Maximizing Network Capacity of MPR-Capable Wireless Networks

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***Abstract*—Multi-packet reception (MPR) technology provides a means of boosting wireless network capacity without requiring additional spectrum. It has received widespread attention over the past two decades from both industry and academic researchers. Despite the huge promise and considerable attention, provable good algorithms for maximizing network capacity in MPR-capable wireless networks are missing in the state of the art. One major technical obstacle is due to the complicated non-binary nature of the link independence; something which appears intractable with existing graph-theoretic methods. In this paper, we present practical polynomial-time approximation algorithms for variants of capacity optimization problems in MPR-capable wireless networks which achieve constant approximation bounds for the first time ever. In addition, polynomial-time approximation schemes are developed for those variants in wireless networks with constant-bounded MPR capabilities.**

I. Introduction

Interference is widely regarded as the fundamental impediment to achieving high network capacity in wireless networks. Traditionally, various multiple access schemes such as TDMA, FDMA, CDMA, OFDMA, and CSMA/CA have been employed to avoid interference among concurrent wireless transmissions. Over the past two decades, there has been rapid technological progress on advanced signal processing for enabling simultaneous decoding of more than one packet at a receiver, a capability known as multi-packet reception (MPR). The MPR technology provides a means of meeting ever-increasing consumer demands for higher speed data transmission without requiring additional spectrum. It has received widespread attention over the past decade from both industry and academic researchers. Several MPR-aware MAC protocols have been developed in [1], [22], [34], [35], [37]. The asymptotics of network throughput in wireless networks over a uniform or Poisson point process were studied in [2], [5], [6], [9], [10], [22], [32], [33]. The queuing-theoretic stable throughput of some simple MPR-capable wireless networks have been investigated in [4], [7], [23], [25]. There are a few of works [3], [11], [15], [20], [21], [31] on the algorithmic aspects of maximizing network capacity in MPR-capable wireless networks. However, only purely heuristic algorithms have been developed in these works, and none of them has any provable performance guarantees.

In a striking contrast, maximizing network capacity in multihop wireless networks without MPR capability under the protocol interference model has been much better studied by [12], [16], [17], [18], [26], [27], [28], [29], [30]. On one hand, both the MAC layer optimization problems such as maximum-weighted independent set and the shortest link schedule and the cross-layer optimization problems such as maximum weighted multiflow and maximum concurrent multiflow are NP-hard [27]. On the other hand, all of them admit a *polynomial-time approximation scheme* (PTAS) even in the more general multi-channel multi-radio setting as long as the number of channels is bounded by a constant [28]. In other words, for any fixed $\varepsilon > 0$, there is a polynomial-time (depending on ε) $(1+\varepsilon)$-approximation algorithm. In addition, a number of faster polynomial-time constant-approximation algorithms have been developed in various setting [29], [30].

The lack of provably good approximation algorithms for maximizing networking capacity in MPR-capable wireless networks in the state of the art stems from the non-binary nature of the link independence in MPR-capable wireless networks: A group of links which can pair-wise transmit successfully at the same time may not transmit successfully at the same time as a group due to the MPR constraint. The non-binary nature of the link independence in MPR-capable wireless networks renders traditional graph-theoretic techniques *inapplicable* and is a major technical impediment to fully exploit the potential benefit of MPR-capability. In order to achieve approximation bounds which do not depend on the MPR-capability, new techniques for both algorithm design and analyses are needed. In order to achieve algorithmic results with approximation guarantees comparable to those developed in wireless networks without MPR-capability, a deeper understanding of the power of MPR and new techniques for both algorithm design and analyses are needed.

In this paper, we conduct a comprehensive algorithmic study of four capacity optimization problems in MPR-capable wireless networks. Among them, **Maximum Weighted Independent Set (MWIS)** and **Shortest Link Schedule (SLS)** are two basic MAC-layer problems; **Maximum Weighted Multiflow** (**MWMF**) and **Maximum Concurrent Multiflow** (**MCMF**) are two important cross-layer problems. For all the four problems in wireless networks with arbitrary and possibly heterogenous MPR capabilities, we have developed

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practical approximation algorithms which achieve constant approximation bounds for the first time ever. For all the four problems in wireless networks with constant-bounded MPR capabilities, we have developed PTAS's which not only fully characterize their approximation hardness but also provide a performance benchmark to other algorithmic solutions. These discoveries successfully fill the present significant knowledge gap between maximizing network capacity in MPR-capable wireless networks and maximizing network capacity in wireless networks without MPR capability.

The remainder of this paper is organized as follows. In Section II, we introduce the network model and give a precise problem description. In Section III, we present polynomial-time exact algorithms for **MWIS**, **SLS**, **MWMF**, and **MCMF** in MPR-capable wireless networks in which all links have mutual conflict. By exploiting these exact algorithms as the basic building blocks, in Section IV we develop polynomial-time constant-approximation algorithms for **MWIS**, **SLS**, **MWMF**, and **MCMF** in general MPR-capable wireless networks. Subsequently in Section V we devise PTAS's for **MWIS**, **SLS**, **MWMF**, and **MCMF** in wireless networks with constant MPR-capabilities. Finally, we conclude this paper in Section VI.

