{r^2}\mathbf{d}$ such dominator nodes. Consequently, the total number of data items of all nodes u such that $\overline{ud(u)}$ interferes with $\overline{vd(v)}$ is at most $\frac{(R+2r)^2}{r^2}\mathbf{d} \cdot \tilde{\Delta}(G) = \Theta(\tilde{\Delta}(G))$. Hence, every such edge $\overline{v_i d(v_i)}$ can be scheduled to transmit n_i times in $\Theta(\tilde{\Delta}(G))$ time-slots using a simple greedy coloring method that colors the nodes sequentially using the smallest available color. □

After data clustering, all data elements are clustered in $T_{\mathbf{C}}$. In other words, each node v_i in the connected dominating set now will have data from all nodes dominated by v_i. The data clustering asymptotically does not incur additional cost for time complexity and message complexity when $n_i = O(1)$. Notice that the total number of messages for data clustering is $\sum_{v_i \notin V_{\mathbf{C}}} n_i$.

3. DATA COLLECTION

In this section, we discuss the complexity of collecting data in wireless sensor networks.

3.1 Message, Energy, and Time Complexity

It is easy to show that the data collection can be done with minimum number of messages $\sum_{i=1}^{n} n_i \cdot h(v_i, v_0)$ using a BFS tree with root v_0. We then study the data collection with the minimum energy cost. Apparently, for any element, it should follow the minimum energy cost path from its origin to the sink node v_0 in order to minimize the energy consumption, where the weight of each link is the energy needed to support a successful transmission using this link. So minimizing the energy is equivalent to the problem of finding the shortest paths from the sink to all nodes, which clearly can be done in time $O(m + n \log n)$ for a communication graph of n nodes and m links. We call the tree formed by minimum energy paths from the root to all nodes as the *minimum energy path tree (MEPT)*. Then we study the time complexity of data collection.

Algorithm 1 presents our efficient data collection method based on a good CDS $\mathbf{C}$. The constructed CDS has the maximum nodal degree at most a constant $\mathbf{d}$, and similar to Theorem 2, all nodes in CDS can be scheduled to transmit once in constant $\beta = \Theta(\mathbf{d})$ time-slots without causing interferences to other nodes in CDS. We take β time-slots as one *round*.

First, the data elements from each dominatee node (a node not in $\mathbf{C}$) are collected to the corresponding dominator node in the connected dominating set $\mathbf{C}$. Here the dominatee nodes that are one-hop away from the sink node v_0 will directly send the data to v_0. Notice that this can be done in time-slots $O(\tilde{\Delta}(G))$.

Now we only consider the dominator nodes and the breadth-first-search spanning tree $T_{\mathbf{C}}$ of nodes in CDS rooted at the sink v_0. Every edge in the tree $T_{\mathbf{C}}$ will be scheduled exactly once in each round. For simplicity, we do not schedule sending an element more than once in the same round. At every round, nodes in CDS push one data item to its parent node until all data are received by v_0.

Algorithm 1 Efficient Data Collection Using CDS

Input: A CDS $\mathbf{C}$ with bounded degree $\mathbf{d}$, tree $T_{\mathbf{C}}$.

```
Every node v_i sends its data to its dominator node d(v_i).
for t = 1 to N do
    for each node v_i ∈ V_C do
        If node v_i has data not forwarded to its parent node in T_C,
        node v_i sends a new data to its parent in round t.
```

THEOREM 4. *Given a connected wireless network G, data collection can be done in time $\Theta(N)$ with $\Theta(\sum_{i=1}^{n} n_i h(v_i, v_0))$ messages.*

PROOF. From Lemma 3, in $O(\tilde{\Delta}(G))$ time-slots, the data elements from each dominatee node are collected to the corresponding dominator node in the connected dominating set. We show that after $N + H(T_{\mathbf{C}})$ rounds, all elements can be scheduled to arrive in the root, where $H(T_{\mathbf{C}})$ is the height of the BFS tree $T_{\mathbf{C}}$. Algorithm 1 illustrates our method to achieve this.

A CDS node v is in level i if the path from v to v_0 in BFS tree $T_{\mathbf{C}}$ has i hops. A level i is said to be *occupied* at a time instance if there exists one CDS node from level i that has at least one data. Assume that originally all levels $i \in [1, H(T_{\mathbf{C}})]$ are occupied, after collecting data from all dominatee nodes. We will show that each round the root will get at least one data item if there are data in the network. We essentially will show that the occupied levels are *continuous*, *i.e.*, before each round t, there is L_t such that all levels in $[1, L_t]$ are occupied and levels in $[L_t+1, H(T_{\mathbf{C}})]$ are not. We prove this by induction. This is clearly true for round 1. Assume that it is true for round t. Then in round t, for each level $i \in [1, L_t - 1]$, every node in level $i+1$ will send its data to its parent in level i. Then every level $i \in [1, L_t - 1]$ will have data for sure before round $t+1$. Then clearly, $L_{t+1} = L_t$ if some nodes in level L_t still have some data; otherwise we set $L_{t+1} = L_t - 1$. Consequently, root will get at least one data item for each round whenever there are data in the network. Since there are at most N data items, Algorithm 1 will take at most N rounds, *i.e.*, $O(N)$ time-slots because each round is composed of constant β time-slots.

When not all levels are occupied initially, then it is easy to show that after at most $H(T_{\mathbf{C}})$ rounds, the occupied levels will be *continuous*. Hence, the collection can be done in at most $N + H(T_{\mathbf{C}})$ rounds. Notice that $H(T_{\mathbf{C}}) = \Theta(D(G))$. Consequently, the total time-slots are at most $O(\tilde{\Delta}(G)) + O(N + D) = O(N)$ since $\tilde{\Delta}(G) \leq N$.

On the other hand, for any data collection algorithm, it needs at least N time slots since the sink can only receive one data item in one time slot and there are N data items.

The total number of messages used by the algorithm is of course at most $4\sum_{i=1}^{n} n_i h(v_i, v_0)$ as the element at node v_i are relayed by at most $4 \times h(v_i, v_0)$ nodes in CDS (since $h(v_i, v_0) \geq 2$). Obviously any algorithm needs at least $\sum_{i=1}^{n} n_i h(v_i, v_0)$ messages. This finishes the proof. □

3.2 Complexity Tradeoffs

One may want to design a universal data collection method whose time-complexity, message-complexity and energy-complexity are all within constant factors of the optimum. Observe that Algorithm 1 is a constant approximation for both time-complexity and message-complexity. However, it is not a constant approximation for energy-complexity. Consider the following line network example: $n+1$ nodes are uniformly distributed in a line segment $[0, r]$; Sink v_0 is the leftmost node and node v_i is at position $i \cdot r/n$ and has one data item. Here we assume $r = 1$. See Figure 2 for illustration. Assume the energy cost for a link uv is $\|uv\|^2$. Then the minimum cost data collection is to let node v_i send all its data to node v_{i-1}. The total energy cost is $\sum_{i=1}^{n} i \cdot \frac{1}{n^2} \simeq 1/2$. While the energy cost of collecting data via CDS is $\sum_{i=1}^{n} (\frac{i}{n})^2 \simeq n/6$. On the other hand, the total number of messages of the minimum-energy data collection scheme is $n(n-1)/2$ and the time slots used by this scheme is also $\Theta(n^2)$; both of which are $\Theta(n)$ times of the corresponding minimum.

