# Localized Low-Weight Graph and Its Applications in Wireless Ad Hoc Networks

Xiang-Yang Li*, Yu Wang*, Peng-Jun Wan*, Wen-Zhan Song* and Ophir Frieder*
*Department of Computer Science
Illinois Institute of Technology, 10 W. 31st Street, Chicago, IL 60616
Email: xli@cs.iit.edu, wangyu1@iit.edu, wan@cs.iit.edu, songwen@iit.edu, ophir@cs.iit.edu

*Abstract*—**We propose a new localized structure, namely, *Incident MST and RNG Graph* (IMRG), for topology control and broadcasting in wireless ad hoc networks. In the construction algorithm, each node first builds a *modified relative neighborhood graph* (RNG'), and then informs its one-hop neighbors its incident edges in RNG'. Each node then collects all its one-hop neighbors and the two-hop neighbors who have RNG edges to some of its one-hop neighbors, and builds an Euclidean minimum spanning tree of these nodes. Each node $u$ keeps an edge $uv$ only if $uv$ is in the constructed minimum spanning tree. We analytically prove that the node degree of the IMRG is at most 6, it is connected and planar, and more importantly, the total edge length of the IMRG is within a constant factor of that of the minimum spanning tree. To the best of our knowledge, this is the first algorithm that can construct a structure with all these properties using small communication messages (at most $13n$ total messages, each with $O(\log n)$ bits) and small computation cost, where $n$ is the number of wireless nodes. Test results are corroborated in the simulation study.**

## I. Introduction

We consider a wireless ad hoc network composed of $n$ nodes distributed in a two-dimensional plane. We assume that all wireless nodes have distinctive identities and each static wireless node knows its position information either through a low-power Global Position System (GPS) receiver or through some other way. More specifically, it is enough for our protocol that each node knows the relative position of its one-hop neighbors. The relative position of neighbors can be estimated by the *direction of arrival* and the *strength of signal*. We assume that each wireless node has an omni-directional antenna and a single transmission of a node can be received by *any* node within its vicinity which, we assume, is a unit disk centered at this node. A wireless node can receive the signal from another node if it is within the transmission range of the sender. Otherwise, they communicate through multi-hop wireless links by using intermediate nodes to relay the message. Consequently, each node in the wireless network also acts as a router, forwarding data packets for other nodes. By one-hop broadcasting, each node $u$ can gather the location information of all nodes within the transmission range of $u$. Consequently, all wireless nodes together define a unit-disk graph (UDG), which has an edge $uv$ if and only if the Euclidean distance $\|uv\|$ is less than one unit.

Wireless ad hoc networks require special treatment as they intrinsically have unavoidable limitations as compared with wired networks. For example, wireless nodes are often powered by batteries only, and they often have limited memories. So wireless ad hoc networks prefer localized and power-efficient algorithms. A transmission by a wireless device is often received by many nodes within its vicinity, called *broadcasting*. We utilize this broadcasting property to reduce the communications needed to send some information. Throughout this paper, a *local broadcast* by a node means it sends the message to all nodes within its transmission range; a *global broadcast* by a node means it tries to send the message to all nodes in the network by the possible relaying of other nodes. Since the main communication cost in wireless networks is to send out the signal while the receiving cost of a message is neglected here, a protocol's message complexity is only measured by how many messages are sent out by all nodes.

In recent years, many research efforts focus on topology control for wireless ad hoc networks [1], [2], [3], [4], [5]. These algorithms are designed for different objectives: minimizing the maximum link length while maintaining the network connectivity [3]; bounding the node degree [5]; bounding the spanning ratio [1], [2]; constructing planar spanner locally [1]. Here a structure $H$ is a spanner of UDG if, for any two nodes, the length of the shortest-path connecting them in $H$ is no more than a constant factor of the length of the shortest-path connecting them in the original UDG. Planar structures are used by several localized routing algorithms [6], [7]. Li and Wang [8] recently also proposed the first localized algorithm to construct a bounded degree planar spanner.

A structure is called *low weight* if its total edge length is within a constant factor of the total edge length of the minimum spanning tree (MST). However, no localized algorithm is known to construct a low-weighted structure. It was recently shown in [9] that a broadcasting based on MST consumes energy within a constant factor of the optimum.

The best distributed algorithm [10], [11] can compute MST in $O(n)$ rounds using $O(m + n \log n)$ communications for a general graph with $m$ edges and $n$ nodes. Since the relative neighborhood graph, the Gabriel graph, and the Yao graph all have $O(n)$ edges and contain the Euclidean MST, we can construct the minimum spanning tree of UDG in a distributed manner using $O(n \log n)$ messages. Unfortunately, even for a wireless network modelled by a ring, the $O(n \log n)$ number of messages is still necessary for constructing MST of UDG.

Recently, Li, Hou, and Sha [12] proposed a novel MST-based method for topology control. Each node $u$ uses its one-hop neighbors to build a *local* minimum spanning tree. They call the final graph *local minimum spanning tree* (LMST). They prove that the graph is connected, and has bounded degree 6. However, it can be shown that LMST is not a low weight structure.

We present the first efficient localized method to construct a bounded degree planar connected structure *Incident MST and RNG Graph* (IMRG) whose total edge length is within a constant factor of MST. The degree of each node is at most 6. The total communication cost of our method is at most $13n$, and every node only uses its partial two-hop information to construct such structure. It was shown in [13], [14] that *some two-hop information is necessary to construct any low-weighted structure*. We also studied the application of this structure in efficient broadcasting in wireless ad hoc networks.

Energy conservation is a critical issue in *ad hoc* wireless network for the node and network life, as the nodes are powered by batteries only. In the most common power-attenuation model, the power needed to support a link $uv$ is $\|uv\|^\beta$, where $\|uv\|$ is the Euclidean distance between $u$ and $v$, $\beta$ is a real constant between 2 and 5 dependent on the wireless transmission environment.

Minimum-energy broadcast/multicast routing in ad hoc network environments is addressed in [15], [16]. To assess the complexities one at a time, the nodes in the network are assumed to be randomly distributed in a two-dimensional plane, and there is no mobility. Three centralized greedy heuristics (as opposed to distributed) algorithms were presented in [16], namely, MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). For illustration purposes, another slight variation of BIP, called BAIP, was discussed in detail in [9]. Wan, *et al.* [9] showed that the approximation ratio of MST-based approach is between 6 and 12, which is the best known method theoretically. Unfortunately, MST cannot be constructed in a localized manner, i.e., each node cannot determine which edge is in the defined structure by purely using the information of the nodes within some constant hops. The relative neighborhood graph was used for broadcasting in wireless ad hoc networks [17]. It is well-known that $MST \subseteq RNG$. The ratio of the weight of RNG over the weight of MST could be $O(n)$ for $n$ points set [18]. As shown in [13], [14], the total energy used by the global broadcasting based on RNG could be about $O(n^\beta)$ times optimum.

Notice that a structure with low-weight cannot guarantee that the broadcasting based on it consumes energy within a constant factor of the optimum. The energy consumption using our new structure IMRG is within $O(n^{\beta-1})$ of the optimum. This improves the previously best known "lightest" structure RNG by an $O(n)$ factor. Our extensive simulations show that the energy consumption of broadcasting based on structure IMRG is within a small constant factor of that based on the MST and has significant improvement over the energy consumption based on RNG.