II. Network Model And Problem Descriptions

In this section, we introduce the network model which has been assumed in the majority of the literature on MPR-capable wireless networks, and variants of the capacity optimization problems in MPR-capable networks. Consider an instance of MPR-capable wireless network on a finite planar set V of networking nodes. Under the protocol interference interference model, all nodes have an interference radius r and a communication radius $r' < r$. A pair of nodes can communicate with each other if their distance is no more than the communication radius r'. Let A denote the set of (directed) communication links. For each node $u \in V$ and any subset B of links, we use $\delta_B^{in}(u)$ (respectively, $\delta_B^{out}(u)$) to denote the set of links with u as a receiver (respectively, sender). If B is A, the subscript B is dropped. Suppose that each link $a \in A$ has transmission data rate $c(a)$. A link a is interfered by another link b if the distance between the receiver of a and the sender of b is at most the interference radius r. Two links are said to have conflict to each other if at least one of them is interfered by the other. Suppose that each node v has MPR-capability $\tau(v)$ and operates in the half-duplex mode, i.e. it cannot transmit and receive at the same time. A set I of links in A may transmit successfully at the same time with the MPR-capability if for each node v which is a receiver of some link in I the following two constraints are satisfied:

1) **MPR Constraint**: v is the receiver of at most $\tau(v)$ links in I. In other words, $\left|\delta_I^{in}(v)\right| \leq \tau(v)$.
2) **Interference-free Constraint**: v is outside the interference range of any link in $I \setminus \delta_I^{in}(v)$.

We remark that the **Interference-free Constraint** implies implicitly that no node can transmit and receive at the same time (i.e., the half-duplex constraint is satisfied) and no node can transmit to two different nodes at the same time. A set I of links is said to be *independent* if all links in I may transmit successfully at the same time. Let $\mathcal{I}$ denote the collection of all independent subsets of A.

Suppose that each link $a \in A$ has a non-negative weight $w(a)$. The weight of any subset B of A is defined to be

$$w(B) = \sum_{a \in B} w(a).$$

The problem **Maximum Weighted Independent Set** (**MWIS**) seeks an independent set $I \in \mathcal{I}$ with maximum weight. Suppose that $d \in \mathbb{R}_+^A$ is a link-demand function. A *link schedule* of d is a set

$$\Pi = \left\{(I_j, x_j) \in \mathcal{I} \times \mathbb{R}^+ : 1 \leq j \leq q\right\}$$

satisfying that for each link $a \in A$,

$$d(a) = c(a) \sum_{j=1}^{q} x_j \left|I_j \cap \{a\}\right|;$$

the two values q and $\sum_{j=1}^{q} x_j$ are referred to as the *size* and *length* (or *latency*) of Π respectively, and are denoted by $|\Pi|$ and $\|\Pi\|$ respectively. The minimum length of all link schedules of d is denoted by $\chi^*(d)$. Given a $d \in \mathbb{R}_+^A$, the problem of finding a shortest link schedule of d is referred to as **Shortest Link Schedule** (**SLS**). The *capacity region* of the network consists of all $d \in \mathbb{R}_+^A$ whose shortest link schedule has length at most one.

Consider k unicast requests in this wireless network specified by source-destination pairs. For each $1 \leq j \leq k$, $\mathcal{F}_j$ denotes the set of flows of the request j, and the value of a flow $f_j \in \mathcal{F}_j$ is denoted by $val(f_j)$. A multiflow is a sequence of flows $f = \langle f_1, f_2, \cdots, f_k \rangle$ with $f_j \in \mathcal{F}_j$ for each $1 \leq j \leq k$. Among all variants of the multiflow problems, the following two are the most basic ones.

- **Maximum Weighted Multiflow** (**MWMF**): Given that each request j has a weight w_j per unit of flow, the problem **MWMF** seeks a multiflow $f = (f_1, \cdots, f_k)$ and a MAC-layer link schedule $\mathcal{S}$ of $\sum_{j=1}^{k} f_j$ such that the length of $\mathcal{S}$ is at most one and the total weight of f defined by
$$\sum_{j=1}^{k} w_j \cdot val(f_j)$$
is maximized.
- **Maximum Concurrent Multiflow** (**MCMF**): Given that each request j has a traffic demand d_j, the problem **MCMF** seeks a multiflow $f = (f_1, \cdots, f_k)$ and a MAC-layer link schedule $\mathcal{S}$ of $\sum_{j=1}^{k} f_j$ such that the length of $\mathcal{S}$ is at most one and the concurrency of f defined by
$$\min_{1 \leq j \leq k} \frac{val(f_j)}{d_j}$$
is maximized.

Both **MCMF** and **MCMF** are cross-layer in nature.