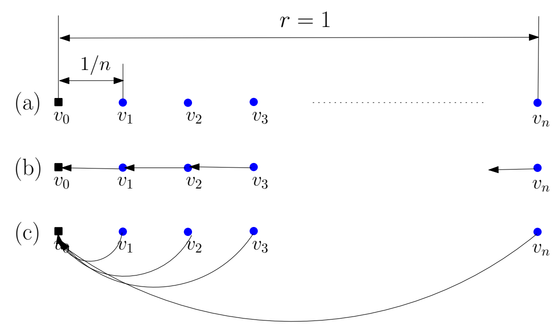


Figure 2: Example: (a) a line network with $n+1$ nodes; (b) the minimum energy data collection tree; (c) the data collection tree via CDS, where v_0 is the only dominator.

Consider any data collecting algorithm $\mathcal{A}$. Let ϱ_M and ϱ_E be the approximation ratio for the message-complexity and energy-complexity of algorithm $\mathcal{A}$. We show that there are graphs of n nodes such that $\varrho_M \cdot \varrho_E = \Omega(n)$.

THEOREM 5. *Assume the energy cost for supporting a link uv is $\|uv\|^2$. For any data collection algorithm $\mathcal{A}$, there are graphs of n nodes, such that $\varrho_M \cdot \varrho_E = \Omega(n)$.*

PROOF. Consider the line graph example defined previously. For a node v_i, assume that the data collection path is composed of k_i hops and the length of the k_i links are $x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}$. Then $\sum_{j=1}^{k_i} x_{i,j} = \frac{i}{n}$. The total energy cost, denoted as e_i, of such data collection path is $e_i = \sum_{j=1}^{k_i} x_{i,j}^2 \geq \frac{(\sum_{j=1}^{k_i} x_{i,j})^2}{k_i}$. Thus,

$$e_i \cdot k_i \geq (\frac{i}{n})^2.$$

Obviously, the total number of messages are $\sum_{i=1}^{n} k_i$ and the total energy cost is $\sum_{i=1}^{n} e_i$. We will use the Holder's inequality:

for positive a_i and b_i, $p > 0$, $q > 0$ with $\frac{1}{p} + \frac{1}{q} = 1$, we have

$$(\sum_{i=1}^{n} a_i^p)^{\frac{1}{p}} (\sum_{i=1}^{n} b_i^q)^{\frac{1}{q}} \geq \sum_{i=1}^{n} a_i \cdot b_i.$$

Equivalently, $(\sum_{i=1}^{n} a_i)^{\frac{1}{p}} (\sum_{i=1}^{n} b_i)^{\frac{1}{q}} \geq \sum_{i=1}^{n} a_i^{\frac{1}{p}} \cdot b_i^{\frac{1}{q}}$. Then

$$(\sum_{i=1}^{n} k_i)(\sum_{i=1}^{n} e_i) \geq (\sum_{i=1}^{n} \sqrt{e_i} \cdot \sqrt{k_i})^2 \geq (\sum_{i=1}^{n} \frac{i}{n})^2 = \frac{(n-1)^2}{4}$$

Clearly, the minimum number of messages is n for any scheme and the minimum energy cost is $1/2$ for any scheme. Thus, we have $\varrho_M \cdot \varrho_E \geq (n-1)^2/(2n) = \Theta(n)$. This finishes the proof. □

Notice that we generally assumed that the energy cost for supporting a link uv is $\|uv\|^\alpha$. Then we can show that

$$(\varrho_M)^{\alpha-1} \varrho_E \geq \frac{n^{\alpha-1}}{2^{\alpha-1}}.$$

Notice that since $\varrho_E \geq 1$ and $\alpha \geq 2$, we have $(\varrho_M)^{\alpha-1}(\varrho_E)^{\alpha-1} \geq (\varrho_M)^{\alpha-1}\varrho_E \geq \frac{n^{\alpha-1}}{2^{\alpha-1}}$. Consequently, $\varrho_M \cdot \varrho_E \geq n/2$ still holds.

When we also take the maximum degree Δ into account, the preceding theorem implies the following corollary (the proof is essentially same).

COROLLARY 6. *For any data collection algorithm $\mathcal{A}$, there are graphs of n nodes with maximum degree Δ, such that $\varrho_M \cdot \varrho_E = \Omega(\Delta)$.*

The preceding theorem also implies that for any data collection algorithm $\mathcal{A}$, $\varrho_M \cdot \varrho_E \cdot \varrho_T = \Omega(\Delta)$, where ϱ_T is the approximation on the time-complexity by algorithm $\mathcal{A}$. We then show that for Algorithm 1, $\varrho_E = O(\Delta(G))$.

THEOREM 7. *Algorithm 1 is $\varrho_E \leq 8\Delta(G)$-approximation for energy cost when the energy to support a link uv is $\|uv\|^\alpha$.*

PROOF. Consider any node v_i and its minimum energy path $P_{v_i v_0}(G) = u_1 u_2 \cdots u_k$ to the sink node v_0 in the original communication graph G, where $u_1 = v_i$ and $u_k = v_0$. Assume that the total Euclidean length of this path is h. Obviously, $k \leq h \cdot \Delta / r$ since any node can have at most Δ neighbors within distance r. Let $x_i = \|u_i u_{i+1}\|$. Then the total energy cost is $\sum_{i=1}^{k-1} x_i^2$. Obviously, $\sum_{i=1}^{k-1} x_i^2 \geq \frac{(\sum_{i=1}^{k-1} x_i)^2}{k-1} \geq \frac{h^2 \cdot r}{h\Delta}$. On the other hand, since the Euclidean distance of the shortest path in G between v_i and v_0 is at most h, the shortest hop path connecting them is at most $2\lceil h/r \rceil$ hops. Thus, we can find a path using CDS to connect v_i and v_0 using at most $2 + 3 \cdot \lceil 2h/r \rceil \leq 4\lceil \frac{2h}{r} \rceil$ hops. The inequality is due to $\lceil h/r \rceil \geq 2$. Consequently, the total energy of the path connecting v_i and v_0 based on CDS is at most $4\lceil \frac{2h}{r} \rceil \cdot r^2$. Observe that our data collection algorithm based on CDS will use the shortest hop path to route the data from v_i to the sink v_0. Thus, the energy cost of data collection using CDS is at most $\frac{4\lceil \frac{2h}{r} \rceil \cdot r^2}{\frac{h^2 \cdot r}{h\Delta}} \leq 8\Delta$ times of the minimum. This finishes the proof. □

Recall that the CDS we used for routing has hop-spanning ratio at most 3. Then similarly we can prove the following theorem.

THEOREM 8. *Algorithm 1 is $\varrho_E \leq 8\Delta(G)$-approximation for energy cost when the energy to support a link uv is $\|uv\|^\alpha + c_2$, where c_2 is the energy cost of a node to receive a packet correctly.*

Consequently, we know that Algorithm 1 is asymptotically optimum if we want to optimize the time-complexity, message-complexity and energy-cost-complexity simultaneously. On the other hand, the minimum energy data-collection based on minimum energy path tree (MEPT) has delay that is at most $O(\Delta^4)$ times of the optimum.

