Grooming of Arbitrary Traffic in SONET/WDM Rings

Peng-Jun Wan   Liwu Liu   Ophir Frieder

Abstract— **SONET add-drop multiplexers (ADMs) are the dominant cost in SONET/WDM rings. They can potentially be reduced by optical bypass via wavelength add-drop multiplexers (WADMs) and traffic grooming. While many works have been done on the grooming of all-to-all uniform traffic and one-to-all traffic, the grooming of arbitrary traffic have not been studied yet. In this paper we first proves the NP-hardness of this problem. We then presents two general lower bounds on the minimum ADM cost. After that we propose a two-phased algorithm. The two subproblems in these two phases are both NP-hard. Various approximation algorithms are proposed to each subproblem, and their performances are briefly discussed.**

I. Introduction

Coupling wavelength division multiplexing (WDM) technology [14] with synchronous optical network (SONET) rings [10] is a very promising network architecture that has attracted much attention recently [5] [6] [8] [9] [13] [15] [17]. In this network architecture, each WDM channel carries a high-speed (e.g., OC-48) SONET ring. Each SONET ring can further carry a number of low-speed (e.g., OC-48) traffic streams. The number of the low-speed streams that can be carried in a SONET ring is referred to as the *traffic granularity*, denote by a parameter g. The key terminating equipments are wavelength add-drop multiplexers (WADMs) and SONET add-drop multiplexers (ADMs). Each node is equipped with one WADM. The WADM can selectively drop wavelengths at a node. Thus if a wavelength does not carry any traffic from or to, a particular node, the WADM allows that wavelength to optically bypass the node rather than being electronically terminated. Thereby in each SONET ring a SONET ADM is required at a node if and only if it carries some traffic terminating at this node. Therefore the SONET/WDM ring architecture can not only greatly increase the capacity, thereby reducing the amount of required fiber and allowing for more graceful upgrades, but also potentially reduce the amount of required SONET ADMs. As SONET ADMs typically cost on the order of hundreds of thousands of dollars, eliminating SONET ADMs potentially represents a significant cost saving.

In general, the minimum ADM cost problem is to partition the set of traffic demands into a number of groups such that each group can be carried in a single SONET ring and the total ADM cost is minimized. Recent studies on the traffic grooming in the SONET/ADM rings have been focused on all-to-all traffic [5] [6] [13] [15] [17] [19] and one-to-all traffic [8] [11] [13] [18]. While the study of these traffic is essential in the planning and design of SONET/WDM rings, the traffic pattern after the deployment of the SONET/WDM rings may become arbitrary. Thus the study of the grooming of arbitrary traffic becomes a necessity. In this paper we will address the grooming of arbitrary traffic. Without loss of generality, we assume that the demand of each traffic is of one unit, e.g. OC-3, for otherwise it can be split into multiple subtraffics of unit demand. To simplify the study, we assume that the routing of all traffic demands are predetermined. Such an instance can be represented by a set A of m circular arcs in *clockwise* direction over a ring of n nodes, and the traffic granularity g.

The remaining of paper is organized as follows. In Section II, we prove the NP-hardness of the optimal grooming of the arbitrary traffic for any fixed grooming granularity. In Section III, we present two lower bounds on the minimum ADM cost for any grooming granularity. In Section IV, we decompose the minimum ADM cost problem into two subproblems, which both remain NP-hard. Subsection IV-A and Subsection IV-B present various approximation algorithms for these two subproblems. Finally we conclude this paper in Section V.

II. Computational Complexities

Let A be any set of circular arcs over a ring. Without loss of generality, we assume that each node in the ring is one endpoint of some arc in A, for otherwise we can remove this node and this removal does not affect the ADM cost. Thus the ring size can be implied from A. With these assumptions, an instance of the minimum ADM problem is simply a set of circular arcs. We use $opt_g(A)$ to denote the minimal number of ADMs required to support the set of traffic demands traffic granularity g represented by A. A is said to be ***uniformly loaded*** if the number of circular arcs passing through each link is the same. In particular, if there are exactly L circular arcs in A passing through each link, A is said to be *L-uniformly loaded.* The following theorem shows the NP-completeness of the minimum ADM problem for any fixed constant g.

Theorem 1: Let g be any fixed integer constant. For any set of uniformly loaded circular arcs A, to find

Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616. Email: wan@cs.iit.edu.
Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616. Email: liwuliu@cs.iit.edu.
Department of Computer Science, Illinois Institute of Technology, Chicago, IL 60616. Email: ophir@cs.iit.edu.

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$opt_g(A)$ is NP-complete.