A wireless network without MPR capability can be treated as a wireless network with MPR capability $\tau(v) = 1$ for all nodes v. In wireless networks without MPR capability, all the four problems **MWIS**, **SLS**, **MWMF**, and **MCMF** are NP-hard [27]. Consequently, they have at least the same computational intractability in wireless networks with MPR capability, as in wireless networks without MPR capability. On the other hand, the traditional graph-theoretic techniques used by the approximation algorithms for them in wireless networks without MPR capability are not applicable to MPR-capable wireless networks due to the non-binary nature of the link independence in wireless MPR-capable networks. In order to achieve approximation bounds for them which do not depend on the MPR-capability, new techniques for both algorithm design and analyses are needed.

III. Exact Algorithms in Fully Conflicted Networks

In this section, we develop polynomial-time exact algorithms for **MWIS**, **SLS**, **MWMF**, and **MCMF** in MPR-capable wireless network in which all links have mutual conflict. The algorithms developed in this section will be utilized in the next section.

A crucial implication of mutual link conflicts is that any independent set is a subset of $\delta^{in}(v)$ for some $v \in V$. Thus, a maximum weighted independent set I is achieved by a set of $\min\left\{\left|\delta^{in}(v)\right|, \tau(v)\right\}$ heaviest links in $\delta^{in}(v)$ for some $v \in V$. Such observation yields a simple algorithm **ExactIS** outlined in Table I for **MWIS**. The correctness of the algorithm is obvious.

Algorithm **ExactIS**

$I \leftarrow \emptyset$;
for each $v \in V$ do
    $I' \leftarrow$ the set of $\min\left\{\left|\delta_B^{in}(v)\right|, \tau(v)\right\}$
        heaviest links in $\delta_B^{in}(v)$;
    if $w(I') > w(I)$ then $I \leftarrow I'$;
return I.

TABLE I. Outline of the algorithm **ExactIS** for MWIS.

Now, we present a polynomial-time exact algorithm **ExactLS** for **SLS**. Let B be the set of links with positive demands. Since any two links with different receivers cannot transmit at the same time due to mutual link conflicts, the minimum schedule length of the demands by B is the summation of the minimum schedule lengths of the demands by $\delta_B^{in}(v)$ for all $v \in V$. Consequently, the concatenation of the shortest link schedules of the demands by $\delta_B^{in}(v)$ for all $v \in V$ yields a shortest link schedule of the demands by B. Subsequently, we develop a simple algorithm **WrapLS** which produces a shortest link schedule of the demands by $\delta_B^{in}(v)$ for each $v \in V$. Such algorithm will be utilized by the algorithm **ExactLS** to produce a shortest link schedule of the demands by B.

Consider a node v with non-empty $\delta_B^{in}(v)$. Since each link a requires a transmission time $\frac{d(a)}{c(a)}$, the minimum schedule length of the demands by $\delta_B^{in}(v)$ is at least

$$\max_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}.$$

Furthermore, since at most $\tau(v)$ links from $\delta_B^{in}(v)$ can transmit at any time, the minimum schedule length of the demands by $\delta_B^{in}(v)$ is at least

$$\frac{\sum_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)}.$$

Thus, the minimum schedule length of the demands by $\delta_B^{in}(v)$ is at least

$$\chi = \max\left\{\max_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)}\right\}.$$

The algorithm **WrapLS** will be able to produce a link schedule Γ of the demands by $\delta_B^{in}(v)$ of length exactly χ and hence is optimal. The output schedule Γ specifies the transmission period $\Gamma(a)$ for each link $a \in \delta_B^{in}(v)$. The basic idea of the algorithm **WrapLS** can be better interpreted as a wrap-around scheme illustrated in Figure 1. We represent each link by a line segment of length $\frac{d(a)}{c(a)}$. By concatenating these line segments in the decreasing order of length, we get a big horizontal line segment of length

$$\sum_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}.$$

Then, we cut the big line segment at the points which are are at a distance of integer multiples of χ from the left end (see Figure 1(a)). Such cuttings split the big line segment into

$$\begin{aligned} &\left\lceil \frac{\sum_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}}{\chi} \right\rceil \\ &\leq \left\lceil \frac{\sum_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}}{\frac{\sum_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)}} \right\rceil \\ &= \tau(v) \end{aligned}$$

pieces. Note that the (original) segment of a link a may be split into two sub-segments, and if such split occurs we must have

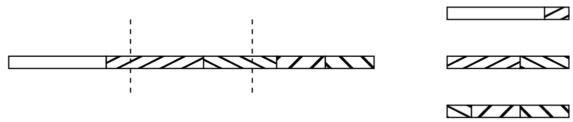
$$\frac{d(a)}{c(a)} < \chi.$$

After the splitting, we align these pieces horizontally by their left ends at the origin of the x-axis (see Figure 1(b)). Such alignment naturally induces a transmission schedule of each link a as follows.

- **Case 1**: The line segment of a is not split. Then $\Gamma(a)$ consists of the single interval of its line segment in the final alignment.
- **Case 2**: The line segment of a is split into two sub-segments. Then $\Gamma(a)$ consists of the two intervals

of its two sub-segments in the final alignment. Since $\frac{d(a)}{c(a)} < \chi$, the two intervals in $\Gamma(a)$ are disjoint.