THEOREM 9. *Data collection using MEPT is $\varrho_E = O(\Delta(G)^4)$-approximation for time complexity.*

PROOF. Consider the node v such that its minimum energy path P to the root has maximum number of hops, which contains data. Assume P has h hops with Euclidean length $y_1, y_2, \cdots, y_h$. Then $\sum_{i=1}^{h} y_i \geq \frac{h \cdot r}{\Delta}$ since every node can have at most Δ nodes within r distance. The total energy of this path is $\sum_{i=1}^{h} y_i^2 \geq \frac{(\sum_{i=1}^{h} y_i)^2}{h} \geq \frac{hr^2}{\Delta^2}$. On the other hand, consider the path from v to root with minimum number of hops h_2. For this path, its energy cost is at most $h_2 r^2$, which should be at least $\sum_{i=1}^{h} y_i^2$ due to optimality of P. Thus, $h_2 r^2 \geq \frac{hr^2}{\Delta^2}$ implies that $h \leq h_2 \Delta^2$.

Now consider an edge in the MEPT, scheduling this edge will interfere $O(\Delta^2)$ nodes. Since $MEPT$ is planar, at most $O(\Delta^2)$ edges in MEPT will be interfered. In other words, if we take one round to be $O(\Delta^2)$ time slots, each edge in the MEPT can be scheduled once. The height of the MEPT to be h. Scheduling the MEPT in a fashion similar to Algorithm 1 can finish the data collection operation in $O(N + h)$ rounds, hence $O(\Delta^2(N + h))$ time slots. On the other hand, any data collection algorithm will take $\Omega(N + h_2)$ time slot. Hence, data collection using MEPT is $\varrho_E = O(\Delta(G)^4)$. □

THEOREM 10. *There is a network example that the delay of data collection by using MEPT is at least $\Delta(G)^2/8$ times of the optimum.*

PROOF. We construct a network example, in which the MEPT has delay that is $\Omega(\Delta^2)$ times of the optimum. Consider a rectangle $uvwz$ with side-length $\|uv\| = p \cdot r$ and $\|uz\| = p\frac{\Delta-2}{8} r(1-\epsilon)$. There are $p+1$ nodes $u = u_1$, u_2, $\cdots$, $u_{p+1} = v$ uniformly distributed over the segment uv and $q = p\Delta^2/8 - 1$ nodes v_1, v_2, $\cdots$, v_q uniformly distributed over the rest of the 3 segments. Then it is easy to show that the MEPT path connecting u and v is $uv_1v_2 \cdots v_q v$, with $q = p\Delta^2/8 - 1$ hops. Obviously, the path $u_1 u_2 \cdots u_p$ connecting u and v has the least delay p. □

4. DATA AGGREGATION

We consider the case when the data aggregation is distributive. In other words, given any node v and its set of children nodes u_1, u_2, $\cdots$, u_d, where d is the number of children nodes of v in a data aggregation tree, the data produced by node v has size same as the size of each of the individual node. Typical examples of such aggregation are *min*, *max*, *average*, or *variance*. In data aggregation, if one node send information twice, it can always save the first transmission. Hence, the data aggregation should be done using a tree.

4.1 Message, Energy, and Time Complexity

Message Complexity: The total message complexity for data aggregation using any tree T is $\Theta(n)$, where n is the number of nodes of the network. The lower bound on the message complexity n is obvious since every node v needs send at least once. The upper bound is also n because we can do the data aggregation using any spanning tree and every node only needs to send once.

Energy Complexity: For distributive aggregation, it seems that we need use some data aggregation tree that is energy efficient since each node needs to send at least once. So the main question now is to construct a tree such that the total cost of all links are minimized. This clearly is the minimum spanning tree, where the link cost of any link uv is the energy cost of sending a unit amount of data over the link uv, which can be computed in polynomial time.

Time Complexity: We will show that the time complexity for such kind of data aggregation is of the order $\Theta(D + \Delta(G))$, where D is the height of the BFS tree rooted at the sink node v_0. Algorithm 2 illustrates our method.

THEOREM 11. *Data aggregation can be done in $\Theta(D + \Delta)$ time with n messages.*

PROOF. The lower bound on the time complexity is obvious since for the node v that is farthest (with the largest hop distance) from the root, it needs at least D time slots to reach the root. Additionally, it obviously needs at least $\Delta(G)$ time-slots to schedule all nodes' transmissions due to interference constraints.

Algorithm 2 Efficient Data Aggregation Using CDS

Input: A CDS **C** with bounded degree **d**, a distributive function f and corresponding function h.

for each dominator node v_i **do**

  For the set of dominatee nodes of the node v_i, we build a minimum spanning tree (MST) rooted at v_i, where the link weight is the energy cost for supporting the link communication. The data elements from all these dominatee nodes are then *aggregated* to the corresponding dominator node v_i along the minimum spanning tree of these dominatee nodes. In other words, any node v_k will compute $h(f(A_i), x_{k,1}, x_{k,2}, \cdots, x_{k,d_k})$ where $x_{k,j}$, for $j \in [1, d_k]$, is the aggregated value node v_k received from its jth child in the minimum spanning tree and d_k is the number of children of node v_k in the MST of all dominatee nodes of v_i. Notice that this aggregation can be done in time-slots $\Theta(\Delta(G))$.

Now we only consider the dominator nodes and the breadth-first-search spanning tree $T_{\mathbf{C}}$ of nodes in CDS rooted at the sink v_0.

for $t = 1$ to D **do**

  for each node $v_i \in V_{\mathbf{C}}$ **do**

    If node v_i has received aggregated data from all its children nodes in $T_{\mathbf{C}}$, it sends the aggregated data (using its own data and all aggregated data from its children) to its parent node in round t.

We then show that Algorithm 2 takes time $\Theta(D + \Delta(G))$. The first step that let each dominatee node send its data to its dominator node will take time-slots at most $\Theta(\Delta(G))$. Then we perform aggregation in round, where each round is composed of β time slots (where constant β is the number of colors needed to color the interference graph induced by all CDS nodes). In round 1, all nodes in level D (all leaves) send a message to their parents. In round t, all nodes in level $D - t + 1$ should have received all the messages from their children, compute the aggregation of all data received so far, and then send the aggregated values to their parents. In all, the total number of rounds to finish data aggregation is D. Recall that each round is composed of β time-slots. This finishes the proof. $\square$

If there are more than one aggregation functions, we can deliver the messages one by one. We call this as sequential aggregation.

COROLLARY 12. *k sequential data aggregations can be done in $O(D + \Delta + k)$ time with kn messages.*

4.2 Complexity Tradeoffs

Again, we may want to design a data aggregation method that has constant approximation ratios for message complexity, time complexity, and energy complexity. First, aggregation based on MST (that is energy optimum for aggregation) is not efficient for time complexity.

THEOREM 13. *The minimum energy data aggregation based on MST is $\varrho_T = \Omega(\min(\frac{n}{\Delta}, \sqrt{n\Delta}))$-approximation for time complexity. On the other hand, $\varrho_T = O(\frac{n}{\Delta})$.*

PROOF. Consider a set of wireless nodes in a grid, with size length r'. Then $\Delta = \Theta((1/r')^2)$ and $D = \Theta(\sqrt{n}r') = \sqrt{n/\Delta}$, assuming the communication range to be 1. There exists a MST T which consists of n sequential line segments. See Figure 3 for an example. In fact, we can perturb the grid slightly, so that this bad MST is the only MST on the grid.