Proof: We will reduce the circular-graph coloring problem to the minimum ADM problem. The circular-arc coloring problem has been proven to be NP-complete in [3]. The proof in [3] actually implies the following stronger result:

Given a L-uniformly loaded circular arcs A, to decide whether its corresponding circular arc graph is L-colorable is *NP-Complete.*

Let A be any L-uniformly loaded circular arcs. Note that the chromatic number of the circular-arc graph corresponding to A, denoted by $\chi(A)$, is at least L, and $\chi(A) = L$ if and only if A can be partitioned into L subsets each of which forming a ring.

We first consider the case that $g = 1$. Obviously, $opt_1(A) \geq |A|$, and the equality holds if and only if A can be partitioned into L subsets each of which forming a ring. Thus $opt_1(A) = |A|$ if and only if $\chi(A) = L$. This implies that the minimum ADM problem is NP-complete when $g = 1$ even if the traffic demands are uniformly loaded.

Now we consider the case that $g > 1$. Let A' be the set of arcs obtained by making g copies of A. We show that $opt_g(A') = |A|$ if and only if $\chi(A) = L$. Consider any optimal grooming of A'. Each ADM can terminate at most $2g$ arcs. As each arc is terminated by two ADMs, cumulatively there are $2|A'| = 2g|A|$ arcs terminated by all ADMs. Hence

$$opt_g(A') \geq \frac{2g|A|}{2g} = |A|,$$

and the equality holds if and only if each ADM terminates exactly $2g$ arcs. Noting that in each wavelength at most g arcs can cross over any link, each ADM terminates exactly $2g$ arcs if and only if there are exactly g copies of one arc between any neighboring ADMs in a wavelength. Thus $opt_g(A') = |A|$ if and only if each aggregated ring is g copies of a subset of A which forms a ring. Therefore $opt_g(A') = |A|$ if and only if A can be partitioned into L subsets each of which forms a ring, or equivalently $\chi(A) = L$. This implies that the minimum ADM problem is NP-complete even if the traffic demands are uniformly loaded. ∎

III. General Lower Bounds

In this section, we present some lower bounds on the minimum ADM cost of any given set of traffic requests. One straightforward lower bound can be derived in the following way. Let σ_i and τ_i denote the total number of circular arcs originating at and terminating at node i respectively. Then the node i must use at least

$$\left\lceil \frac{\max(\sigma_i, \tau_i)}{g} \right\rceil$$

ADMs. Hence the total ADM cost is at least

$$\alpha_{lb} = \sum_{i=0}^{n-1} \left\lceil \frac{\max(\sigma_i, \tau_i)}{g} \right\rceil.$$

However, this lower bound is in general very loose.

When no two circular arcs share the same source and destination, we can develop another bound by calculating the *maximum ADM efficiency*, the maximal number of circular arcs that an ADM can carry out in average for any given traffic capacity of the wavelength. In the next we will introduce a stronger result. We define the *load* of a set of arcs over a ring to be the maximal number of arcs that share a link. Given any two positive integers n and g, let $A(n,g)$ denote the maximal number of different arcs over a ring of n nodes whose load is c. Then the *maximal node efficiency* over an n-node ring with link capacity g is defined by

$$E(n,g) = \frac{A(n,g)}{n},$$

and the maximal node efficiency with link capacity g is defined by

$$E(g) = \max_{n \geq 2} E(n,g).$$

To illustrate the concepts, let's look the cases that the link capacity g is one or two. It's obvious that when $g = 1$, $A(n,1) = n$ and thereby $E(n,1) = E(1) = 1$. When $g = 2$, it's easy to verify that

$$A(n,2) = \begin{cases} \left\lfloor \frac{3n}{2} \right\rfloor & \text{if } n \geq 5 \\ \left\lfloor \frac{3n}{2} \right\rfloor - 1 & \text{if } n \leq 4 \end{cases}$$

and thereby $E(2) = \frac{3}{2}$.

In general, we call a set of arcs to be a *canonical* set of arcs if it satisfies the following property: if an arc of length ℓ is in this set, then all arcs of length less than ℓ are also in this set. It's easy to show that any set of arcs over a ring can be transformed to a canonical set of arcs of the same cardinality over the same ring and with the same or less load. Thus there is always a canonical set of $A(n,g)$ arcs over a ring of n nodes whose load is g. Such canonical set of $A(n,g)$ arcs can be generated in the following greedy manner: we first add all arcs of length one, then we add all arcs of length two and so on until we can not add all arcs of some length; in this case we add as much arcs of such length as possible. Based on this greedy selection, we calculate the $A(n,g)$, $E(n,g)$ and $E(g)$ for any n and g in the next.