Such link schedule Γ is valid since at any moment each link transmits at most once and at most $\tau(v)$ transmissions occur. Furthermore, it is optimal as its length is χ.



(a) (b)

Fig. 1. The illustration of the wrap-around scheme with five segments of length $4, 4, 3, 2, 2$ respectively and $\chi = 5$.

An efficient implementation of the algorithm **WrapLS** is outlined in Table II. The algorithm **ExactLS** first applies the algorithm **WrapLS** to compute a shortest link schedule of the demands by $\delta_B^{in}(v)$ for each $v \in V$, and then concatenates them into a shortest link schedule of the demands by B. The length of the output schedule is

$$\sum_{v \in V} \max\left\{ \max_{a \in \delta_B^{in}(v)} d(a), \frac{\sum_{a \in \delta_B^{in}(v)} d(a)}{\tau(v)} \right\}$$
$$= \sum_{v \in V} \max\left\{ \max_{a \in \delta^{in}(v)} d(a), \frac{\sum_{a \in \delta^{in}(v)} d(a)}{\tau(v)} \right\}.$$

Algorithm **WrapLS**

$\chi \leftarrow \max\left\{ \max_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a \in \delta_B^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\}$;
$s \leftarrow 0$;
for each $a \in B$ in the decreasing order of $\frac{d(a)}{c(a)}$ do
  if $s + \frac{d(a)}{c(a)} \le \chi$ then
    $\Gamma(a) \leftarrow [s, s + \frac{d(a)}{c(a)})$;
    $s \leftarrow s + \frac{d(a)}{c(a)}$;
  else
    $\Gamma(a) \leftarrow [s, \chi) \cup [0, s + \frac{d(a)}{c(a)} - \chi)$;
    $s \leftarrow s + \frac{d(a)}{c(a)} - \chi$;
  if $s = \chi$ then $s \leftarrow 0$;
return Γ.

TABLE II. OUTLINE OF THE ALGORITHM **WrapLS** FOR SLS.

Finally, we present polynomial-time exact algorithms for **MWMF** and **MCMF**. The optimality of the algorithm **ExactLS** for **SLS** implies that the capacity region Ω of the network consists of exactly all link demands $d \in \mathbb{R}_+^A$ satisfying that

$$\sum_{v \in V} \max\left\{ \max_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a \in \delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\} \le 1.$$

By a straightforward linearization with auxiliary variables, Ω can be represented as a system of a linear number of linear inequalities. For the problem **MWMF**, we first compute a multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ by solving the following linear program (LP)

$$\begin{array}{ll} \max & \sum_{j=1}^{k} w_j \cdot val(f_j) \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \le j \le k; \\ & \sum_{j=1}^{k} f_j \in \Omega. \end{array}$$

Then, we compute a link schedule Γ of $\sum_{j=1}^{k} f_j$. As $\sum_{j=1}^{k} f_j \in \Omega$, the length of Γ is at most one. So, the multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ together with the schedule Γ is an optimal solution to the problem **MWMF**.

Similarly, for the problem **MCMF** we first compute a multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ by solving the following LP

$$\begin{array}{ll} \max & \phi \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \le j \le k; \\ & val(f_j) \ge \phi d_j, \forall 1 \le i \le k; \\ & \sum_{j=1}^{k} f_j \in \Omega. \end{array}$$

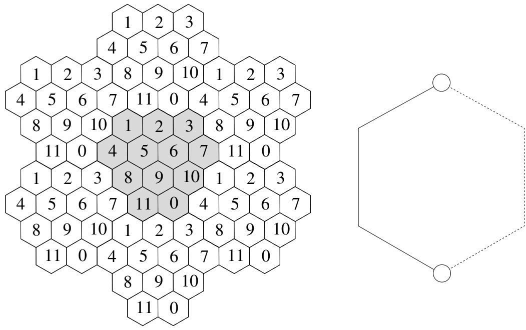
Then, we compute a link schedule of $\sum_{j=1}^{k} f_j$. As $\sum_{j=1}^{k} f_j \in \Omega$, the length of Γ is at most one. So, the multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ together with the schedule Γ is an optimal solution to the problem **MWMF**.

IV. Approximation Algorithms in General Networks

In this section, we present polynomial-time constant-approximation algorithms for **MWIS**, **SLS**, **MWMF**, and **MCMF** in general wireless networks. Our approximation algorithms **DC-IS** and **DC-LS** for **MWIS** and **SLS** respectively both take a divide-and-conquer approach. The division part of both algorithm is described in Subsection IV-A. The conquer part and the combination part of the algorithm **DC-IS** (respectively, **DC-LS**) are described and analyzed in Subsection IV-B (respectively, Subsection IV-C). Our approximation algorithms **R-WMF** and **R-CMF** for **MWMF** and **MCMF** respectively both take a restriction-based approach, and are described and analyzed in Subsection IV-D. By proper scaling, we assume that the maximum link length is one and interference radius r is accordingly greater than one.