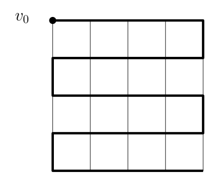


Figure 3: Example of a bad MST.

Clearly, the data aggregation on T takes $\Theta(n)$ time slots. On the other hand, Algorithm 2 takes $O(\Delta + D)$ time slots. Notices that the diameter of the CDS is a constant factor of the original graph. Hence, $\varrho_T \geq \Omega(\frac{n}{\Delta + D}) = \Omega(\frac{n}{\Delta + \sqrt{n/\Delta}})$. The lower bound follows by considering the cases that $n \geq \Delta^3$ and $n \leq \Delta^3$.

Now consider the upper bound, the data aggregation on a MST takes at most $n - 1$ time slots. However, any optimal solution should take $\Omega(\Delta + D)$. Hence $\varrho_T = O(n/\Delta)$. $\square$

Observe that our method (Algorithm 2) has constant ratio for both message complexity and time complexity. However, it is not always energy efficient due to the following theorem.

THEOREM 14. *Algorithm 2 is $\varrho_E = (\mathbf{d}+6)\Delta(G))$-approximation for energy cost, where **d** is the maximum nodal degree of CDS.*

PROOF. First, consider any dominator u and let $v_1, v_2, \cdots, v_d$ be the d dominatee nodes associated with u, where $d \leq \Delta$, and $\|v_i u\| \leq r$. Let $x_1, x_2, \cdots, x_d$ be the d edges of the minimum spanning tree connecting u and its associated dominatees. It was proved in [3] that

$$\sum_{i=1}^{d} x_i^2 \leq 6r^2.$$

Assume that there are n_d dominator nodes in CDS (*e.g.*, we can use a maximal independent set as a dominating set). Obviously, $n_d \cdot \Delta \geq n$. Then the total energy cost of aggregating data from all dominatee nodes to dominators is at most $6n_d r^2$. Recall that in our CDS, all nodes in $V_{\mathbf{C}}$ will have at most **d** neighbors. It is easy to show that the total energy cost of aggregating data over CDS is at most $\mathbf{d} \cdot n_d \cdot r^2$. Thus, the total energy cost of aggregating data using Algorithm 2 is at most $(\mathbf{d} + 6) \cdot n_d \cdot r^2 = \Theta(n_d \cdot r^2)$.

We then study the minimum cost of data aggregation, which is to use the minimum spanning tree of original communication graph G. Let $y_1, y_2, \cdots, y_n$ be the length of the n edges of the MST connecting $n+1$ nodes $v_0, v_1, \cdots, v_n$. Since there are n_d independent nodes, it is easy to show that the MST will have length at least $n_d \cdot r$, *i.e.*, $\sum_{i=1}^{n} y_i \geq n_d r$. The total energy cost of data aggregation based on minimum spanning tree is $\sum_{i=1}^{n} y_i^2$. Notice, $\sum_{i=1}^{n} y_i^2 \geq (\sum_{i=1}^{n} y_i)^2/n \geq n_d^2 r^2/n$.

Then our algorithm has approximation ratio on energy cost at most $\frac{(\mathbf{d}+6)\cdot n_d \cdot r^2}{\frac{n_d^2 r^2}{n}} \leq (\mathbf{d}+6)\Delta = O(\Delta)$, because of $n_d \cdot \Delta \geq n$.

We then show that there are examples of networks (with maximum degree Δ) such that Algorithm 2 is $\varrho_E = \Theta(\Delta(G))$-approximation for energy cost. Consider a line graph composed of $n+1$ nodes evenly distributed in a segment $[0, \frac{2r}{\Delta}n]$, *i.e.*, node v_i is at position $\frac{2r}{\Delta} \cdot i$, for $i \in [0, n]$. See Figure 4 for illustration. It is easy to show that the minimum energy data aggregation tree is simply the path $v_0 v_1 \cdots v_{n-1} v_n$, whose total energy cost is $n(\frac{2r}{\Delta})^2$. On the other hand, the energy cost of using tree T_{C} is $\Theta(\frac{2n}{\Delta} r^2)$ since the CDS will have $\frac{2n}{\Delta}$ nodes. The energy cost using tree T_{C} is $\Theta(\Delta)$ times of the minimum. This finishes the proof. □

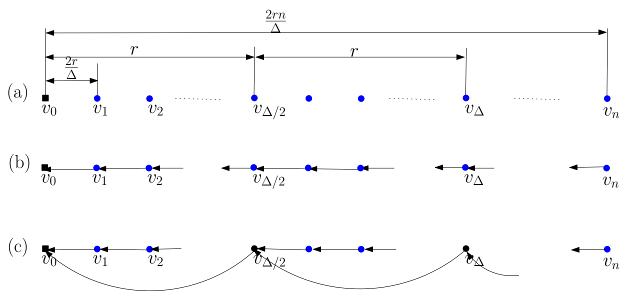


Figure 4: Example: (a) the line network with $n+1$ nodes; (b) the minimum energy data aggregation tree; (c) the tree T_{C}.

Although our method is not energy efficient in the worst case (with approximation ratio up to $\Theta(\Delta)$ in the worst case), we show that it is the best we can do if we want to achieve $\Theta(1)$ ratio in delay. Again, given a data aggregation method $\mathcal{A}$, let ϱ_E, ϱ_T and ϱ_M be the approximation ratios of $\mathcal{A}$ over all networks with n nodes and maximum degree Δ. We prove the following theorem.

THEOREM 15. *For any data aggregation algorithm $\mathcal{A}$, there are graphs of n nodes with maximum degree Δ, such that $\varrho_T \cdot \varrho_E = \Omega(\Delta)$.*

PROOF. Again consider the line network example used in the proof of Theorem 14. Assume that we choose a tree T for data aggregation. Consider the unique path P from v_0 to v_n in T. Assume P has k edges. Then the data aggregation using T takes at least k time slots. Let $\{x_i\}$ be the Euclidean lengths of the k edges in P. Clearly, $\sum_1^k x_i \geq 2rn/\Delta$. Then, the energy cost of this path is $\sum_1^k x_i^2 \geq (\sum_1^k x_i)^2/k \geq 4r^2n^2/(k\Delta^2)$. Notice that for any algorithm, the minimum delay is $2n/\Delta$ and the minimum energy cost is nr^2/Δ^2 (using MST). Thus, the approximation ratio ϱ_T and ϱ_E satisfy that: $\varrho_T \cdot \varrho_E \geq \frac{k}{\frac{2n}{\Delta}} \cdot \frac{\frac{4n^2r^2}{k\Delta^2}}{\frac{nr^2}{\Delta^2}} = 2\Delta$. □

Consequently, our method for data aggregation is asymptotically optimum in terms of the tradeoffs between time-complexity, and energy-complexity when the energy needed to support a link uv is proportional to $\|uv\|^\alpha$. It remains a challenging question to design algorithms with best tradeoffs the energy needed to support a link uv is $\|uv\|^\alpha + c_2(v)$ or, more generally, an arbitrary function.