The rest of the paper is organized as follows. In Section II, we review the related work on network topology control and minimum energy broadcasting. In Section III, we present our communication and computation efficient localized method that can construct a connected, planar, bounded degree, low-weight structure IMRG. The total communication cost to build it is at most $13n$. We compare the performance of this structure with previously best-known structures in Section IV. We conclude our paper with a discussion of possible future research directions in Section V.

## II. Related Work

Before reviewing the related work, we first introduce the formal definition of *low weight*. Given a structure $G$ over a set of points, let $\omega(G)$ be the total length of the links in $G$ and $\omega_\beta(G)$ be the total power needed to support all links in $G$, i.e., $\omega_\beta(G) = \sum_{uv \in G} \|uv\|^\beta$. Then, a structure $G$ is called *low weight* if $\omega(G)$ is within a constant of $\omega(MST)$.

### A. Topology Control

Recently, topology control for wireless ad hoc networks has attracted considerable attention [3], [19], [20], [21], [22], [23], [24]. Rajaraman [25] conducted an excellent survey. Several geometrical structures are used for topology control. Here we review the definitions of some of them.

The *relative neighborhood graph*, denoted by RNG, is a geometric concept proposed by Toussaint [26]. It consists of all edges $uv$ such that there is no point $w$ with $uw$ and $wv$ satisfying $\|uw\| < \|uv\|$ and $\|wv\| < \|uv\|$. Let $disk(u, v)$ be the disk with diameter $uv$. Then, the *Gabriel graph* [27] (GG) contains an edge $uv$ from $G$ if and only if $disk(u, v)$ contains no other vertex $w$ inside. It is easy to show that RNG is a subgraph of the Gabriel graph GG. For unit disk graph, the relative neighborhood graph and the Gabriel graph only contain the edges in UDG and satisfying the respective definitions. Both GG and RNG are used as network topology in wireless ad hoc networks.

The *Yao graph* with an integer parameter $k \geq 6$, denoted by $\overrightarrow{YG}_k$, is defined as follows. At each node $u$, any $k$ equally-separated rays originated at $u$ define $k$ cones. In each cone, choose the shortest edge $uv$, if there is any, and add a directed link $\overrightarrow{uv}$. Ties are broken arbitrarily or by the smallest ID. The resulting directed graph is called the Yao graph. Some researchers used a similar construction named $\theta$-graph [28]. Recently, the Yao structure was re-discovered by several researchers for topology control in wireless ad hoc networks of directional antennas.

Li, *et al.* [18] extended the definitions of these structures on top of any given graph $G$. They proposed to apply the Yao structure on top of the Gabriel graph structure, and apply the Gabriel graph structure on top of the Yao structure. These structures are sparser than the Yao structure and the Gabriel graph, and they still have a constant bounded power stretch factor. These two structures are connected graphs. Wattenhofer, *et al.* [24] also proposed a two-phased approach that consists of a variation of the Yao graph followed by a variation of the Gabriel graph.

Li, *et al.* [21] proposed a structure that is similar to the Yao structure for topology control. Each node $u$ finds a power $p_{u,\alpha}$ such that in every cone of degree $\alpha$ surrounding $u$, there is some node that $u$ can reach with power $p_{u,\alpha}$. Notice that the number of cones to be considered in the traditional Yao structure is a constant $k$. However, unlike the Yao structure, for each node $u$, the number of cones needed to be considered in the method proposed in [21] is about $2n$, where each node $v$ could contribute two cones on both side of segment $uv$. Then the graph $G_\alpha$ contains all edges $uv$ such that $u$ can communicate with $v$ using power $p_{u,\alpha}$. They proved that, if $\alpha \leq \frac{5\pi}{6}$ and the UDG is connected, then graph $G_\alpha$ is a connected graph. Unlike the Yao structure, the final topology $G_\alpha$ is not necessarily a bounded degree graph.

Li, *et al.* [18] also proposed another structure called *YaoYao graph* $\overrightarrow{YY}_k$ by applying a *reverse* Yao structure on $\overrightarrow{YG}_k$. They proved that the directed graph $\overrightarrow{YY}_k$ is strongly connected if UDG is connected and $k > 6$. In [5], Wang, *et al.* considered another undirected structure, called *symmetric Yao graph* $YS_k$. An edge $uv$ is selected if and only if both directed edges $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are in the $\overrightarrow{YG}_k$. Then it is obvious that the maximum node degree is $k$. They showed that the graph $YS_k$ is strongly connected if UDG is connected and $k \geq 6$.

Recently, Li, Hou, and Sha [12] proposed a novel local MST-based method for topology control. Each node $u$ first collects its one-hop neighbors $N_1(u)$. Node $u$ then computes the minimum spanning tree $MST(N_1(u))$ of the induced unit disk graph on its one-hop neighbors $N_1(u)$. Node $u$ keeps a directed edge $uv$ if and only if $uv$ is an edge in $MST(N_1(u))$. They call the union of all directed edges of all nodes the *local minimum spanning tree*, denoted by $G_0$. If only symmetric edges are kept, then the graph is called $G_0^-$, i.e., it has an edge $uv$ if and only if both directed edge $uv$ and directed edge $vu$ exists. If we ignore the directions of the edges in $G_0$, they call the graph $G_0^+$, i.e., it has an edge $uv$ if and only if either directed edge $uv$ or directed edge $vu$ exists. They prove that the graph is connected, and has bounded degree 6.

Here, we also show that graph $G_0^-$ is actually planar. For the sake of contradiction, assume that $G_0^-$ is not a planar graph and two edges $uv$ and $xy$ intersect each other. Assume that the clockwise order of these four nodes are $u, y, v, x$. Obviously, one of the four angles $\angle uxv$, $\angle xvy$, $\angle vyu$, and $\angle yux$ is at least $\pi/2$. Without loss of generality, assume that $\angle uxv \geq \pi/2$. Thus, edge $uv$ is the longest edge among triangle $\triangle uvx$. Thus, in the local minimum spanning tree $MST(N_1(u))$, edge $uv$ cannot appear since there is already a path $uxv$ whose edges are all shorter than $uv$. Similarly, graph $G_0^+$ is a planar graph (by replacing the undirected edges with directed edges in the above proof).

Inspired by the local minimum spanning tree structure in [12], we propose another structure, called IMRG, that has an additional property: the total edge length of the structure is no more than a constant factor of that of the minimum spanning tree. We call this property *low weight*. Notice that the total edge length is related to the total power of all nodes used to keep the network connected. It is not difficult to construct an example such that the structure $G_0^-$ and $G_0^+$ are not low-weight (the same example in [13], [14] for RNG). We also show that our structures $IMRG^+$ and $IMRG^-$ are always subgraphs of the structures $G_0^+$ and $G_0^-$ constructed in [12].

### *B. Power Assignment*

Assume that each node can adjust its transmission power according to its neighbors' positions for a possible energy conservation. A natural question is then how to assign the transmission power for each node such that the wireless network is connected with the optimization criteria being minimizing the maximum or total transmission power assigned.

A transmission power assignment on the vertices in $V$ is a function $f$ from $V$ into real numbers. The *communication graph*, denoted by $G_f$, associated with a transmission power assignment $f$, is a directed graph with $V$ as its vertices and has a directed edge $\overrightarrow{v_iv_j}$ if and only if $||v_iv_j||^\beta + c \leq f(v_i)$. We call a transmission power assignment $f$ *complete* if the communication graph $G_f$ is strongly connected. Here $c$ is the fixed overhead cost of a node receiving and processing the signal, which is assumed to be same for all nodes.