A. A Spatial Division

We tile the plane into regular hexagons of diameter $r - 1$ (see Figure 2(a)). Each hexagon, or cell, is considered to be left-closed and right-open, with only the left-most pair of vertices included (see Figure 2(b)). A link is said to be associated with a cell if its receiver lies in this cell. A cell is said to be non-empty if at least one link is associated with this cell. We shall introduce a labelling of the cells satisfying that all cells with the same label are apart from each other at a distance of greater than $r + 1$. Such labelling ensures that any pair of links associated with two cells having the same label have no interference to each other. Figure 2(a) gives one such labelling for $r = 3$.



(a) (b)

Fig. 2. Tiling of the plane into half-open half closed hexagons of diameter $r-1$.

The labelling of the cells can be reduced to the labelling of lattice points. Specifically, we pick an arbitrary non-empty cell and use its center as the origin o. Let e_1 and e_2 be the centers of the straight right cell and the upper right cell respectively. Then, the centers of all the cells are integer combinations of e_1 and e_2. In other words, the centers of all cells form a lattice with e_1 and e_2 as a base. We adopt an oblique coordinate system with e_1 and e_2 as the base vectors, and represent each point $xe_1 + ye_2$ with the oblique coordinates $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$. It's easy to verify that the squared distance from the origin to a point $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ is

$$\frac{3}{4}\left(x^2+xy+y^2\right)(r-1)^2.$$

A number of the format x^2+xy+y^2 with integers x and y is called *rhombic* number. Rhombic numbers can be characterized in an elegant way by their prime decomposition (see, e.g., [24]): A positive integer greater than one is rhombic if and only if, after removing all square factors, its prime decomposition contains no prime other than 3 and primes of the form $6i+1$ with $i \in \mathbb{Z}$. A representation of a rhombic number λ is a point $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ satisfying that

$$x^2+xy+y^2=\lambda.$$

Among all representations of a rhombic number λ, there is at least one lattice point $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ with $x \geq y \geq 0$, which is called a *positive representation* of λ. The rhombic numbers between 7 and 12 are $7, 9, 12$, and their positive representations are

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

respectively.

Now, we describe the labelling of all the centers of the cells, which also gives rise to a labelling of the cells. Let λ be the smallest rhombic number at least

$$\frac{16}{3}\left(\frac{r}{r-1}\right)^2$$

, which decreases with r. It's easy to verify that for the practical applications with $r \geq 3$,

$$7 \leq \lambda \leq 12.$$

We then pick a positive representation $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ of λ from the list in the previous paragraph. Let

$$p_1 = \begin{bmatrix} x \\ y \end{bmatrix}, p_2 = \begin{bmatrix} -y \\ x+y \end{bmatrix}.$$

Let P be the half-open half-closed rhombus extended by p_1 and p_2, i.e.,

$$P = \left\{t_1 p_1 + t_2 p_1 : 0 \leq t_1, t_2 < 1\right\}.$$

It's well-known that P contains λ lattice points. All these lattice points in P receive distinct labels from the set $\{0, 1, \cdots, \lambda-1\}$. Repeat the same assignment in other translates of P by the lattice points with p_1 and p_2 as the base. Thus, λ labels are used.

Note that the distance between any pair of lattice points with the same label is at least

$$\begin{aligned}
&\sqrt{x^2+xy+y^2} \cdot \frac{\sqrt{3}}{2}(r-1) \\
&= \sqrt{\lambda} \cdot \frac{\sqrt{3}}{2}(r-1) \\
&\geq \frac{4}{\sqrt{3}} \frac{r}{r-1} \cdot \frac{\sqrt{3}}{2}(r-1) \\
&\geq 2r.
\end{aligned}$$

Thus, for any pair of points in two cells with the same label, their distance is greater than

$$2r-(r-1)=r+1.$$

For each $0 \leq i < \lambda$, we use $\mathcal{S}_i$ to denote the sets of non-empty cells with label i. For each non-empty cell S, we use V_S to denote the the set of receiving nodes lying in S. Note that the set of links associated with a non-empty cell S is exactly $\bigcup_{v \in V_S} \delta^{in}(v)$, and all links associated with S have mutual conflict.

B. *Divide-And-Conquer Algorithm for* ***MWIS***

With the spatial division described in the previous subsection, the conquer part and combination part of the algorithm **DC-IS** for **MWIS** are described as follows:

- **Conquer**: For each non-empty cell S, we apply the algorithm **ExactIS** to compute a maximum-weighted independent set I_S of all links associated with S.
- **Combination**: We compute a label $0 \leq i < \lambda$ with maximum $\sum_{S \in \mathcal{S}_i} w(I_S)$, and output $\bigcup_{S \in \mathcal{S}_i} I_S$ as the final output I.

Clearly, I is independent. The theorem below provides an approximation bound of I.

Theorem 4.1: The output I is a λ-approximate solution.