The following lemma gives the load of the set of arcs of length no more than k.

Lemma 2: For any $\ell \leq n-1$, the load of all arcs of length no more than ℓ over a ring of size n is $\frac{\ell(\ell+1)}{2}$.

Proof: For any $k \leq n-1$, there are n arcs of length k. These n arcs of length k contribute a load of k to each link. Thus the load contributed to each link by those $n\ell$ arcs of length at most ℓ is

$$1 + 2 + \cdots + \ell = \frac{\ell(\ell+1)}{2}.$$

∎

Let ℓ be the largest integer satisfying that $\frac{\ell(\ell+1)}{2} \le g$. If $n \le \ell + 1$, then the load of all $n(n-1)$ arcs over a ring of size n is no more than g, and thereby

$$A(n,g) = n(n-1),$$
$$E(n,g) = n - 1 \le \ell - 1$$

Now we assume that $n \ge \ell + 2$. If $g = \frac{\ell(\ell+1)}{2}$, then the load of all $n\ell$ arcs of length at ℓ over a ring of size n is exactly g, and thereby

$$A(n,g) = n\ell,$$
$$E(n,g) = \ell.$$

If $g > \frac{\ell(\ell+1)}{2}$, then $A(n,g) - n\ell$ is equal to the maximal number of arcs of length $\ell+1$ which contribute a load of no more than $g - \frac{\ell(\ell+1)}{2}$ to each link. The cumulative link load contributed by these arcs of length $\ell+1$ is at most $n\left(g - \frac{\ell(\ell+1)}{2}\right)$. As each arc of length $\ell+1$ contributes a unit load to $\ell+1$ links, the total number of such arcs is

$$A(n,g) - n\ell \le \frac{n\left(g - \frac{\ell(\ell+1)}{2}\right)}{\ell+1} = n\left(\frac{g}{\ell+1} - \frac{\ell}{2}\right).$$

On the other hand, it's obvious that

$$A(n,g) - n\ell \ge \left\lfloor \frac{n}{\ell+1} \right\rfloor \left(g - \frac{\ell(\ell+1)}{2}\right).$$

Hence

$$n\ell + \left\lfloor \frac{n}{\ell+1} \right\rfloor \left(g - \frac{\ell(\ell+1)}{2}\right) \le A(n,g) \le n\left(\frac{g}{\ell+1} + \frac{\ell}{2}\right)$$
$$\ell + \frac{1}{n}\left\lfloor \frac{n}{\ell+1} \right\rfloor \left(g - \frac{\ell(\ell+1)}{2}\right) \le E(n,g) \le \frac{g}{\ell+1} + \frac{\ell}{2}.$$

In particular, when n is a multiple of $\ell+1$,

$$A(n,g) = n\left(\frac{g}{\ell+1} + \frac{\ell}{2}\right),$$
$$E(n,g) = \frac{g}{\ell+1} + \frac{\ell}{2}.$$

Therefore

$$E(g) = \frac{c}{\ell+1} + \frac{\ell}{2}.$$

The next lemma summarizes the above discussion.

Lemma 3: For any positive integer g, the maximal node efficiency is $\frac{g}{\ell+1} + \frac{\ell}{2}$ where ℓ is the largest integer satisfying that $\frac{\ell(\ell+1)}{2} \le g$.

The maximal node efficiency provides an upper bound on the maximal ADM efficiency. As the total number of ADMs required is at least the total traffic divided by the maximal ADM efficiency, we have the following lower bound on the minimal ADMs required.

Lemma 4: Let g be the transmission capacity of each wavelength. Then the minimum ADM cost of any set of traffic demands given by D different circular arcs is at least $\left\lceil \frac{D}{\frac{g}{\ell+1} + \frac{\ell}{2}} \right\rceil$ where ℓ is the largest integer satisfying that $\frac{\ell(\ell+1)}{2} \le g$.

IV. Two-Phased Approach

Due to complexity of the problem, we propose the following two-phased approach.

Generation of Primitive Rings: Partition the given set of circular arcs into groups such that the circular arcs in any group do not overlap. Thus the circular arcs in each group can be arranged in a single ring, called as a *primitive ring*. The cost of each group (or primitive ring) is defined as the number of nodes appearing as the endpoints of the circular arcs contained in this primitive ring, and the cost of a partition is defined as the sum of the costs of the primitive rings within this partition. The objective is then to find a valid partition with minimum cost.