The *maximum-cost* of a transmission power assignment $f$ is defined as $mc(f) = \max_{v_i \in V} f(v_i)$. And the *total-cost* of a transmission power assignment $f$ is defined as $sc(f) = \sum_{v_i \in V} f(v_i)$. The min-max assignment problem is then to find a complete transmission power assignment $f$ whose cost $mc(f)$ is the least among all complete assignments. The min-total assignment problem is to find a complete transmission power assignment $f$ whose cost $sc(f)$ is the least among all complete assignments.

Given a graph $H = (V, E)$, we say the power assignment $f$ is induced by $H$ if

$$f(v) = \max_{(v,u) \in E} ||vu||^\beta + c,$$

where $E$ is the set of edges of $H$. In other words, the power assigned to a node $v$ is the largest power needed to reach all neighbors of $v$ in $H$. Clearly, when graph $H$ is connected, the induced power assignment $f$ is complete.

Transmission power control is well-studied. Monks, *et al.* [29] conducted simulations which show that implementing power control in a multiple access environment can improve the throughput of the non-power controlled IEEE 802.11 by a factor of 2. Therefore, it provides a compelling reason for adopting the power controlled MAC protocol in wireless network.

The min-max assignment problem was studied by several researchers [3], [30]. Let EMST be the Euclidean minimum spanning tree over a point set $V$. Both [3] and [30] use the power assignment induced by EMST. It was proved in [3] that the longest edge of the Euclidean minimum spanning tree EMST is always the critical link for min-max assignment. Here, for an optimum transmission power assignment $f_{opt}$, call a link $uv$ the *critical link* if $||uv||^\beta + c = mc(f_{opt})$. Both algorithms presented in [3] and [30] compute the minimum spanning tree from the fully connected graph with possible very large communication cost. Notice that the best distributed algorithm [10], [11], [31] can compute the minimum spanning tree in $O(n)$ rounds using $O(m + n \log n)$ communications for a general graph with $m$ edges and $n$ nodes. Using the fact that RNG, GG and the $YG_k$ all have $O(n)$ edges and contain the EMST, a simple $O(n \log n)$ time complexity centralized algorithm can be developed and can be implemented efficiently in a distributed manner.

The min-total assignment problem was studied by Kiroustis, *et al.* [32] and by Clementi, *et al.* [33], [34], [35]. Kiroustis, *et al.* [32] first proved that the min-total assignment problem is *NP-hard* when the mobile nodes are deployed in a three-dimensional space. A simple 2-approximation algorithm based on the Euclidean minimum spanning tree was also given in [32]. The algorithm guarantees the same approximation ratio in any dimensions. Clementi, *et al.* [33], [34], [35] proved that the min-total assignment problem is still NP-hard when nodes are deployed in a two dimensional space.

So far, we generate asymmetric communication graph from the power assignment. For the symmetric communication, several methods also guarantee a good performance. It is easy to show that the minimum spanning tree method still gives the optimum solution for the min-max assignment and a 2-approximation for the min-total assignment. Recently, Călinescu, *et al.* [36] gave a method that achieves better approximation ratio $\frac{5}{3}$ by using an idea from the minimum Steiner tree. Like the minimum spanning tree method, it works for any power definition.

### C. Minimum Energy Broadcasting

Minimum-energy broadcast/multicast routing in an ad hoc network environment is addressed in [15], [16]. Any broadcast routing is viewed as an arborescence (a directed tree) $T$, rooted at the source node of the broadcasting, that spans all nodes. Let $f_T(p)$ denote the transmission power of the node $p$ required by $T$. For any leaf node $p$ of $T$, $f_T(p) = 0$. For any internal node $p$ of $T$, $f_T(p) = \max_{pq \in T} \|pq\|^{\beta}$, i.e., the $\beta$-th power of the longest distance between $p$ and its children in $T$. The total energy required by $T$ is $\sum_{p \in V} f_T(p)$. Thus, the minimum-energy broadcast routing problem is different from the conventional link-based minimum spanning tree problem. Indeed, while the MST can be solved in polynomial time by algorithms such as Prim's algorithm and Kruskal's algorithm, it is known [37] that the minimum-energy broadcast routing problem cannot be solved in polynomial time if $P \neq NP$. Three greedy heuristics were proposed in [16] for the minimum-energy broadcast routing problem: MST (minimum spanning tree), SPT (shortest-path tree), and BIP (broadcasting incremental power). For a pure illustration purpose, another variation of BIP (called BAIP) was discussed in detail in [9]. Wan, *et al.* [9] showed that the approximation ratio of the MST based approach is between 6 and 12; the approximation ratio of the BIP is between $\frac{13}{2}$ and 12; on the other hand, the approximation ratios of SPT and BAIP are at least $\frac{n}{2}$ and $\frac{4n}{\ln n} - o(1)$ respectively, where $n$ is the number of nodes. The following lemma was proved in [9].

*Lemma 1:* For any point set $V$ in the plane, the total energy required by any broadcasting among $V$ is at least $\omega_\beta(MST)/C_{mst}$, where $6 \leq C_{mst} \leq 12$ is a constant related to the geometry minimum spanning tree.

RNG is used for broadcasting in wireless ad hoc networks [17]. Obviously, the ratio of the total edge length of RNG over that of MST could be $O(n)$ for $n$ points set [18]. An example was given in [13], [14] to show that the total energy used by broadcasting on RNG could be about $O(n^{\beta})$ times of the minimum-energy used by an optimum method. We can prove that the $\omega_\beta(IMRG) \leq O(n^{\beta-1}) \cdot \omega_\beta(MST)$ which improves the previously known structure RNG by $O(n)$ factor.

## III. Construct Low Weighted Structure Locally

In this section, we present our efficient localized method to construct a connected, low-weighted, bounded degree planar structure.

### A. Modified RNG

Let $\|xy\|$ denote the Euclidean distance between two points $x$ and $y$. A disk centered at a point $x$ with radius $r$, denoted by $disk(x, r)$, is the set of points whose distance to $x$ is at most $r$, i.e., $disk(x, r) = \{y \mid \|xy\| \leq r\}$. Let $lune(u, v)$ defined by two points $u$ and $v$ be the intersection of two disks with radius $\|uv\|$ and centered at $u$ and $v$ respectively, i.e., $lune(u, v) = disk(u, \|uv\|) \cap disk(v, \|uv\|)$. The *relative neighborhood graph* [26], denoted by RNG, consists of all edges $uv$ such that the *interior* of $lune(u, v)$ contains no point $w \in V$. Notice here if only the boundary of $lune(u, v)$ contains a point from $V$, edge $uv$ is still included in RNG. A minimum spanning tree of a set of points $V$ is a connected graph whose weight is the minimum among all connected graphs spanning $V$. It is known that the relative neighborhood graph always contains the minimum spanning tree as a subgraph.

Our low-weight structure is based on a modified relative neighborhood graph. Notice that, traditionally, the relative neighborhood graph will always select an edge $uv$ even if there is some node on the boundary of $lune(u, v)$. Thus, RNG may have unbounded node degree, e.g., considering $n - 1$ points equally distributed on the circle centered at the $n$th point $v$, the degree of $v$ is $n - 1$. Notice that for the sake of lowing the weight of a structure, the structure should contain as less edges as possible without breaking the connectivity. Li [13], [14] then naturally extended the traditional definition of RNG as follows.