Proof: Let O be an optimal solution. For each label i between 0 and $\lambda-1$, let O_i be the subset of the links in O

associated with the non-empty cells with label i. Since I_S is a maximum weighted-independent set of all links associated with S for each non-empty cell S, $\bigcup_{S\in\mathcal{S}_i} I_S$ is a a maximum weighted-independent set of all links associated with the non-empty cells with label i for each $0 \le i < \lambda$. Thus, for $0 \le i < \lambda$ we have

$$w(O_i) \le w\left(\bigcup_{S\in\mathcal{S}_i} I_S\right).$$

By the selection of I, we have

$$\begin{aligned} w(O) &= \sum_{i=0}^{\lambda-1} w(O_i) \\ &\le \sum_{i=0}^{\lambda-1} w\left(\bigcup_{S\in\mathcal{S}_i} I_S\right) \\ &\le \lambda w(I). \end{aligned}$$

So, the theorem holds. ■

C. Divide-And-Conquer Algorithm for **SLS**

With the spatial division described in Subsection IV-A, the conquer part and combination part of the algorithm **DC-LS** for **SLS** are described as follows:

- **Conquer**: For each non-empty cell S, we apply the algorithm **ExactLS** to compute a shortest link schedule Γ_S of the demands by all links associated with S.
- **Combination**: For each $0 \le i < \lambda$, we merge the link schedules Γ_S for all $S \in \mathcal{S}_i$ into a single link schedule Π_i of length $\max_{S\in\mathcal{S}_i} \|\Gamma_S\|$. Then, we concatenate the link schedules Π_i for all $0 \le i < \lambda$ into a single link schedule Π as the final output.

Clearly, Π is a link schedule of all link demands. The theorem below provides an explicit expression of $\|\Pi\|$ and an approximation bound of Π.

Theorem 4.2: The output Π has length

$$\sum_{i=0}^{\lambda-1} \max_{S\in\mathcal{S}_i} \sum_{v\in V_S} \max\left\{ \max_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\}$$

and is a λ-approximate solution.

Proof: By the property of the algorithm **ExactLS**, for each non-empty cell S we have

$$\|\Gamma_S\| = \sum_{v\in V_S} \max\left\{ \max_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\}.$$

Thus, for each $0 \le i < \lambda$ we have

$$\|\Pi_i\| = \max_{S\in\mathcal{S}_i} \|\Gamma_S\| \le \lambda opt.$$

Hence,

$$\begin{aligned} \|\Pi\| &= \sum_{i=0}^{\lambda-1} \|\Pi_i\| \\ &= \sum_{i=0}^{\lambda-1} \max_{S\in\mathcal{S}_i} \|\Gamma_S\| \\ &= \sum_{i=0}^{\lambda-1} \max_{S\in\mathcal{S}_i} \sum_{v\in V_S} \max\left\{ \max_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\}. \end{aligned}$$

Now, let *opt* be the length of a shortest link schedule. For each non-empty cell S, since Γ_S is a shortest link schedule of the demands by all links associated with S we have $\|\Gamma_S\| \le opt$. Hence,

$$\|\Pi\| = \sum_{i=0}^{\lambda-1} \max_{S\in\mathcal{S}_i} \|\Gamma_S\| \le \lambda opt.$$

So, the theorem holds. ■

D. Restricted Mutiflows

Let Ω be the capacity region of the underlying network, and Φ be the set of all link demands $d \in \mathbb{R}_+^A$ satisfying that

$$\sum_{i=0}^{\lambda-1} \max_{S\in\mathcal{S}_i} \sum_{v\in V_S} \max\left\{ \max_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}, \frac{\sum_{a\in\delta^{in}(v)} \frac{d(a)}{c(a)}}{\tau(v)} \right\} \le 1.$$

By Theorem 4.2,

$$\Phi \subseteq \Omega \subseteq \lambda\Phi.$$

In other words, Φ is a polynomial λ-approximate capacity subregion. In addition, by a straightforward linearization with auxiliary variables Φ can be represented as a system of a linear number of linear inequalities.

A multiflow $\langle f_1, f_2, \cdots, f_k\rangle$ is said to be Φ-*restricted* if $\sum_{j=1}^{k} f_j \in \Phi$. A Φ-restricted multiflow $\langle f_1, f_2, \cdots, f_k\rangle$ with maximum total weight $\sum_{j=1}^{k} w_i \cdot val(f_j)$ is referred to as maximum weighted Φ-restricted multiflow. A Φ-restricted multiflow $\langle f_1, f_2, \cdots, f_k\rangle$ with maximum concurrency

$$\min_{1\le j\le k} \frac{val(f_j)}{d_j}$$

is referred to as maximum concurrent Φ-restricted multiflow. Since $\Omega \subseteq \lambda\Phi$, for any multiflow $\langle f_1^*, f_2^*, \cdots, f_k^*\rangle \in \Omega$, the multilfow $\left\langle \frac{1}{\lambda}f_1^*, \frac{1}{\lambda}f_2^*, \cdots, \frac{1}{\lambda}f_k^*\right\rangle$ is Φ-restricted. Hence, the total weight of a maximum weighted Φ-restricted multiflow is at least $\frac{1}{\lambda}$ times the total weight of a maximum weighted multiflow, and the concurrency of a maximum concurrent Φ-restricted multiflow is at least $\frac{1}{\lambda}$ times the the concurrency of a maximum concurrent multiflow. These properties of Φ-restricted multiflows motivate us to take a restriction-based approach for approximating **MWMF** and **MCMF**.