5. DATA SELECTION

In this section, we consider the scenario when we want to find the kth smallest data (or median when $k = N/2$) among all N data items stored in n wireless sensor nodes. Here we assume that each wireless sensor node will store at least one data item, and may store more than one data items. We further assume that all data items have a complete order. In most results here, we use the selection of median as an example to study the complexity.

5.1 Time Complexity

First we give a lower bound on the time complexity of any *deterministic* distributed algorithm.

THEOREM 16. *Any deterministic distributed algorithm needs at least $\Omega(\Delta + D\log_D N)$ time to find the median of all data items.*

PROOF. For any deterministic algorithm, each node in the wireless network need send at least one message. In fact, if a node does not announce at least once, the adversary could place the median (or the kth largest item) in it. Hence, the time complexity is $\Omega(\Delta)$ due to wireless interferences. On the other hand, the time complexity of finding the median for a wireless network is at least as expensive as that of finding the median at a corresponding wired network (by assuming that no interferences exist among all transmission links). It has been proved in [18] that any deterministic algorithm for finding median in a wired network G of n nodes with diameter D, and total N data items has time complexity at least $\Omega(D\log_D N)$. Finding kth smallest element need time at least $\Omega(D\log_D k)$ when $k \leq N/2$. Consequently, for wireless network G of n nodes with diameter D, any deterministic distributed algorithm needs at least $\Omega(\Delta + D\log_D N)$ time to find the median of all data items. □

We then present our method (Algorithm 3) for distributed data ranking in wireless ad hoc networks. In our method, we first collect data from dominatee nodes to corresponding dominator nodes, then we will run the distributed selection method for wired networks over the CDS (from [18] and is summarized in Algorithm 4 for completeness of presentation). The Algorithm 4 will be run by the sink node and the basic idea is as follows:

1. Initially, let $L = -\infty$ and $U = \infty$. The sink node will first broadcast control message **getRndElementsInRange**$(t, (L, U))$ to all nodes, asking for t *independent random* elements from *all* elements in the interval (L, U).
2. All nodes with data in this range together will return t random elements using t sequential findings of *one* random element. This clearly can be done in time $O(D+t)$. Let x_1, $x_2, \cdots, x_t$ be the t random elements in the increasing order.
3. The sink node then broadcasts control message **countElementsInRange** to count the total number of items in the range of $(x_{i-1}, x_i]$ for $i \in [2, t]$. This can be done using simple counting aggregation in time $O(D)$ with message n_{C}.
4. The sink node can then find the interval $(x_{j-1}, x_j]$ where the globally kth smallest element locate. We find the kth smallest element if x_j is. Otherwise, repeat the preceding steps using the new interval $(L, U) \Leftarrow (x_{j-1}, x_j)$.

THEOREM 17. *There is a randomized distributed algorithm that can find the median of all data items in expected time $O(\tilde{\Delta} + D\log_D n)$ and also in time $O(\tilde{\Delta} + D\log_D n)$ with high probability.*

PROOF. Clearly, the time-costs of our algorithm are as follows (1) the first step has time-complexity $\Theta(\tilde{\Delta})$; (2) the second step will cost $O(D\log_D N)$ rounds of communications with high probability [18], *i.e.*, $O(D\log_D N)$ time-slots *w.h.p.*, since each round is composed of β time-slots. This finishes the proof. □

Algorithm 3 Data Selection With Low Delay

Input: A CDS with bounded degree **d**.

1. Each dominatee node sends its data to its dominator node. This can take place in time $\Theta(\Delta)$ when we consider wireless interferences and each node v_i has one data item.
2. Then the median is found using only the connected dominating set, *i.e.*, only nodes in CDS will participate. We run the randomized Algorithm 4 with $t = 8\lambda D$ with a constant $\lambda > 1$ (see [18] for details). This method has time complexity $O(D \log_D N)$ in wired communication model. Notice that for wired networks, a node v_i can send a message to *each* of its neighboring nodes in one time-slot. This clearly cannot be done in wireless networks. We will mimic the wired communication of CDS nodes using wireless links: one round of wireless communications corresponds to one time-slot in the corresponding wired network.

Notice that, if each node has single data item, then the time complexity of Algorithm 3 is $O(\Delta(G) + D \log_D n)$ with high probability. Similarly, if we run the best deterministic algorithm for data selection for wired networks [18], we have the following theorem.

THEOREM 18. *There is a deterministic distributed algorithm that can find the median of all data items in time $O(\tilde{\Delta}(G) + D \log_D^2 N)$ for wireless ad hoc networks of n nodes with diameter D and maximum weighted degree $\tilde{\Delta}(G)$.*

Observe that the lower bound $D \log_D N$ on the time complexity for wired networks is not tight for wireless networks. Consider a network formed by a sink node with coordinate $(0, 0)$, and other n nodes evenly distributed in the circle centered at $(0, 0)$ with radius r. Then $D = 2$ and $D \log_D N$ is only $2 \log n$. On the other hand, $\Delta = n$, thus, data selection needs time at least n due to wireless interferences.

Algorithm 4 Random Data Selection $RDS(t, k)$

$L \leftarrow -\infty$; $U \leftarrow \infty$; phase$\leftarrow 0$;
repeat
  $x_0 \leftarrow L$; $x_{t+1} \leftarrow U$; phase$\leftarrow$phase+1;
  $\{x_1, \ldots, x_t\} \leftarrow$ **getRndElementsInRange**$(t, (L, U))$
  for $i = 1, \ldots, t$ in parallel **do**
    $r_i =$**countElementsInRange**$((x_{i-1}, x_i])$
  if $x_0 \neq -\infty$ **then**
    $r_1 \leftarrow r_1 + 1$
  $j \leftarrow \min_{l \in \{1,\ldots,t+1\}} \sum_{i=1}^{l} r_i > k$
  $k \leftarrow k - \sum_{i=1}^{j-1} r_i$
  if $k \neq 0$ and $j \neq 1$ **then**
    $k \leftarrow k + 1$
until $r_j \leq t$ or $k = 0$
if $k = 0$ **then**
  return x_j
else
  $\{x_1, \ldots, x_s\}$ =getElementsInRange$([x_{j-1}, x_j])$;
  return x_k

5.2 Message Complexity

We then study the message complexity of finding median of all numbers stored in the network.

5.2.1 Lower Bound

Our lower bound on message complexity is based on the result on two-party model. For two nodes connected by a link, each with $N/2$ data, finding the median need $\Theta(\log N)$ messages [7, 25]; or generally, the kth smallest element ($k < \frac{N}{2}$) can be found using $\Theta(\log k)$ messages. In [18], Kuhn *et al.* studied the lower bound of the time complexity for the selection problem. Especially, they proved the following result on the two-party problem where both nodes have n elements. This result concludes the number of rounds (thus, an obvious lower bound on the number of messages) needed to compute the k^{th} smallest element.

THEOREM 19 ([18]). *Let $h = \min\{k, 2N - k\}$. Every, possibly randomized, generic two-party protocol needs at least $\Omega(\log h)$ rounds(messages) to find the element with rank k in expectation and with probability at least $1/h^{\delta}$ for any constant $\delta \leq 1/2$.*

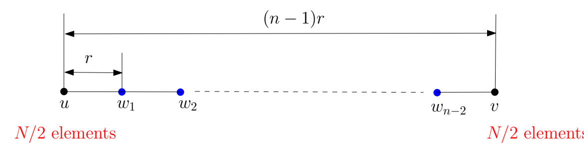


Figure 5: A network example in which $\Omega(n \log h)$ messages are required to compute the k^{th} smallest element in G.