The *modified relative neighborhood graph* consists of all edges $uv$ such that (1) the *interior* of $lune(u, v)$ contains no point $w \in V$ and, (2) there is no point $w \in V$ with $ID(w) < ID(v)$ on the boundary of $lune(u, v)$ and $\|wv\| < \|uv\|$, and (3) there is no point $w \in V$ with $ID(w) < ID(u)$ on the boundary of $lune(u, v)$ and $\|wu\| < \|uv\|$, and (4) there is no point $w \in V$ on the boundary of $lune(u, v)$ with $ID(w) < ID(u)$, $ID(w) < ID(v)$, and $\|wu\| = \|uv\|$. See Figure

1 for an illustration when an edge $uv$ is *not* included in the modified relative neighborhood graph. Li called such structure

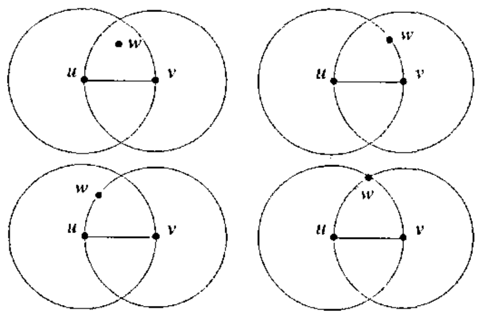


Fig. 1. Four cases when edges are not in the modified RNG.

by RNG'. Obviously, RNG' is a subgraph of traditional RNG. It was proved in [13], [14] that RNG' has a maximum node degree 6 and still contains a MST as a subgraph. However, RNG' is still not a low weight structure.

Obviously, graph RNG' still can be constructed using $n$ messages. Each node first locally broadcasts its location and ID to its one-hop neighbors. Then every node decides which edge to keep solely based on the one-hop neighbors' location information collected. Since the definition is still symmetric, the edges constructed by different nodes are consistent, i.e., an edge $uv$ is kept by a node $u$ if and only if it is also kept by node $v$. The computational cost of a node $u$ is still $O(d \log d)$, where $d$ is its degree in UDG. A simple edge by edge testing method has time complexity $O(d^2)$.

### B. Bound the Weight

We now peovide a communication efficient method to construct a sparse topology from RNG' whose total edge weight is within a constant factor of $\omega(MST)$. In [13], [14], Li gave the first localized method to construct a structure with weight $O(\omega(MST))$ using total $O(n)$ local-broadcast messages, but the computation at each node is expensive. Notice that it is well-known that the communication complexity of constructing a minimum spanning tree of a $n$-vertex graph $G$ with $m$ edges is $O(m + n \log n)$; while the communication complexity of constructing MST for UDG is $O(n \log n)$ even under the local broadcasting communication model in wireless networks. It was shown in [13], [14] that it is *impossible* to construct a low-weighted structure using only one hop neighbor information.

We first review the localized algorithm given in [14] that constructs a low-weighted structure using only some two hops information.

*Algorithm 1:* Construct Low Weight Structure LRNG

1) All nodes together construct the graph RNG' in a localized manner.
2) Each node $u$ locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node $u$ listens to the messages from its one-hop neighbors.
3) Assume node $u$ received a message informing existence of edge $xy \in RNG'$ from its neighbor $x$. For each edge $uv \in RNG'$, if $uv$ is the longest among $uv$, $xy$, $ux$, and $vy$, node $u$ removes edge $uv$. Ties are broken by the label of the edges. Here assume that $uvyx$ is the convex hull of $u$, $v$, $x$, and $y$.
4) Let $LRNG$ be the final structure formed by all remaining edges in RNG'.

Obviously, if an edge $uv$ is kept by node $u$, then it is also kept by node $v$. It was shown in [13], [14] that the structure $LRNG$ has total edge length $\Theta(\omega(MST))$.

Clearly, the communication cost of Algorithm 1 is at most $7n$: initially each node spends one message to tell its one-hop neighbors its position information, then each node $uv$ tells its one-hop neighbors all its incident edges $uv \in RNG'$ (there are at most total $6n$ such messages since $RNG'$ has at most $3n$ edges). The computational cost of Algorithm 1 could be high since for each link $uv \in RNG'$, node $u$ has to test whether there is an edge $xy \in RNG'$ and $x \in N_1(u)$ such that $uv$ is the longest among $uv$, $xy$, $ux$, and $vy$. We continue to present our new algorithms that improve the computational complexity of each node while still maintains low communication costs.

*Algorithm 2:* Construct Low Weight Structure by MST of 2-hop Neighbors

1) Each node $u$ collects its two hop neighbors information $N_2(u)$ using a communication efficient protocol described in [38].
2) Each node $u$ computes the Euclidean minimum spanning tree $MST(N_2(u))$ of all nodes $N_2(u)$, including $u$ itself.
3) For each edge $uv \in MST(N_2(u))$, node $u$ tells node $v$ about this directed edge.
4) Node $u$ keeps an edge $uv$ if $uv \in MST(N_2(u))$ or $vu \in MST(N_2(v))$. Let $LMST_2^+$ be the final structure formed by all edges kept. [1]

We then prove that structures $LMST_2^+$ and $LMST_2^-$ are connected, planar, low-weighted, and has bounded node degree at most 6.

*Lemma 2:* MST is a subgraph of $LMST_2^-$ and $LMST_2^+$.

Proof. We prove MST is a subgraph of $LMST_2^-$ by induction on the length of the edges in MST.

Consider the shortest edge $uv$ in the original unit disk graph. Clearly, the edge $uv$ belongs to MST, and $uv$ belongs to $MST(N_2(w))$ for any node $w$. Thus, $uv$ belongs to $LMST_2^-$.

Assume that the first $k$th shortest edges from MST are in $LMST_2^-$. Then consider the $(k+1)$th shortest edge $uv$ from MST. For the sake of contradiction, assume that some node $w$ removes edge $uv$ because $uv$ does not belong to $MST(N_2(w))$ and $u \in N_1(w)$. From the property of minimum spanning tree, we know that there is a path in the unit disk graph formed on $N_2(w)$ connecting $u$ and $v$ using

[1] It keeps an edge if either node $u$ or node $v$ wants to keep it. Another option is to keep an edge only if both nodes want to keep it. Let $LMST_2^-$ be the structure formed by such edges.

edges with length at most $\|uv\|$ (ties are broken by rank). Clearly, these edges are also in the original UDG and thus it is a contradiction to the fact that $uv$ belongs to MST. Thus, edge $uv$ is also kept $LMST_2^-$.

Thus, MST is a subgraph of $LMST_2^-$. Since $LMST_2^-$ is a subgraph $LMST_2^+$, MST is a subgraph of $LMST_2^+$. ∎

The above lemma immediately implies that

*Lemma 3:* Structures $LMST_2^-$ and $LMST_2^+$ are connected.

*Lemma 4:* Structures $LMST_2^-$ and $LMST_2^+$ are subgraphs of $RNG'$.

Proof. We prove the above by contradiction. Assume that a node $u$ adds an edge $uv \notin RNG'$ to $LMST_2$. Since edge $uv \notin RNG'$, there is a node $w$ inside the lune defined by segment $uv$ or a node $w$ on the boundary of the lune with smaller ID. Remember that the minimum spanning tree of the node set $N_1(u)$ can be constructed by adding edges in ascending order (using IDs to break the ties) whenever it does not create a cycle with previously added edges. Clearly, when we try to add edge $uv$, there is already a path connecting $u$ and $w$ and a path connecting $w$ and $v$ since $uw$ and $wv$ are not longer than $uv$ (or have same length but with smaller IDs). It implies that node $u$ cannot have edge $uv$ in its $MST(N_2(u))$. Consequently, both graph $LMST_2^+$ and graph $LMST_2^-$ are subgraphs of RNG'. ∎

Since RNG' is a planar graph with bounded node degree at most 6, the above lemma immediately implies that

*Lemma 5:* Structures $LMST_2^-$ and $LMST_2^+$ are planar graphs and with bounded node degree at most 6.