Our approximation algorithm **R-WMF** for **MWMF** first computes a maximum weighted Φ-restricted multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ by solving the following LP

$$\begin{array}{ll} \max & \sum_{j=1}^{k} w_j \cdot val(f_j) \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k; \\ & \sum_{j=1}^{k} f_j \in \Phi. \end{array}$$

Then, a link schedule Π of $\sum_{j=1}^{k} f_j$ is computed by applying the algorithm **DC-LS**. As $\sum_{j=1}^{k} f_j \in \Phi$, the length of Π is at most one. So, the multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ together with the schedule Π is a λ-approximate solution to the problem **MWMF**.

Similarly, our approximation algorithm **R-CMF** for **MCMF** first computes a maximum concurrent Φ-restricted multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ by solving the following LP

$$\begin{array}{ll} \max & \phi \\ s.t. & f_j \in \mathcal{F}_j, \forall 1 \leq j \leq k; \\ & val(f_j) \geq \phi d_j, \forall 1 \leq i \leq k; \\ & \sum_{j=1}^{k} f_j \in \Phi. \end{array}$$

Then, a link schedule Π of $\sum_{j=1}^{k} f_j$ is computed by applying the algorithm **DC-LS**. As $\sum_{j=1}^{k} f_j \in \Phi$, the length of Π is at most one. So, the multiflow $\langle f_1, f_2, \cdots, f_k \rangle$ together with the schedule Π is a λ-approximate solution to the problem **MCMF**.

In summary, we have the following theorem on the performance of the two algorithms **R-WMF** and **R-CMF**.

Theorem 4.3: Both the algorithm **R-WMF** and **R-CMF** have approximation bound λ.

V. Polynomial-Time Approximation Schemes

In practical MPR-capable wireless networks, the maximum MPR capability

$$\overline{\tau} := \max_{v \in V} \tau(v)$$

is typically bounded by a constant. In this section, we show that the four problems **MWIS**, **SLS**, **MWMF**, and **MCMF** in those networks with constant-bounded MPR capability all admit a PTAS. A PTAS for **MWIS** is presented in Subsection V-A. The PTAS's for other three problems are presented in Subsection V-B.

A. PTAS for MWIS

In this subsection, we give a PTAS for **MWIS**. By proper scaling, we assume that the interference radius $r = 1/2$. The interference range of a link a is the disk of radius r centered at the sender of a. Let η be the ratio of the interference radius $r = 1/2$ to the maximum link length, and

$$\mu = \left\lceil \pi / \arcsin \frac{\eta - 1}{2\eta} \right\rceil - 1.$$

The following sparsity of the independent sets of links is the cornerstone to our PTAS for **MWIS**.

Lemma 5.1: Suppose that S is a (closed) square of side L and I is a set of independent links whose interference ranges are contained in S. Then,

$$|I| \leq \frac{4L^2 \mu \overline{\tau}}{\pi}.$$

Proof: We first claim that each point $o \in S$ lies in the interference ranges of at most $\mu\overline{\tau}$ links in I. Indeed, if o is also the receiver of some link in I, then

$$|I| = \left|\delta_I^{in}(o)\right| \leq \overline{\tau}.$$

So, we assume that o is not a receiver of any link in I. Let I' be the set of links whose interference ranges contain o, and let V' be the set of receiving nodes of the links in I'. It was as proved in [27] that the angle separation between any two receivers in V' at o is greater than $2 \arcsin \frac{\eta-1}{2\eta}$. Hence,

$$|V'| \leq \left\lceil \pi / \arcsin \frac{\eta - 1}{2\eta} \right\rceil - 1 = \mu.$$

Thus,

$$|I'| = \sum_{v \in V'} \left|\delta_{I'}^{in}(v)\right| \leq |V'| \overline{\tau} \leq \mu\overline{\tau}.$$

So, our claim holds.

Next, we bound $|I|$ using the standard disk packing argument. The above claim implies that the summation of the areas of the interference ranges of all links in I is at most $\mu\overline{\tau}$ times the area of S. In other words,

$$|I| \cdot \pi (1/2)^2 \leq \mu\overline{\tau} \cdot L^2.$$

Thus,

$$|I| \leq \frac{4L^2 \mu \overline{\tau}}{\pi}.$$

So, the lemma holds. ■

In the remaining of subsection, we fix an integer $L \geq 3$ and present a PTAS which utilizes the shifting strategy [13], [14] to produce an independent set I in polynomial time (depending on L) satisfying that

$$w(I) \geq \left(1 - \frac{2}{L}\right) opt,$$

where *opt* is the weight of an optimal solution.