Grooming of Primitive Rings: Group those primitive rings into high-speed *aggregated rings* such that the number of primitive rings in each aggregated ring is no more than the traffic granularity g. Each aggregated ring is then assigned a unique wavelength or equivalently a SONET ring. As the size of each aggregated ring represents the ADM cost contributed by this aggregated ring, the sum of the sizes of these aggregated rings are exactly the total ADM cost required by the grooming. The objective is thus to find a grooming of a set of primitive rings with minimum ADM cost.

This two-phased approach tries to minimize the cost of the entire system by solving two individual optimization problems. However, the combination of an optimal solution in each phase, even if it could be found, does not necessarily lead an overall optimal solution. In the next, we briefly discuss the approaches to both phases.

A. Generation of Primitive Rings

The optimal generation of primitive rings is essentially the same problem studied in [7], which considers how to assign wavelengths to a set of lightpaths to minimize the total ADM cost. Both problems can be interpreted as the minimum ADM cost problem when the grooming granularity is one. From Theorem 1, it is NP-hard in general. A simple lower bound on the minimum ADM cost was derived in [7], which was later improved in [12] by the authors. Two heuristics were developed in [7]: **Cut-First**, and **Assign-First**. The former allows splitting of circular arcs while and the latter does not. The original design and analysis of **Assign-First** presented in [7] contains some bugs, which were fixed in[12]. In addition, three new greedy heuristics were proposed in [12]: **Iterative Merging**, **Iterative Matching**, and **Euler Cycle De-**

composition. Their approximation ratios were shown to be at most $\frac{7}{4}$. Recently, another greedy algorithm called **Close Segment First** was proposed in [2], whose approximation ration was shown to be between $\frac{4}{3}$ and $\frac{3}{2}$. To describe the design of these algorithms, we first introduce some terminologies.

We call a sequence of circular arcs as a *chain of arcs*, or a *chain* in short, if the termination of each circular arc (except the last one) is the origin of the subsequent circular arc. For each chain c, the number of nodes in c is referred to as the *cost* of c, and the number of arcs in c is referred to the *size* of c denoted by $|c|$. A chain c is said to be *odd* (or *even*) if its size is odd (or even respectively). A chain is said to be *close* if the termination of the last circular arc is also the origin of the first circular arc, or *open* otherwise. If the circular arcs in a chain do not overlap with each other, then the chain is called as a *segment*. A chain which is not a segment can be split into a number of segments by walking along the chain from some starting arc and generating a segment whenever there is an overlap. If the chain is open, the first arc in the chain is chosen as the starting arc. If the chain is close, any arc in the chain can be chosen as the staring arc.

Given a set of primitive rings, its cost can be calculated as follows. Each primitive ring can be treated as a collection disjoint segments. As the cost of a close segment is equal to the number of circular arcs inside the segment and the cost of an open segment is one plus the number of circular arcs inside the segment, the cost of a primitive ring is equal to the number of open segments plus the total number of circular arcs in this primitive ring. Consequently, the cost of a set of primitive rings is equal to the total number of open segments in all the primitive rings plus the total number of circular arcs.

Based on this observation, two sets of primitive rings would have the same cost if they contain the same number of open segments. The optimal set of primitive rings must contain the minimal number of open segments, and vice versa. Thus the optimal generation of primitive rings can be solved in two phases: in the first phase, called as *segmenting phase*, the circular arcs are grouped into segments such that the number of open segments is as few as possible; in the second phase, called as *coloring phase*, these segments are grouped into minimum number of primitive rings. Note that the second phase is exactly the well-studied circular-arc coloring problem [16]. The second phase only affects the number of primitive rings, but has no impact on the total cost of these primitive rings. So we only have to consider the optimal segmenting problem.

All the algorithms proposed in [12] and [2] are all approximation algorithms for the optimal segmenting. Due to the space limitation, we briefly describe the designs of **Iterative Merging, Iterative Matching** and **Close Segment First**. In **Iterative Merging**, each segment consists of one circular arc. At each iteration, one of the following three possible operations is performed in decreasing priority:

Operation 1. Merge two open segments into a close segment.

Operation 2. Split an open segment into two open segments and then merge one of them with another open segment into a close segment.

Operation 3. Merge two open segments into a larger open segment.

Such iteration is repeated until no merging can be obtained any more.

In **Iterative Matching**, initially each segment consists of one circular arc. At each iteration, we construct a weighted graph over the current set of segments as follows. There is an edge between two segments if and only they do not overlap but share at least one endpoint. The weight of an edge is the number of endpoints shared by the two segments incident to this edge. We then find the maximum weighted matching in the graph. The two segments incident to each edge in the obtained matching are then merged into a larger segment. Such iteration is repeated until no matching can be found any more.

The **Close Segment First** consists of two stages. In the first stage, it finds as many close segments as possible. In the second stage, it uses the iterative matching to obtain as few open segments as possible from the remaining circular arcs.

