TWDM Multihop Lightwave Networks Based on Rotator Digraphs

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Abstract

Time and Wavelength Division Multiplexed (TWDM) is one of the most promising ways to exploit the enormous bandwidth in a single-mode optical fiber. In this paper, we focus on constructing scalable TWDM networks based on rotator digraphs. To make the network be more economically and technically feasible and to improve the network performance, each station is equipped with multiple fixed transmitters and multiple fixed receivers. For such network architecture, we present the optimal wavelength assignment and transmission schedule. We also study cost performance relation and provide a scheme to to support large network size with multiple passive star couplers.

1 Introduction

Emerging lightwave networks are expected to provide end users with the integrated services at ultra-high speed[2]. The lightwave networks may be implemented via an optical passive star coupler[4, 6] for high speed local and metropolitan area networks (LANs/MANs). Each station has a set of transmitters and receivers. This paper focuses on the configuration that each station has multiple fixed transmitters and multiple fixed receivers. We assume that each station a has T transmitters $\{(a,t) \mid 0 \leq t \leq T-1\}$ and R receivers $\{< a, r >\mid 0 \leq r \leq R-1\}$. For a transmitter (a,t), a and t are called its node index and local index respectfully. For a receiver $< a, r >$, a and r are also called its node index and local index respectfully.

A wavelength assignment induces a *virtual topology* [9, 8]. The regular topologies such as re-circulating p-Shuffle [5], the de Bruijn graph [7], the Bus-Mesh [3], etc., have been extensively considered as a potential candidate for virtual topology. For other regular communication patterns including generalized de Bruijn digraphs, generalized Kautz digraphs, hypercubes and star graphs, the reader can refer to [8]. In this paper, we use rotator digraphs as the virtual topology. One of the main motivations to propose the rotator digraph as the regular communication pattern is its extremely small diameter and average routing distance.

The *Time and Wavelength Division Multiplexed* (TWDM) media access protocol is conceived as one of the most promising techniques for lightwave networks [5]. The challenges of TWDM are that given N stations, W_{avail} wavelengths, and T transmitters and R receivers in each station, how do we optimally assign wavelengths for each transmitter and receiver to maximize the transmission concurrence, how do we schedule the transmission (note that since receivers always listen on the pre-determined wavelength, there is no need for scheduling receptions) to minimize the transmission cycle length? Furthermore, how to scalable the networks if the number of station is more than the dimension of the passive star couplers by using multiple star couplers?

This paper is organized as follows. Section 2 introduces the rotator digraphs and presents a wavelength assignment scheme to realize it. Section 3 analyzes the network structure of the TWDM network. Section 4 applies the structural properties of the TWDM network to derive the optimal wavelength assignment and transmission schedule and give an scheme for interconnection of multiple star couplers to support scalable network size. Finally, section 5 concludes this paper.

2 Network Description

An n-dimensional Rotator graph R_n, also referred to as n-rotator, is the Cayley digraph $Cay(S : P_n)$ where P_n is the *symmetric group of degree* n which consists of all permutations on n symbols $\{1, 2, \cdots, n\}$, and S consists of $n-1$ left rotations $\{\alpha_2, \alpha_3, \cdots, \alpha_n\}$, where $\alpha_k = 23\cdots k1(k+1)\cdots n$ is called the *left rotation* of length k for $2 \leq k \leq n$. The permutation $\beta_k = k12\cdots(k-1)(k+1)\cdots n$ is the inverse of α_k and is called the *right rotation* of length k. The n-rotator R_n consists of $n!$ nodes. At each node x, the link $x \to x\alpha_{i+2}$ is called the i-th outgoing link of x, and the link $x\beta_{i+2} \to x$ is called the i-th incoming link of x, where $0 \leq i < n-1$.

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The rotator digraphs have a lot of attractive properties. All rotator digraphs are vertex and edge symmetric. In [1], it is proved that the diameter of the n-rotator is $n-1$, and has a simple and optimal routing algorithm. They share the fault handling capacity of hierarchical Cayley graphs, including the Star and Pancake graphs and the binary hypercubes, with good performance possible because of the small diameter and average routing distance.