We first introduce some terms and notations. The lines $x = i$ for $i \in \mathbb{Z}$ are called vertical grid lines; the lines $y = j$ for $j \in \mathbb{Z}$ are called horizontal grid lines. A vertical grid line $x = i$ is said to be *l-active* for some integer $0 \leq l < L$ if $i \bmod L = l$; similarly, a horizontal grid line $y = j$ is said to be *l-active* if $j \bmod L = l$. For each $0 \leq l < L$, all the l-active grid lines decompose the whole plane into half-open half-closed squares of side L

$$(l, l) + [iL, (i+1)L) \times [jL, (j+1)L)$$

for all $i, j \in \mathbb{Z}$. All these squares are called *l-active* squares, and we use $\mathcal{S}_l$ to denote those l-active squares which contain the interference range of at least one link. A link a is said to be *l-active* is its interference range is contained in some $S \in \mathcal{S}_l$, and *l-inactive* otherwise. If a is l-inactive, then one of the following two cases must occur to the interference range of a:

- the interference range of a either crosses a l-active horizontal grid line or it touches a horizontal l-active line from below;
- the interference range of a either crosses a l-active vertical grid line or it touches a vertical l-active line from left.

Since two consecutive horizontal (respectively, vertical) lines are separated by distance one which is the diameter of the interference range of a, the first (respectively, second) case may occur with at most one l. Thus, a link a can be l-inactive for at most two l's.

For each $0 \leq l < L$, we use A_l to denote the set of l-active links. We describe a divide-and-conquer algorithm to compute a maximum weighted independent set I_l of A_l in polynomial time.

- **Division**: For each $S \in \mathcal{S}_l$, Let B_S be the set of links whose interference ranges are contained in S. Then, the sets B_S for all $S \in \mathcal{S}_l$ form a partition of A_l. In addition, for any distinct S and S' in $\mathcal{S}_l$, every link in B_S and every link in $B_{S'}$ are interference-free.
- **Conquer**: For each $S \in \mathcal{S}_l$, we compute enumeratively a maximum weighted independent set J_S of B_S whose size is at most $\frac{4L^2\mu\overline{\tau}}{\pi}$. By Lemma 5.1, J_S is a maximum weighted independent set of B_S.
- **Combination**: The union of all J_S for $S \in \mathcal{S}_l$ is returned as I_l. Clearly, I_l is a a maximum weighted independent set of A_l.

Our PTAS applies the divide-and-conquer algorithm described in the previous paragraph to compute a maximum weighted independent set I_l of A_l for each $0 \leq l < L$, and then return the heaviest one among them as the output I. We claim that

$$w(I) \geq \left(1 - \frac{2}{L}\right) opt.$$

Indeed, let O be a maximum weighted independent set of A. For each $0 \leq l < L$, let A'_l denote the set of l-inactive links in A. Then, for each $a \in A$,

$$|\{0 \leq l < L : a \in A'_l\}| \leq 2.$$

So,

$$\begin{aligned}
&\textstyle\sum_{l=0}^{L-1} w(O \cap A'_l) \\
&= \textstyle\sum_{l=0}^{L-1} \sum_{a \in O} w(a) |\{a\} \cap A'_l| \\
&= \textstyle\sum_{a \in O} w(a) \sum_{l=0}^{L-1} |\{a\} \cap A'_l| \\
&= \textstyle\sum_{a \in O} w(a) |\{0 \leq l < L : a \in A'_l\}| \\
&\leq 2 \textstyle\sum_{a \in O} w(a) \\
&= 2w(O) \\
&= 2opt.
\end{aligned}$$

Thus,

$$\min_{0 \leq l < L} w(O \cap A'_l) \geq \frac{2}{L} opt.$$

Consequently,

$$\begin{aligned}
w(I) &= \max_{0 \leq l < L} w(I_l) \\
&\geq \max_{0 \leq l < L} w(O \cap A_l) \\
&= w(O) - \min_{0 \leq l < L} w(O \cap A'_l) \\
&\geq \left(1 - \frac{2}{L}\right) opt.
\end{aligned}$$

So, our claim holds.

B. PTAS for Others

The PTAS for other three problems **MWMF**, **MCMF**, and **SLS** are developed by establishing approximation-preserving reductions from them to **MWIS**. The reductions make use of the ellipsoid method [8] for linear programming in the way similar to those developed in [27]. The details are omitted due to the limit of the space.

VI. Conclusion

In this paper, we have developed the first-ever polynomial-time approximation algorithms for **MWIS**, **SLS**, **MWMF**, and **MCMF** in wireless networks with arbitrary and possibly heterogenous MPR capabilities. We have also devised PTAS's for **MWIS**, **SLS**, **MWMF**, and **MCMF** in wireless networks with constant-bounded MPR capabilities. In addition, the new techniques and tools developed in this paper are expected to have wide applications in future algorithmic and queuing-theoretic studies of other interesting network capacity and cross-layer design and optimizations in MPR-capable wireless networks.

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