The details of their designs and performance analyses can be found in [12] and [2].

B. Grooming of Primitive Rings

An instance of the optimal grooming of primitive rings is the grooming granularity g, and a collection of primitive rings represented by a collection of sets $A_1, A_2, \ldots, A_m$ from the universe $\{0, 1, \ldots, n-1\}$. A solution is a partition of the collection of primitive rings (or sets) into groups of size at most g. A group of size k is referred to as a k-group. If all groups in a partition are k-groups, the partition is referred to as a k-grouping. The ADM cost of each group is the cardinality of the union of its component primitive rings in it, and the total ADM cost of a grooming is thus the sum of the costs of the all groups. The objective is to find an grooming with minimal total ADM costs.

Two versions of Ring Grooming are considered. In *restricted ring grooming*, the number of groups has to be the minimum $\left\lceil \frac{m}{g} \right\rceil$ so as to minimize the wavelength requirement. In *unrestricted ring grooming*, there is no constraint on the number of aggregated rings. However, one can show that there is always existing an optimal unrestricted ring grooming in which at most one group contains $\left\lfloor \frac{g}{2} \right\rfloor$ or less primitive rings, and thus the number of aggregated ring is at most one plus twice the minimum. In particular, when $g = 2$, there is one optimal unrestricted ring grooming which is also an optimal restricted ring grooming.

It was shown in [1] that optimal restricted ring grooming is NP-hard for any fixed $g > 2$. When $g = 2$, the optimal ring grooming can be solved in polynomial time. The optimal solution is reduced to a maximum weighted-matching. For any collection of sets Π, its *intersection graph*, denoted by $G(\Pi)$, is a weighted complete graph constructed as follows: the vertex set is Π; the weight of each edge (A, B) is equal to $|A \cap B|$. A matching of $G(\Pi)$ is also simply called as a matching of Π. When $g = 2$, any optimal restricted grooming corresponds to a maximum-weighted perfect matching of the given primitive rings.

When g is a power of two and $m \bmod g = 0$, we propose the following algorithm called iterative matching for optimal restricted ring grooming. It consists of $\log g$ iterations. Let Π_1 be the original sets. The i-th iteration starts with Π_i, a 2^{i-1}-grouping of Π_1, and finds a maximum-weighted *perfect* matching of Π_i. Then for each edge in the obtained matching, the two sets incident to the edge are merged. Thus the i-th iteration outputs a 2^i-grouping of Π_1, denoted by Π_{i+1}. A trivial upper bound on the approximation ratio of the iterative matching is $\frac{g}{2}$. By more sophisticated analysis, the approximation ratio was shown in [1] to be exactly 1.5 when $g = 4$ and at most 2.5 when $g = 8$. In general, its approximation ratio was conjectured to be at most $\frac{g}{4} + \frac{1}{2}$.

The iterative matching can be extended to the case that g is not a power of two. Even when g is a power of two, we can still make some potential improvement. For an example, if two sets have an empty intersection, we can leave them alone so that each of them can potentially be matched with some other sets in the future to save some ADMs. With such modification, the number of original sets in each group, referred to as the group size, might be different from each other. So in the subsequent iteration, any two groups can be merged only if the sum of their group sizes is no more than g. Another observation is that when the maximum matching is zero, further grouping can still be conducted in order to reduce the number of wavelengths. The optimal grouping that uses the minimal number of wavelengths can be formulated into the well-known *bin-packing problem*, which is an NP-complete problem but has many well-known approximation algorithms [4].

Based on above discussions, the general iterative matching is described follows. The algorithm maintains the group size for each group. At each iteration, a weighted graph is constructed as follows. Each vertex corresponds a current group, and there is an edge between two groups if and only if the their intersection is not empty and the sum of their group sizes is no more than g. The weight of each edge is then equal to the cardinality of the intersection of the two groups incident to this edge. If the edge set is not empty, we find a maximum weighted matching in this weighted graph, and then merge the two groups incident to each edge in the obtained matching. The group size of each group is updated accordingly. The algorithm then starts the next iteration. If the edge set is empty, we merge the groups by applying any approximation algorithm for the bin-packing problem to reduce the number of wavelengths used.

V. Conclusion

This paper addresses the traffic grooming of arbitrary traffic in SONET/WDM rings. We first proves the NP-hardness of this problem. We then presents two general lower bounds on the minimum ADM cost. After that we decompose the minimum ADM cost problem into two subproblems. Both subproblems remains NP-hard. Various approximation algorithms are proposed to each subproblem, and their performances are briefly discussed.

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