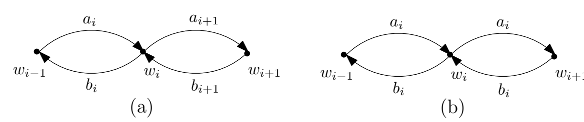


Figure 6: Assume $a_i + b_i \leq a_{i+1} + b_{i+1}$. The intermediate vertex w_i can just forward messages between w_{i-1} and w_{i+1} without increase the total message complexity.

Based on the result, we show that there exist graphs that require $\Omega(n \log h)$ messages to compute the median. Our construction is similar to the lower bound of time complexity obtained in [18].

We first construct a line graph G as follows. See Figure 5 for illustration. The left and right vertices are u and v, each having $N/2$ elements. The other $n - 2$ intermediate vertices $w_1, w_2, \cdots, w_{n-2}$ do not contain any element. Vertex u is connected to w_1; w_i is connected to w_{i+1} for $1 \leq i \leq n - 2$, and w_{n-2} is connected to v. The distance between consecutive nodes is r. We can construct a wireless communication graph which can be contracted to this example.

For simplicity, we first assume all intermediate vertices can only duplicate messages without any computation. This is exactly the case for the general two-party protocol. The following theorem is directly implied by Theorem 19.

THEOREM 20. *Assume that all intermediate vertices are only allowed to relay messages in G. $\Omega(n \log h)$ messages are required to compute the k^{th} smallest element in G in expectation and with probability at least $1/n^{\delta}$ for every constant $\delta < 1/2$.*

Of course, in practice, we may allow intermediate vertices to perform certain computation on the messages it received before it sends out messages. However, we show that this additional freedom does not reduce the message complexity required. In particular, we argue that it is not necessary for any intermediate vertex to preform computation. For notational simplicity, let $w_0 = u$ and $w_{n-1} = v$. Consider an intermediate vertex w_i, and its left vertex w_{i-1} and right vertex w_{i+1}. For each $i \in [1, n - 2]$, assume that during the computation the vertex w_i receives a_i messages from w_{i-1} and sends b_i messages to w_{i-1}. Now consider the vertex w_i, without loss of generality, we assume $a_i + b_i \leq a_{i+1} + b_{i+1}$. Instead of performing computation, we let w_i forward all a_i messages from w_{i-1} to w_{i+1}. Because all b_i messages from w_i to w_{i-1} are computed from the a_i and b_{i+1} messages which w_{i+1} already poses after the forwarding. Hence, we can just send the b_i messages from w_{i+1} to w_{i-1} by passing w_i. See Figure 6 for illustration. In all, the number of messages does not increase. On the other hand, all the information w_i original obtained now available on w_{i+1}. Hence, this change does not affect the computation process.

We can pick i so that $a_i + b_i$ is (one of) the smallest. The preceding procedure can propagate from w_i to both leaves u and v, so that each intermediate vertex will forward $a_i + b_i$ messages. As we argued, the total number of messages does not increase.

THEOREM 21. *Let $h = \min\{k, N-k\}$. There is a wireless network with n nodes, such that $\Omega(n \log h)$ messages are needed to compute the k^{th} smallest element in expectation and with probability at least $1/h^{\delta}$ for every constant $\delta < 1/2$.*

In Figure 6, its diameter $D = n$. Therefore, the lower bound stated in previous theorem can directly imply next result. The $\Omega(n)$ lower bound comes from the fact that each node need send at least one message.

In the preceding study of the lower bound on the message complexity of distributed selection, we only use the graph size n and number of data items N as parameters. We then extend this idea to get a more precise lower bound for finding median for all graphs with size n and diameter D. We construct a graph G as follows. Let $p = \frac{n-D+1}{2}$. On the left side, there are p vertices $u_1, u_2, \cdots, u_p$. On the right side, there are p vertices $v_1, v_2, \cdots, v_p$. Each of the vertices u_i and v_i has $\frac{N}{2p}$ elements, where N is total number of data items. The other $D-1$ intermediate vertices $w_1, w_2, \cdots, w_{D-2}, w_{D-1}$ do not contain any elements. Graph G only has following edges $u_i w_1$, $w_{D-1} v_i$, for $1 \leq i \leq p$, and $w_j w_{j+1}$ for $j \in [1, D-2]$. See Figure 7 for illustration. Similarly, we can show that for a graph G of n nodes with diameter D, finding the kth smallest element requires $\Omega(D \log h)$ messages using a line graph of diameter D.

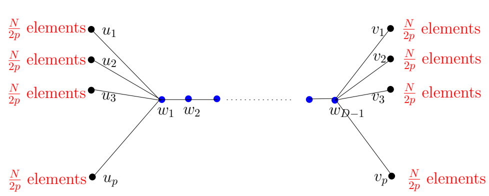


Figure 7: A network example in which any algorithm finding median needs at least $\Omega(n + D \log N)$ messages.

THEOREM 22. *Let $h = \min\{k, N-k\}$. There is a graph (wired or wireless communication model) with n nodes and diameter D, any algorithm finding kth smallest (or median) needs at least $\Omega(n + D \log h)$ (or $\Omega(n + D \log N)$) messages.*

5.2.2 *Upper Bound*

We then present a randomized algorithm that will find the median of all N data items using expected number $O(n \log N)$ of messages, and also $O(n \log N)$ messages with high probability. The algorithm essentially is to find a random element x and then count the number of elements that are less than x. It is likely that a considerable fraction of all nodes no longer need be considered. By iterating this procedure on the remaining candidate nodes, the kth smallest element can be found quickly for all k. Algorithm 5 illustrates our basic method.

Algorithm 5 Data Selection With Less Messages

Input: A CDS with bounded degree **d**.

1: The dominatee node will send its data to its dominator node. This can take place using $O(N)$ total messages. Then only nodes in CDS will participate the second step.

2: We run the randomized data selection Algorithm 4 with $t = \lambda$ for some constant integer $\lambda \geq 1$.

THEOREM 23. *Let $h = \min\{k, N-k\}$. Given a wireless UDG network with n nodes (each with one data item) and diameter D, Algorithm 5 can find the median with $O(N + n_{\mathbf{C}} \log N)$ messages with high probability.*

PROOF. Clearly, the first step costs at most N messages. Let $n_{\mathbf{C}}$ be the number of nodes in CDS. Then we will prove that variable *phase* (defined in Algorithm 4) is at most $2\log_{1/c} N$ (for a constant $c < 1$) with high probability when the median is found. Obviously, in each "phase" of Algorithm 4, the total number of messages is $2\lambda n_{\mathbf{C}}$: for each randomly selected data, each node in CDS will forward at most one control message from the sink and at most one data message back to the sink. Thus, the total number of messages used, with high probability, is at most $N + 4\lambda n_{\mathbf{C}} \log_{1/c} N$.