To realize the n-rotator in a network with size $N = n!$, we fisrt partition the $n-1$ outgoing (incoming) links at each station a into T (R) groups, and associate the t-th (r-th) group with transmitter (a, t) (receiver $< a, r >$). Then for each link in the rotator digraph, we tune the pair of transmitters and receivers that are associated with it in the second step to the same wavelength channels. This realization scheme can be intuitively interpreted as virtually breaking each source (destination) station into T (R) small nodes, with each small node implementing $(n-1)/T$ ($(n-1)/R$) outgoing (incoming) links. This realization induces a *transmission graph* $G(N, T, R)$, a bipartite digraph, in which the vertex sets consists of all the transmitters and receivers and each link is from transmitter to receiver.

The above realization scheme only specifies which transmitters and receivers should be assigned with the same wavelength channels. It does not specify which transmitters and receivers can have different wavelength channels. In fact, in the extreme case, all the transmitters and receivers can be assigned the same wavelength channel. However, this trivial wavelength assignment provides no transmission concurrence. As the transmission concurrence is equal to the number of wavelengths employed by the network, it's desirable to maximize the number of wavelengths used subject to the number of available wavelengths. To achieve this objective, we first characterize the structures of the all transmitters and receivers that are required to have the same wavelength channels by the above realization.

Since all the transmitters and receivers are fixtuned, for any transmitter (receiver) in the transmission graph, all receivers (transmitters) that it connects to are forced to have the same wavelength channels of the transmitter (receiver). Therefore for any pairs of transceivers, if there is path between them assuming the links in the transmission graph are bidirectional, they must have the same wavelength channel. This key observation leads to the concept of *subnetworks.* In the transmission graph, a set of transmitters and receivers form a subnetwork if there is a path between any two of them if we ignore the unidirectional nature the links.

In summary, we have the following lemma.

Lemma 1 *All transmitters and receivers constituting a subnetwork in the transmission graph are assigned to the same wavelength.*

From the above lemma, the maximum number of wavelengths that can be employed is equal to the minimum of the number of available wavelengths W and the number of subnetworks in the transmission graph. We denote by $W(T, R)$ equal to the number of subnetworks in the transmission graph. In the next section, we will determine the structure of the subnetwork structures completely.

3 Subnetwork Structures

When $\max(T, R) = p$, the structure of the subnetworks is very simple.

Theorem 1 *Suppose that* $\max(T, R) = n-1$, *then*

$$W(T, R) = N \min(T, R)$$

However, when $\max(T, R) < n-1$, the structure of the subnetworks is very complicated. In the remaining of this section, we assume that $T, R < n-1$. The general frame of our analysis is as follows.

- **Step 1**: Characterize the structure of the set of local indices of all transmitters (receivers) in the same subnetwork.
- **Step 2**: Characterize the structure of the set of node indices of all transmitters (receivers) which are in the same subnetwork and have the same local indices.

We begin with **Step 1**. Let m denote the least common multiple of $\frac{n-1}{T}$ and $\frac{n-1}{R}$, and

$$T' = \frac{m}{\frac{n-1}{T}} = \frac{T}{\frac{n-1}{m}}, R' = \frac{m}{\frac{n-1}{R}} = \frac{R}{\frac{n-1}{m}}.$$

Then

$$\frac{T}{T'} = \frac{R}{R'} = \frac{n-1}{m}.$$

The next lemma gives the result in **Step 1**.

Lemma 2 *For any subnetwork, there exists a unique* $0 \leq k < \frac{n-1}{m}$ *such that in this subnetwork*

- *the set of local indices of all transmitters is* $\{t \mid kT' \leq t < (k+1)T'\}$;

- *the set of local indices of all receivers is* $\{r \mid kR' \leq r < (k+1)R'\}$.