To prove that structure $LMST_2^+$ is low-weighted, we need the following result proved in [13], [14].

*Lemma 6:* A subgraph $G$ of RNG' is low-weighted if for any two edges $uv \in G$ and $xy \in G$, neither $uv$ nor $xy$ is the longest edge of quadrilateral $uvyx$.

We then prove following lemma.

*Lemma 7:* Structures $LMST_2^-$ and $LMST_2^+$ are low-weighted.

Proof. Consider any quadrilateral $uvyx$ formed by two edges $uv \in LMST_2^+$ and $xy \in LMST_2^+$. W.l.o.g. assume that $uv$ is the longest edge, then $\|ux\| \leq 1$, $\|yv\| \leq 1$. Thus, the four edges of quadrilateral $uvyx$ are in the UDG induced on $N_2(u)$. Consequently, edge $uv$ will be removed when constructing the local minimum spanning tree $MST(N_2(u))$. Together with Lemma 6, we know that $LMST_2^+$ is low-weighted. Structure $LMST_2^-$ is low-weighted directly from $LMST_2^- \subseteq LMST_2^+$. ∎

Although the constructed structures $LMST_2^-$ and $LMST_2^+$ have several nice properties such as being bounded degree, planar, and low-weighted, the communication cost of Algorithm 2 could be very large to save the computational cost of each node. The large communication costs are from collecting the two hop neighbors information $N_2(u)$ for each node $u$, although the total communication of the protocol described in [38] is $O(n)$, the hidden constant is large.

We could improve its communication cost of collecting $N_2(u)$ by using a subset of two hop information without sacrificing any properties. Define

$$N_2^{RNG'}(u) = \{w \mid vw \in RNG' \text{ and } v \in N_1(u)\} \cup N_1(u).$$

We will first build $RNG'$ to collect $N_2^{RNG'}(u)$ for each node $u$, then apply local MST based on $N_2^{RNG'}(u)$. We describe our modified algorithm as follows.

*Algorithm 3:* Construct Low Weight Structure by 2-hop Neighbors in RNG'

1) Each node $u$ tells its position information to its one-hop neighbors $N_1(u)$ using a local broadcast model. All nodes together construct the graph RNG' in a localized manner.
2) Each node $u$ locally broadcasts its incident edges in RNG' to its one-hop neighbors. Node $u$ listens to the messages from its one-hop neighbors.
3) Each node $u$ computes the Euclidean minimum spanning tree $MST(N_2^{RNG'}(u))$ of all nodes $N_2^{RNG'}(u)$, including $u$ itself.
4) For each edge $uv \in MST(N_2^{RNG'}(u))$, node $u$ tells node $v$ about this directed edge.
5) Node $u$ keeps an edge $uv$ if $uv \in MST(N_2^{RNG'}(u))$ or $vu \in MST(N_2^{RNG'}(v))$. Let $IMRG^+$ be the final structure formed by all edges kept. Similarly, the final structure is called $IMRG^-$ when edge $uv$ is kept if and only if $uv \in MST(N_2^{RNG'}(u))$ and $uv \in MST(N_2^{RNG'}(v))$. Here IMRG is the abbreviation of *Incident MST and RNG Graph*.

Notice that in the algorithm, node $u$ constructs the local minimum spanning tree $MST(N_2^{RNG'}(u))$ based on the induced UDG of the point sets $N_2^{RNG'}(u)$. As seen later (Lemma 8), the constructed structures are subgraphs of the modified RNG graph. Thus, these structures are planar and have at most $3n$ edges. In addition, the total communication cost of Algorithm 3 is at most $13n$ when either structure $IMRG^-$ or $IMRG^+$ is needed; the total communication cost is at most $7n$ if the directed structure IMRG is needed. (Step 1 takes $n$ messages; Step 2 takes $6n$ messages since each edge is broadcasted by at most its 2 end-points and the total number of edges is at most $3n$; similarly Step 4 takes $6n$ messages.)

*Lemma 8:* Structure IMRG is a subgraph of modified RNG.

Proof. Consider any edge $uv \notin RNG'$. We show that node $u$ will not propose $uv$. From the definition of RNG', we know that there is a node $w$ inside the lune defined by segment $uv$ and edge $uw$ and $wv$ has a label less than $uv$. Considering the process of constructing $MST(N_2^{RNG'}(u))$, when we decide whether to add edge $uv$ after processing edges with smaller label, there is already a path connecting $u$ and $w$, and a path connecting $w$ and $v$. Thus, edge $uv$ cannot be added by node $u$ to $MST(N_2^{RNG'}(u))$. This finishes the proof. ∎

The above lemma immediately implies that all structures $IMRG^+$ and $IMRG^-$ are planar graph, and have bounded node degree at most 6.

We then show that structures $IMRG^+$ and $IMRG^-$ are still connected and low-weighted. Clearly, the constructed structures are a supergraph of the previous structures, i.e., $LMST_2^+ \subseteq IMRG^+$ and $LMST_2^- \subseteq IMRG^-$, since Algorithm 3 uses less information than Algorithm 2 in constructing the local minimum spanning tree. If an edge $uv$ is removed from $MST(N_2^{RNG'}(u))$, it means that there is a path connecting $u$ and $v$ using shorter edges when we process $uv$. By simple induction, we can show that there is also a path connecting $u$ and $v$ when we process $uv$ in constructing $MST(N_2(u))$. Thus, these two structures $IMRG^+$ and $IMRG^-$ are still connected.

We then prove the following lemma.

*Lemma 9:* Structures $IMRG^-$ and $IMRG^+$ are still low-weighted.

Proof. We only need to prove that $IMRG^+$ is still low-weighted since $IMRG^- \subseteq IMRG^+$. Consider any quadrilateral $uvyx$ formed by two edges $uv \in IMRG^+$ and $xy \in IMRG^+$. By the construction algorithm, we know that both edges $uv$ and $xy$ are in RNG'. W.l.o.g, assume that $uv$ is the longest edge of the quadrilateral, then $\|ux\| \leq 1$, $\|yv\| \leq 1$. Thus, the four edges of quadrilateral $uvyx$ are in the UDG induced on $N_2^{RNG'}(u)$: node $u$ will know the existence of edge $xy \in RNG'$ through node $x$, node $v$ will know the existence of edge $xy \in RNG'$ through node $y$. Consequently, edge $uv$ will be removed when constructing the local minimum spanning tree $MST(N_2(u))$. Together with Lemma 6, we know that $IMRG^+$ is low-weighted. Structure $IMRG^-$ is low-weighted directly from $IMRG^- \subseteq IMRG^+$. □

*Theorem 10:* Algorithm 3 constructs structures $IMRG^-$ or $IMRG^+$ using at most $13n$ messages. The structures $IMRG^-$ or $IMRG^+$ are connected, planar, bounded degree, and low-weighted. Both $IMRG^-$ and $IMRG^+$ have node degree at most 6.

We show that the constructed structure $IMRG^-$ is always a subgraph of the structure $G_0^-$ constructed in [12].

*Lemma 11:* The constructed structure $IMRG^-$ is always a subgraph of the structure $G_0^-$ constructed in [12].