We then prove that variable *phase* is at most $2\log_{1/c} N$ (say for a constant $c = 1/2$) with high probability when the median is found. First, we compute an upper bound on the probability that after any phase i the wanted element is in a fraction of size at least c times the size of the fraction after phase $i-1$ for a suitable constant c, *i.e.*, $n^{(i)} \geq c \cdot n^{(i-1)}$. Here $n^{(i)}$ is the size of the all data items we have to check to find the kth smallest data before the phase i starts. Notice $n^{(0)} = N$. Let $\{a_1, a_2, \cdots, a_{n^{(i)}}\}$ be the sorted list of the $n^{(i)}$ data items that we will check for the kth smallest element in phase $i+1$. The probability that none of the λ randomly selected elements is in $\{a_k, a_{k+1}, \cdots, a_{k+cn^{(i)}/2}\}$ is at most $(1-c/2)^{\lambda}$. Same argument holds for data items $\{a_{k-cn^{(i)}/2}, \cdots, a_{k-1}, a_k\}$. Thus,

$$\Pr(n^{(i)} \geq c \cdot n^{(i-1)}) \leq 2(1-c/2)^{\lambda} \leq 2e^{-c\lambda/2}.$$

If $n^{(i)} \leq c \cdot n^{(i-1)}$ the phase i is called *successful*; otherwise it is called *failed*. Clearly, we need at most $S = \log_{1/c} N$ successful phases to find the kth smallest element. A phase i will fail with probability at most $p = 2e^{-c\lambda/2}$. Then among $2S$ phases, the probability that we have less than S successful phases (*i.e.*, at least $S+1$ failed phases) is at most

$$\sum_{i=S+1}^{2S} \binom{2S}{i} p^i (1-p)^{2S-i} \leq \binom{2S}{S} p^S \leq (\frac{2eS}{S})^S \cdot p^S$$
$$= \quad (4e^{1-\frac{c\lambda}{2}})^{\log_{1/c} n} = 1/n^{(\frac{c\lambda}{2}-1-\ln 4)/\ln\frac{1}{c}}$$

When $(\frac{c\lambda}{2} - 1 - \ln 4)/\ln\frac{1}{c} > 1$ (equivalently, $\lambda \geq \frac{2\ln\frac{4e}{c}}{c}$), this probability is at most $1/n$. For example, we can set $c = 1/2$, then $\lambda = \lceil 4\ln(8e) \rceil = 7$. Then, with probability at least $1 - \frac{1}{n}$, Algorithm 5 will terminate in $2\log_2 N$ phases. Each phase will cost at most $2\lambda n_{\mathbf{C}}$ messages. This finishes the proof. $\square$

Instead of collecting data from dominatee nodes to the dominator nodes, we can directly run Algorithm 4 on the wireless network G.

By an argument similar to Theorem 23, the algorithm will find the median with $\Theta(n \log N)$ messages with high probability. Notice that this could be better than Algorithm 5 when N is very large, *e.g.*, $N = \Omega(n \log N)$.

We then consider the message complexity when n sensor nodes are randomly and uniformly deployed in a square of $[0, a] \times [0, a]$ and each sensor node has one data item. It has been proved in [13] that, to guarantee the random wireless sensor network is connected with high probability, the transmission range r should satisfy that $n\pi r^2 = \Theta(a^2 \cdot \log n)$. Thus, the number of dominators n_d, using a maximal independent set, is of order $\frac{a^2}{r^2} = \Theta(\frac{n}{\log n})$. Thus, size of CDS $n_{\mathbf{C}} = \Theta(n_d) = \Theta(\frac{n}{\log n})$. Consequently, the message complexity of Algorithm 5 for random networks, with high probability, is $\Theta(n + n_{\mathbf{C}} \cdot \log n) = \Theta(n)$ when total data items is $N = O(n)$. This is clearly asymptotically minimum.

5.3 Other Models

In previous discussions, we only consider the *comparison model*, *i.e.*, we assume the only operation between data items is to *compare their values*. A number of additional information can be used to improve the message and/or time complexities. For example, we may know that the values of all data items are positive integers or integers in range $[L, U]$.

Value Sensitive Query: We first consider the case that all data items are *positive integers*. We show that the message complexity of finding the median is no more than $\min\{N + 4n_{\mathbf{C}} \log f_k, 4n \log f_k\}$ based on methods in [6, 23] for wired networks. Here $n_{\mathbf{C}}$ is the size of the connected dominating set and f_k is the value of the kth smallest data. We assume synchronized communications are used by all wireless nodes. The method is essentially to solve the unbounded search: we first iterate to find i (starting from $i = 0$) such that the kth smallest element is in the range $(2^i, 2^{i+1}]$; we then use binary search to locate the kth smallest element in this range. It is easy to show that we need at most $2 \log f_k$ such rounds and each round will cost us at most $2n_{\mathbf{C}}$ messages.

Known Intervals: When we know the interval $[L, U]$, then the message complexity is no more than $\min\{N + 2n_{\mathbf{C}} \log \frac{U}{L}, 2n \log \frac{U}{L}\}$, where U is the largest possible value and L is the lowest possible value among all data, by using a simple distributed binary search method. Observe that both U and L can be found using a simple distributive function $\max$ and $\min$ with n messages.

Notice that we can combine the preceding two techniques as follows. We first call $\min$ function to find L. Then we iterate to find i (starting from $i = \lfloor \log L \rfloor$) such that the kth smallest element is in the range $(2^i, 2^{i+1}]$; we then use binary search to locate the kth smallest element in this range. It is easy to show that we need at most $2 \log \frac{f_k}{L}$ such rounds and each round will cost us at most $2n_{\mathbf{C}}$ messages when CDS is used or $2n$ messages if original network G is used. Thus, the total messages complexity is at most

$$\min\{N + n + 4n_{\mathbf{C}} \log \frac{f_k}{L}, 2n \log \frac{f_k}{L}\}.$$

5.4 Energy Complexity

At last, we study the energy cost of finding the median in any wireless networks by presenting some lower bound and upper bound.

THEOREM 24. *Any algorithm that can correctly find the median needs energy cost at least $\omega(MST) = \sum_{uv \in MST} E(u, v)$, where MST is the minimum spanning tree of G with weight of a link uv defined as the energy cost $E(u, v)$ for supporting link uv.*

PROOF. First of all, using adversary argument, we can show that every node needs to send at least one message to reveal some information about the data item it has. If it did not, adversary can put the median at this node to prevent the algorithm from finding the correct median. Let H be the graph over V and its set of edges are edges used by an optimum algorithm for communications. Clearly, graph H must be a connected graph; otherwise, the adversary can put the median at a connected component that does not contain the sink node. Consequently, the total link weight of minimum spanning tree is the lower bound for the energy consumption of any data selection algorithm. □

Assume that we are given the minimum spanning tree a prior. To minimize the energy consumption, we will directly run Algorithm 4, or value sensitive query methods discussed in previous subsection, on top of MST. Then we have the following theorem.

THEOREM 25. *There are algorithms that can correctly find the median with energy cost at most $O(\omega(MST) \cdot \log N)$ or $O(\omega(MST) \cdot \log \frac{f_k}{L})$ where L is the smallest value of all data items.*

PROOF. We showed that Algorithm 4 will terminate after at most $2 \log N$ phases with high probability. At each phase, the sink node need broadcast a control message and then all related nodes will reply with a certain answer. Obviously, both the broadcast from the sink along the MST and convergecast of the answer back to the sink cost energy $\omega(MST)$. Thus, Algorithm 4, run on top of MST has energy cost $O(\omega(MST) \cdot \log N)$.