The lemma can be proved as follows. First it's easy to show that for any $0 \leq t \leq T-1$ and $0 \leq i \leq \frac{n-1}{T}-1$,

$$\lfloor \frac{\lfloor \frac{t\frac{n-1}{T}+i}{\frac{n-1}{R}} \rfloor}{R'} \rfloor = \lfloor \frac{t\frac{n-1}{T}+i}{m} \rfloor = \lfloor \frac{t}{T'} \rfloor,$$

and for any $0 \leq r \leq R-1$ and $0 \leq i \leq \frac{n-1}{R}-1$,

$$\lfloor \frac{\lfloor \frac{r\frac{n-1}{R}+i}{\frac{n-1}{T}} \rfloor}{T'} \rfloor = \lfloor \frac{r\frac{n-1}{R}+i}{m} \rfloor = \lfloor \frac{r}{R'} \rfloor.$$

Therefore, for any subnetwork there exists a unique integer $0 \leq k < \frac{n-1}{m}$ such that for any transmitter (a,t) and receiver (b,r) in this subnetwork,

$$\lfloor \frac{t}{T'} \rfloor = \lfloor \frac{r}{R'} \rfloor = k.$$

Secondly, on the other hand, we can prove the following lemma.

Lemma 3 *Let a be any permutation in P_n.*

1. *If $t \bmod T' > 0$, then for any $\lfloor \frac{t}{T'} \rfloor \leq t' < t$, the two transmitters (a,t) and $(a\prod_{i=t'+1}^{t}(i\frac{n-1}{T}+2, i\frac{n-1}{T}+1), r')$ are in the same subnetwork.*

2. *If $r \bmod R' > 0$, then for any $\lfloor \frac{r}{R'} \rfloor \leq r' < r$, the two receivers $< a, r >$ and $< a(r\frac{n-1}{R}+2, (r-1)\frac{n-1}{R}+2, \cdots, (r'+1)\frac{n-1}{R}+2, 1), r' >$ are in the same subnetwork.*

Therefore Lemma 2 is true.

Now we consider **Step 2**. The following lemma states that all subnetworks which have the same set of local indices of transmitters (or receivers) are isomorphic to each other.

Lemma 4 *Let σ be any permutations in P_n.*

1. *Suppose that the two transmitters (a,t) and (a',t') are in the same subnetwork. Then the two transmitters $(\sigma a, t)$ and $(\sigma a', t')$ are also in the same subnetwork.*

2. *Suppose that the two receivers $< b, r >$ and $< b', r' >$ are in the same subnetwork. Then the two receivers $< \sigma b, r >$ and $< \sigma b', r' >$ are also in the same subnetwork.*

Therefore, to study the subnetwork structure, we only need to consider the subnetwork C_k that contains the transmitter (e, kT') where e is the identity permutation. For convenience of description, we introduce some new terminologies. Let π be any permutation in P_n. A symbol i is called *invariant* under π if $\pi(i) = i$. A set S of symbols is called *closed* under π if for any symbol $i \in S$, $\pi(i) \in S$. Given a set S of symbols, π is called *local* to S if S is closed under π and other symbol not in S is invariant under π.

For any $0 \leq r < R$, let

$$N_r = \{i+2 \mid r\frac{n-1}{R} \leq i < (r+1)\frac{n-1}{R}, i \bmod \frac{n-1}{T} > 0\}.$$

The following lemma characterizes the structure of node indices of the transmitters in C_k.

Lemma 5 *Let a be any permutation in P_n. Then the transmitter (a, kT') is in C_k if and only if*

1. *a is local to the set $\cup_{r=kR'}^{(k+1)R'-1} N_r$;*

2. *the set $N_{kR'} \cup \{km+2\}$ is closed closed under a;*

3. *for any $kR' < r < (k+1)R'$, the set N_r is closed under a.*

The proof is very complicated and is omitted here. From Lemma 2 and Lemma 5, the number of transmitters with local indices kT' in C_k is

$$(|N_{km}|+1)! \prod_{r=kR'+1}^{(k+1)R'-1} |N_r|!.$$

The following two lemma count $|N_r|$ in the case $T \leq R$ and in the case $T > R$ respectively.