Proof. Consider any edge $uv$ from UDG that does not belong to $G_0^-$. Remember that $G_0^-$ contains an edge $xy$ if and only if the edge $xy$ belongs to the local minimum spanning tree $MST(N_1(u))$ and $MST(N_2(u))$. Without loss of generality, assume that edge $uv$ is removed because it is not in the local minimum spanning tree $MST(N_1(u))$. Thus, there is a path connecting $u$ and $v$ in the induced unit disk graph on $N_1(u)$, whose edges have length less than $\|uv\|$ (Ties are broken by IDs). Clearly, this path is still in the induced unit disk graph on $N_2^{RNG'}(u)$ since $N_1(u) \subset N_2^{RNG'}(u)$. Consequently, edge $uv$ cannot appear in the Euclidean minimum spanning tree $MST(N_2^{RNG'}(u))$. It further implies that $uv$ is not in $IMRG^-$. □

Similarly, the constructed structure $IMRG^+$ is always a subgraph of the structure $G_0^+$ constructed in [12].

In summary, we have the following relations among these structures:

$$MST \subseteq LMST_2 \subseteq IMRG \subseteq G_0 \subseteq RNG' \subseteq RNG$$
$$MST \subseteq LMST_2 \subseteq IMRG \subseteq LRNG \subseteq RNG' \subseteq RNG$$

## C. Bound the Longest Edge Length

Notice that the **min-max assignment** problem is basically to find a connected structure whose longest edge is minimum. It was proved in [3] that the longest edge of the Euclidean minimum spanning tree is always the critical link for **min-max assignment**. However, it is communication expensive to construct MST in a distributed manner. Thus, it is natural to ask whether we can construct a structure in a localized manner such that the longest edge of this structure is within a constant factor of that of MST.

We show by example that there is unfortunately *no* such deterministic localized algorithm. Assume that there is such a deterministic localized algorithm A that uses $k$-hop information. Figure 2 illustrates such an example that algorithm A cannot approximate the longest edge of the MST within a constant factor. In the example, the distance between nodes $u$ and $x$ is more than $k$ hops. Then algorithm A will have the same information at node $u$ for both configurations (a) and (b). If A decides to keep edge $uv$, then the the longest edge kept by A could be arbitrarily larger than that of MST for configuration (a). If A decides not to keep edge $uv$, then the structure constructed by A is not connected for configuration (b).

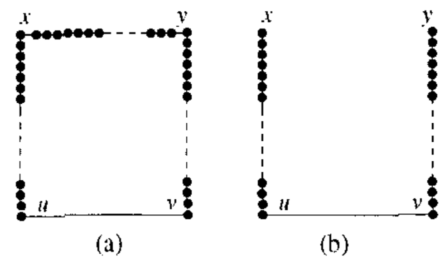

Fig. 2. No localized algorithm approximates the longest edge.

Thus, we have the following theorem.

*Theorem 12:* It is *impossible* to have a deterministic localized algorithm to construct a connected structure such that the maximum node power based on this structure is within a constant factor of that based on MST.

## IV. EXPERIMENTS

We conducted extensive simulations to study the performance of our structure in terms of the longest edge length and the total edge length. Although network throughput is an important performance metric, it is influenced by many other factors such as the MAC protocol, routing protocol and so on. Therefore, most related work does not test the throughput performance. As almost all previously related work did, we will use the following metrics to compare the performance:

1) **Total Messages**: In wireless networks, less messages to construct a topology will saves energy consumption. We already showed that the total messages of IMRG is at most $13n$.
2) **Max Messages**: We also test what is the maximum number of messages a node will send in building this

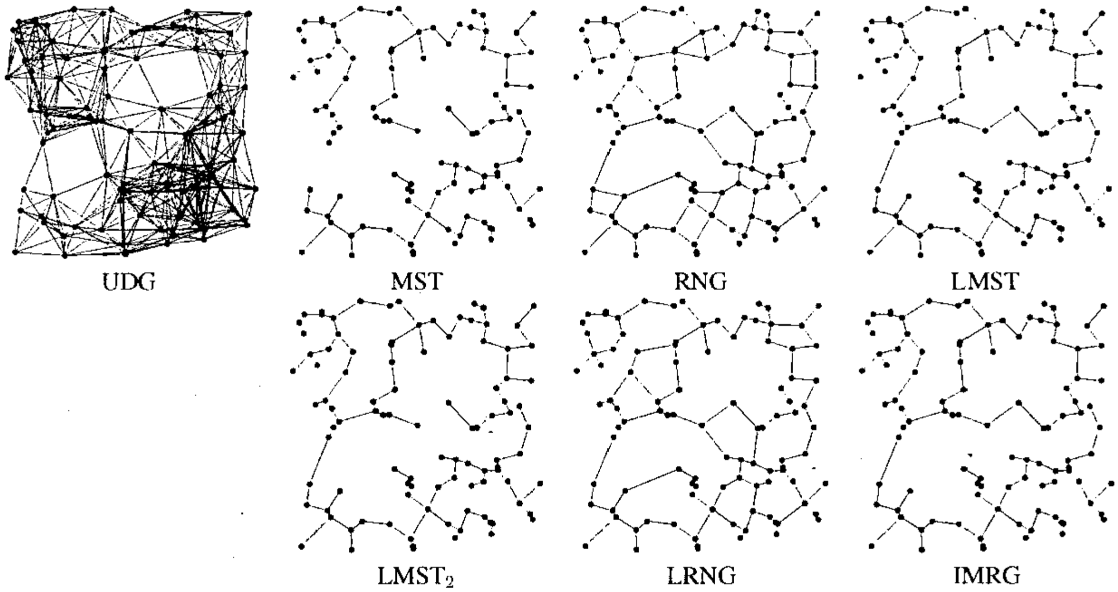


Fig. 3. Different structures from a UDG.

structure. A large number of messages at some node will delay the topology updating and drain out its battery power quickly.

3) **Average Node Degree**: A smaller average node degree often implies less contention and interference for signal and thus a better frequency spatial reuse, which in turn will improve the throughput of the network.
4) **Max Node Degree**: We also test the maximum node degree. A larger node degree at some node will cause more contention and interference for signal, and also may drain out its battery power quickly. Here, in all our simulations, we set the constant $\beta = 2$, so that the power needed to support a link $uv$ is $\|uv\|^2$.
5) **Max Node Power**: Notice that each user $u$ will set its transmission range equal to the length of the longest edge incident on $u$, called *node power*. Thus, a smaller node power will always save the power consumption. The max-node-power captures the maximum power used by all nodes. It is known that the maximum node power based on MST is the optimum to guarantee the network connectivity. We would like to compare the maximum node power induced from our structure $IMRG^-$ compared with that based on MST.
6) **Total Node Power**: The total node power approximates the total power used by all nodes to keep the connectivity.
7) **Total Node Power for Broadcasting**: The total node power approximates the total power used by doing broadcasting. The difference with total node power is not considering the powers of leaves.
8) **Total Edge Length**: We proved that all structures proposed have the total edge length within a constant factor of MST, while no previously known structures having this property.
9) **Total Link Power**: It was also proved in [9] that a broadcasting based on MST consumes energy within a constant factor of the optimum. We thus compare the total link power used by our structure with previously known structures.

In the simulations, since we already showed that structure $IMRG^-$ is a subgraph of $IMRG^+$ and $LMST_2^-$ is a subgraph of $LMST_2^+$, we will only test the performances of structure $IMRG^-$ and $LMST_2^-$, compare them with previously known structure $LRNG$ in [13], [14], $G_0^-$ in [12], RNG in terms of the above metrics. The reason for only selecting $G_0^-$ and RNG is that in [12], their simulations already show that $G_0^-$ out-performs other previously known structures in terms of the node degree, max node power, and the total node power. Hereafter, we use LMST, $\text{LMST}_2$ and IMRG instead of $G_0^-$, $LMST_2^-$ and $IMRG^-$ in the experiments, if it is clear from the context.