For value sensitive query method, we first find L (which has energy cost at most $\omega(MST)$) and then then query will terminate after at most $2 \log \frac{f_k}{L}$ phases, where each phase cost energy at most $2\omega(MST)$. This finishes the proof. □

If we interleave the preceding two methods (a phase is an atomic step), then clearly, we have algorithm whose energy cost is at most $O(\min\{\log N, \log \frac{f_k}{L}\})$ times of the minimum for data selection. Observe that from the network example illustrated by Figure 7 we can show that

THEOREM 26. *There are networks G of n nodes and diameter D, and placement of data items such that, the minimum energy required by* any *data selection algorithm is $\Omega(\omega(MST(G)) \log N)$.*

However, this does not mean that, for *any* graph, it is always the case. In particular, it does not give the bound on ϱ_E for our algorithm on the MST. We make the following conjecture

CONJECTURE 1. *For any algorithm that can correctly find the median and* any *network, there exists a* placement of data items *such that the algorithm will cost energy at least*

$$O(\omega(MST) \cdot \min\{\log N, \log \frac{f_k}{L}\})$$

where L is the smallest value of all data items, f_k is the value of the kth smallest element.

6. RELATED WORKS

As the fundamental many-to-one communication pattern in sensor network applications, convergecast has been studied in both networking and database communities in recent years.

Most existing convergecast methods [4, 12, 28] are based on a tree structure and with minimum either energy or data latency as the objective. For example, [28] first constructs a tree using greedy approach and then allocates DSSS or FHSS codes for its nodes to achieve collision-free, while [4, 12] uses TDMA to avoid collisions. In [4], the authors did *not* give any theoretical tradeoffs between energy cost and latency. Gandham [12] mainly studied the minimum time convergecast for linear networks and tree networks. They presented a lower bound $3n - 2$ for time-complexity for convergecast in linear networks and proposed a distributed convergecast scheduling algorithm that requires at most $3n$ timeslots for tree networks. They perform convergecast based on BFS, whose internal nodes implicitly forms a CDS structure, which is used here. However, BFS structure cannot guarantee the best theoretical performance in terms energy consumption. Furthermore, they did not provide theoretical results for general network topologies. Zhang and Huang [30] proposed a hop-distance based temporal coordination heuristic for adding transmission delays to avoid collisions. They studied the effectiveness of packet aggregation and duplication mechanisms with such convergecast framework. Kesselman and Kowalski [17] proposed a randomized distributed algorithm for convergecast that has the expected running time $O(\log n)$ and uses $O(n \log n)$ times of minimum energy in the worst case, where n is the number of nodes. They also showed the lower bound of running time of any algorithm in an arbitrary network is $\Omega(\log n)$. However, they assume that all nodes can dynamically adjust its transmission power from 0 to any arbitrary value and a data message by a node can contain *all* data it has collected from other nodes. In [8], Chu *et al.* studied how to provide approximate and bounded-loss data collection in sensor networks instead of accurate data. Their method used replicated dynamic probabilistic models to minimize communication from sensor nodes to the base station.

To significantly reduce communication cost in sensor networks, in-network aggregation has been studied and implemented. In TAG (Tiny AGgregation service) [19], besides the basic aggregation types (such as *count*, *min*, *max*, *sum*, *average*) provided by SQL, five groups of possible sensor aggregates are summarized: distributive aggregates (*e.g.*, *count*, *min*, *max*, *sum*), algebraic aggregates (*e.g.*, *average*), holistic aggregates (*e.g.*, *median*), unique aggregates (*e.g.*, *count distinct*), and content-sensitive aggregates (*e.g.*, *fixed-width histograms* and *wavelets*). Notice that the first two groups aggregates are very easy to achieve by a tree-based method. To overcome the severe robustness problems of the tree approaches [19, 20, 29], multipath routing for in-network aggregation has been proposed [9,22]. Then recently Manjhi *et al.* [21] combined the advantages of the tree and multi-path approaches by running them simultaneously in different regions of the network. In [14], Kashyap *et al.* studied a randomized (gossip-based) scheme using which all the nodes in a complete overlay network can compute the common aggregates of min, max, sum, $average$, and $rank$ of their values using $O(n \log \log n)$ messages within $O(\log n \log \log n)$ rounds of communication. Kempe *et al.* [15] earlier presented a gossip-based method which can get the average in $O(\log n)$ rounds with $O(n \log n)$ messages.

Data selection (*e.g.*, *median* or k-th *smallest element*) is much harder than general distributive and algebraic aggregates. Distributed selection has been studied in general networks [23]. Recently, Kuhn *et al.* [18] studied the distributed selection for general networks with n nodes and diameter D. They proved that distributed selection is strictly harder than convergecast by giving a lower bound of $\Omega(D \log_D n)$ on the time complexity. They then present a novel randomized algorithm which matches this lower bound with high probability and de-randomized it to a deterministic distributed selection algorithm with a time complexity of $O(D \log_D^2 n)$ which constitutes a substantial improvement over prior art. However, there are no many results on distributed selection in wireless networks. In [24], Patt-Shamir presented a deterministic algorithm that computes the median value such that each node transmits only $O((\log n)^2)$ bits and a randomized algorithm that computes an approximate median in which each node transmits $O((\log \log n)^3)$ bits. He also proved that computing the exact number of distinct elements in the data set indeed requires linear communication in the worst case. His method implies total $O(n \log n)$ messages for finding median when each node has one data item, while our method can find the median in $O(n_{\mathbf{C}} \log n)$ messages. However, no lower bound on message complexity or time complexity is given in [24].

In [27], Shrivastava *et al.* proposed a novel data structure, *quantile digest*, to provide provable guarantees on approximation error and maximum resource consumption for approximate quantile queries. If the values returned by the sensors are integers in the range of $[1, \delta]$, then using quantile digest, they can answer quantile queries using message size m within an error of $O(\log(\delta)/m)$. Their method can also support range queries, most frequent items and histograms. Recently, there are several papers studied the various aggregations on streaming data, such as thresholded counts [16], threshold function query [26], approximate quantile queries [11], top-k monitoring [5].

7. CONCLUSION

In this paper, we study the time complexity, message complexity, and energy complexity of data collection, algebraic data aggregation, and data selection in wireless sensor networks. We first study the lower bound of the complexities for these problems and then present efficient algorithms that achieve asymptotically optimal time complexity, and message complexity. We are currently testing the performances of our methods via simulations and testbeds. There are still a number of interesting questions left for future research. One is to design efficient algorithms when each node will produce a data stream. The second challenge is what is the best algorithm when we do not require that the found data item to be precise, *i.e.*, we allow certain relative errors, or additive errors on the found answer. We also need to study the lower bound on energy cost and design energy efficient algorithm for holistic data operations. Another question is to study the time complexity and message complexity for other holistic queries such as *most frequent items*, *number of distinctive items*. The last, but not the least important is to study the lower bound on complexities, and to design efficient algorithms to address these questions when the communication links are not reliable.

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