Lemma 6 *If $T \leq R$, Then*

$$|N_r| = \begin{cases} \frac{n-1}{R} - 1 & \text{if } r \in \{\lfloor \frac{tR}{T} \rfloor \mid 0 \leq t < T\}, \\ \frac{n-1}{R} & \text{otherwise.} \end{cases}$$

If $T > R$, then

$$|N_r| = \frac{n-1}{R} - \lfloor \frac{(r \bmod R' + 1)T}{R} \rfloor + \lfloor \frac{(r \bmod R')T}{R} \rfloor.$$

Now are able to calculate $W(T,R)$.

Theorem 2 *Supposer that $T, R < n-1$. If $T \leq R$, then $W(T,R)$ is equal to*

$$\frac{n-1}{m} \frac{n!}{(\frac{n-1}{R}-1)!)^{T'-1}(\frac{n-1}{R}!)^{R'-T'+1}}.$$

If $T > R$, then $W(T,R)$ is equal to

$$\frac{n-1}{m} \frac{n!}{(\frac{n-1}{R} - \lfloor \frac{T}{R} \rfloor + 1)! \prod_{r=1}^{R'-1}(\frac{n-1}{R} - \lfloor \frac{(r+1)T}{R} \rfloor + \lfloor \frac{rT}{R} \rfloor)!}.$$

4 Applications of Subnetwork Structures

In this section, we will apply the structural properties of the subnetworks to the optimal wavelength assignment, the optimal transmission schedule and the scalability issues.

We first study the optimal wavelength assignment and transmission schedule given N, T, R and W_{avail}. It's easy to see that the transmission concurrence equals to the number of wavelengths employed by the network, and the transmission cycle length equals to the maximal number of transmitters that share a particular wavelength. Since all transmitters and receivers in a subnetwork must have the same wavelength channel, to maximize the transmission concurrence, the network should exploit $\min(W_{avail}, W(T,R))$ wavelength channels. To minimize the transmission cycle length, the subnetworks should share W_{avail} available wavelength channels *evenly.* Once we have the optimal wavelength assignment, the optimal transmission schedule become straightforward as we discussed previously. In summary, we have the following corollary regarding the optimal wavelength assignment and transmission schedule.

Corollary 1 *If $W_{avail} \geq W(T,R)$, each subnetwork has a unique wavelength channel in any optimal wavelength assignment. Otherwise, the $W(T,R)$ subnetworks should share the W_{avail} wavelength channels evenly. Therefore, the maximal transmission concurrence is $\min(W_{avail}, W(T,R))$, and the minimal transmission cycle length is $\frac{NT}{W(T,R)} \cdot \lceil \frac{W(T,R)}{W_{avail}} \rceil$.*

In a single star broadcast-and-select optical network, the network size is limited by the dimension of the optical passive star coupler. To support a larger network size, one possible solution is to use multiple star couplers, each of which interconnects a subset of the stations in the network. The structure of subnetworks can provide a straightforward implementation of multi-star broadcast-and-select optical networks. If we carefully select the values of T and R, the number of transmitters and receivers in each subnetwork can be far less than the dimension of each individual star coupler. Therefore it's possible to use one star coupler to interconnect multiple subnetworks. To reduce the number of star couplers, each coupler should interconnect as many subnetworks as possible.

5 Conclusion

One potential problem for multihop lightwave networks is the long delay of the message. To reduce such delay, we propose the rotator digraphs as the virtual topology as the rotator digraphs has extremely short diameter and average routing distance. To improve the network throughput and reduce the transmission cycle length, we use multiple fixed transmitters and multiple fixed receivers at each station. In this paper, we have studied the network structure. Based on the elegant structural properties, we have derived the optimal wavelength assignment and transmission schedule and given an scheme for interconnection of multiple star couplers to support scalable network size.

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