In the first simulation, we randomly generate 100 nodes uniformly in a $1000m \times 1000m$ region. The maximum transmission range of each node is set as $250m$ for all the nodes. The topology (i.e., UDG) derived using the maximum transmission power, MST, RNG, LMST (i.e., $G_0^-$), $\text{LMST}_2$ (i.e., $LMST_2^-$), LRNG, and IMRG (i.e., $IMRG^-$) are shown in Figure 3 respectively. To make the performance testing precise, we generate 100 samples of 100-node sets and compute the performance metrics accordingly. The corresponding performances are illustrated in the following Table IV. Here for max node degree, max message and max node power, we show both the maximum and average values over the 100 sets.

As we proved, our structures $\text{LMST}_2$ and IMRG outperform the structure LMST in all aspects except the number of messages used. The maximum node power used to guar-

TABLE I

THE PERFORMANCES COMPARISON OF SEVERAL STRUCTURES. NUMBER OF MESSAGES WITH * DOES NOT COUNT MESSAGES FOR COLLECTING 2-HOP NEIGHBORS WHEN BUILDING $LMST_2$.

| | MST | RNG | LMST | $LMST_2$ | LRNG | IMRG |
|---|---|---|---|---|---|---|
| MaxMaxMsg | - | 1.00 | 5.00 | 5.00* | 5.00 | 9.00 |
| AvgMaxMsg | - | 1.00 | 4.50 | 4.50* | 4.92 | 8.42 |
| TotMsg | - | 100.00 | 305.72 | 299.88* | 334.76 | 538.68 |
| MaxMaxDeg | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 | 4.00 |
| AvgMaxDeg | 3.50 | 3.92 | 3.50 | 3.50 | 3.92 | 3.50 |
| AvgDeg | 1.98 | 2.35 | 2.06 | 2.00 | 2.30 | 2.04 |
| MaxMaxNPow | 4.13 | 5.40 | 4.69 | 4.13 | 5.40 | 4.69 |
| AvgMaxNPow | 2.93 | 4.17 | 3.77 | 3.03 | 4.17 | 3.55 |
| TotNPow | 79.85 | 122.80 | 92.79 | 82.56 | 119.69 | 90.10 |
| TotNPowBrdcst | 66.48 | 118.21 | 83.26 | 70.08 | 114.74 | 79.43 |
| TotLength | 132.79 | 183.59 | 144.86 | 135.55 | 175.52 | 141.99 |
| TotLPow | 112.47 | 187.37 | 131.85 | 116.56 | 177.29 | 127.13 |

antee the network connectivity by structure LMST is higher than those by our structure $LMST_2$ and IMRG. The total node power used to guarantee the network connectivity by LMST is also much higher than that by $LMST_2$ and IMRG in average. Among the structures $LMST_2$ and IMRG, we prefer IMRG in practice though its power consumption is slightly higher. The reason is that the construction of $LMST_2$ has large communication costs (it is still $O(n)$ but the hidden constant here is large). Notice in the experiments, we do not count the number of messages used to collect the information of 2-hop neighbors when building $LMST_2$. Notice, if we simply ask each node to broadcast its one-hop neighbors to collect the two-hop neighbors, it will cost $\sum d_i$ messages, where $d_i$ is the number of one-hop neighbors of node $v_i$ in the UDG. Clearly, $\sum d_i = 2m$, where $m$ is the number of links in UDG, which could be as large as $n^2$ for dense graphs. The total number of messages used by this simple approach clearly could be much higher than those by IMRG and LMST. On the other hand, even the method given in [38] can collect two-hop neighbors for all nodes with total $O(n)$ messages using geometry information, the hidden theoretical constant could be as large as several hundreds.

We then vary the number of nodes in the region from 50 to 500. The transmission range of each node is still set as $250m$. We plotted the performances of all structures in Figure 4. We observed that our structure has the best performance among all locally constructed structures such as LMST, RNG, and IMRG. For example, the broadcasting based on RNG consumes almost twice the energy than that based on structure IMRG. More importantly, the broadcasting based on structure IMRG is almost as good as that based on MST. Remember that it is proven in [9] that the broadcasting based on MST consumes energy no more than 12 times of the optimum.

Finally, we fix the number of nodes in the region as 500 and grow the transmission range of each node from $100m$ to $300m$. We plotted the performances of all structures in Figure 5. We found that our structures still out-perform the previously best known structures significantly.

All the results show that IMRG has better performance than LMST and RNG. In other words, IMRG has less length cost and power cost for broadcasting; it has smaller node power to keep the connectivity. The messages used for construction of IMRG are slightly more than the one of LMST. The simulation results confirm all of our theoretical analysis. Remember that IMRG maybe spend $O(n^{\beta-1})$ times of power of the optimum for broadcasting. However, our simulations show that the energy consumption of broadcasting based on IMRG is within a small constant factor (about 10% more) of that based on the MST and is much better than the energy consumed based on RNG, or LMST. In summary, the IMRG is the best among all these known local structures; additionally, it can approximate MST theoretically and be used for energy efficient broadcasting.

## V. CONCLUSION

We consider a wireless network composed of a set of $n$ wireless nodes distributed in a two dimensional plane. We presented the first localized method to construct a bounded degree planar connected structure $IMRG^-$ whose total edge length is within a constant factor of that of the minimum spanning tree. The total communication cost of our method is at most $13n$, and every node only uses its partial two-hop information to construct such structure. Notice that some two-hop information is necessary to construct any low-weighted structure [13], [14]. We conducted extensive simulations to study the performance of our structures compared with previously known structures and it out-performs all previously known structures (with small message overhead).

The constructed structure is planar, and has bounded degree, low-weight. Li and Wang [40], [14] recently gave an $O(n \log n)$-time centralized algorithm constructing a bounded degree, planar, and low-weighted spanner. However, we do not have a distributed algorithm using $O(n)$ communications without sacrificing the spanner property. On the other hand, we [8] showed how to construct a planar spanner with bounded degree in a localized manner (using $O(n)$ messages) for unit disk graph. However, the constructed structure does not seem to have low-weight. It remains open how to construct a bounded degree, planar, and *low-weighted spanner* in a distributed manner using only $O(n)$ communications under the local broadcasting communication model.

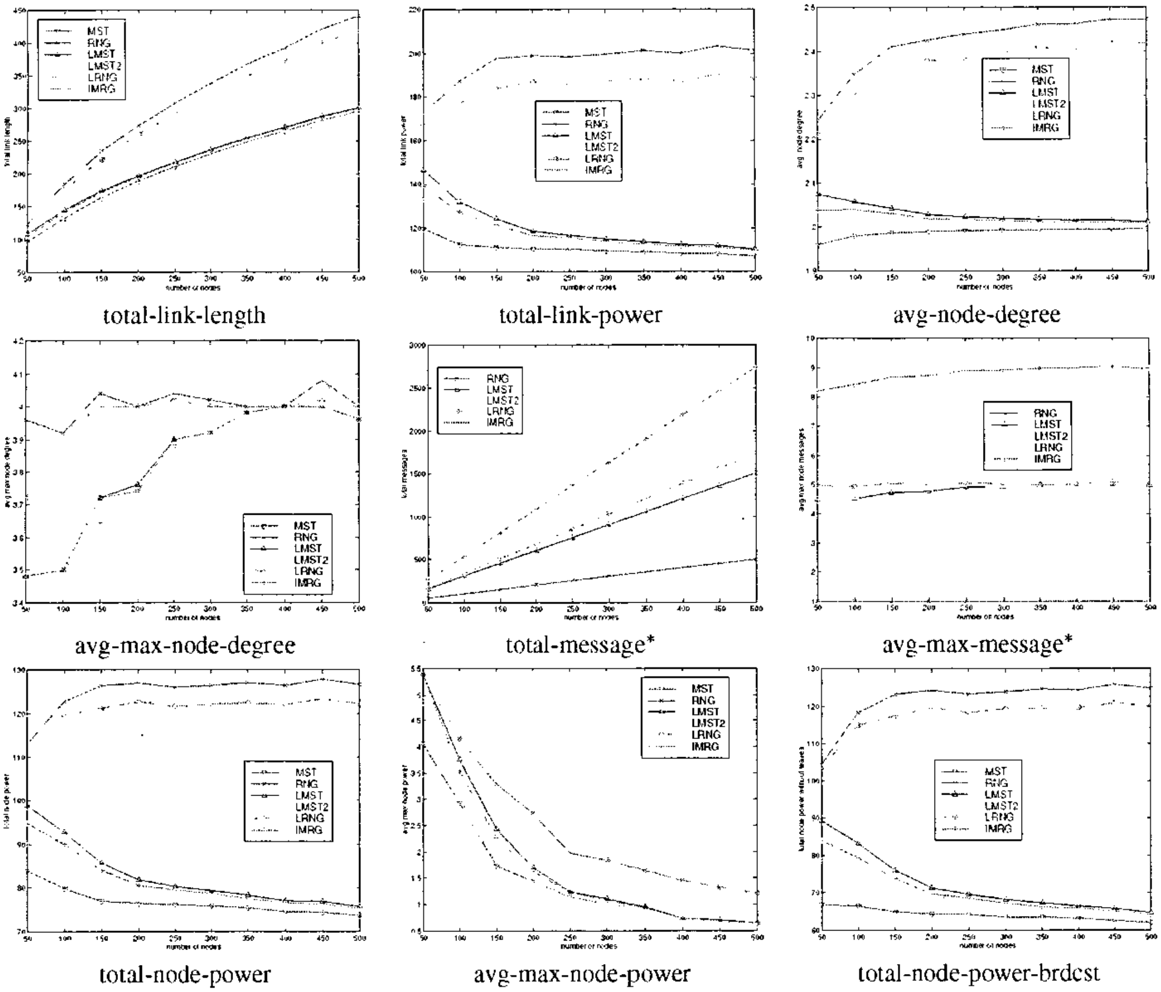


Fig. 4. Results when the number of nodes in the networks are different (from 50 to 500). Here the transmission range is set as $250m$. Number of messages with * does not count messages for collecting 2-hop neighbors when building $\text{LMST}_2$.

For topology control of the wireless network, there are two objectives: either minimize the maximum node power or minimize the total node power needed to guarantee the network connectivity. We showed that it is impossible to have a deterministic localized algorithm to construct a connected structure such that the maximum node power based on this structure is within a constant factor of optimum. Our structure $IMRG$ has total edge length within a small constant factor of that MST. However, its total link power (or node power) could still be $O(n^{\beta-1})$ times of the optimum to guarantee the network connectivity. We leave it as future research whether there is a deterministic localized algorithm to construct a connected structure whose total link power (or node power) is within a constant factor of that of MST.

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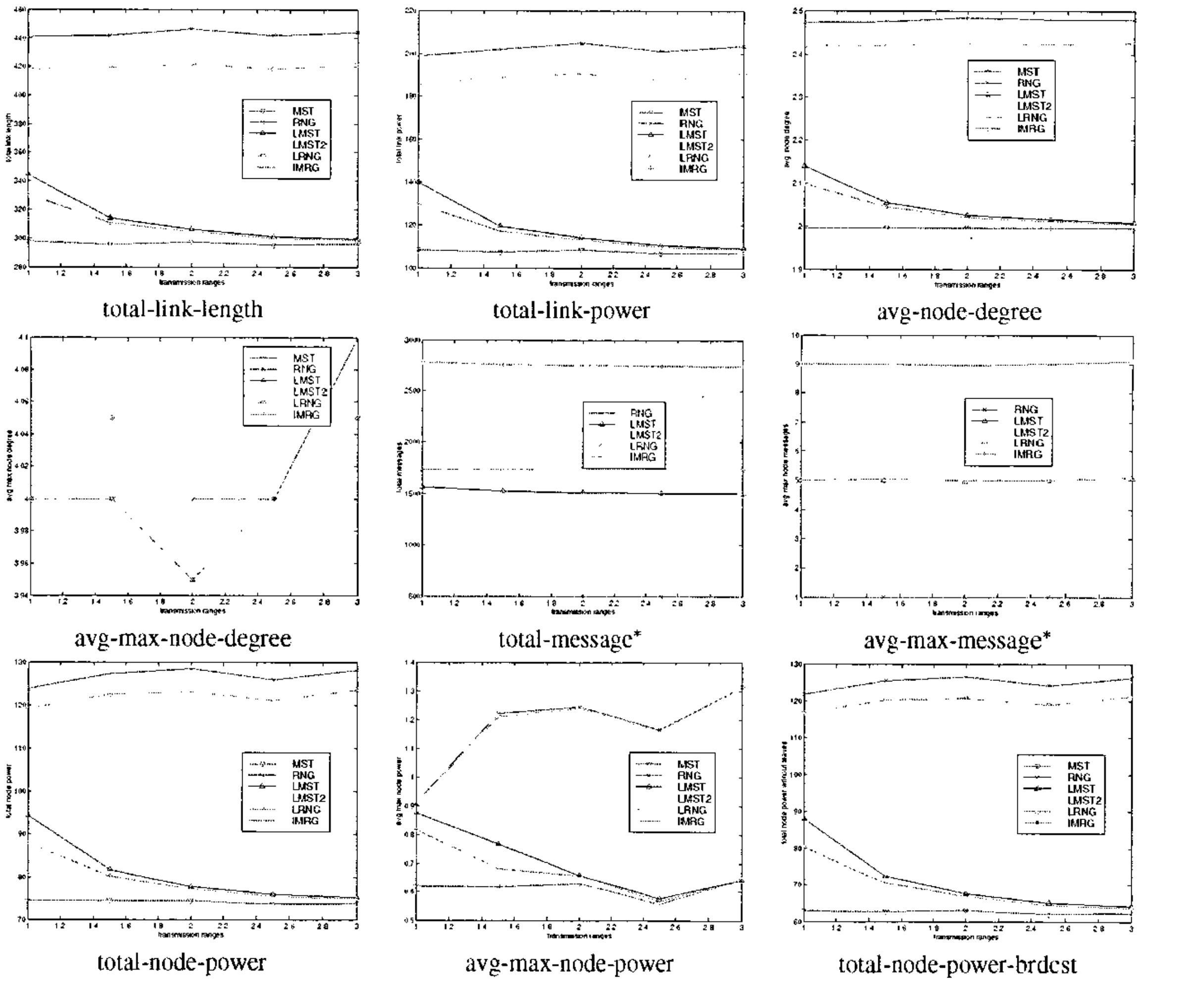


Fig. 5. Results when the transmission range are different (from $100m$ to $300m$). Here the number of nodes is 500. Number of messages with * does not count messages for collecting 2-hop neighbors when building $LMST_2$.

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