Maximum Weighted Independent Set of Links under Physical Interference Model*

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Abstract. Interference-aware scheduling for wireless communications is crucial to improve the network throughput. In this paper, we study the problem of Maximum Weighted Independent Set of Links (MWISL) under the physical interference model in wireless networks. Given a set of communication links distributed in a two-dimensional Euclidean plane, assume each link is associated with a positive weight which represents the benefit of transmitting along the link, the objective is to seek an independent set of links subject to the physical interference constraints with maximum weighted sum. To the best of our knowledge, no algorithm for MWISL under physical interference model has been proposed. We focus on MWISL in the oblivious power assignment setting.

1 Introduction

Link scheduling in wireless networks plays a critical role for wireless networking performances, especially when the network has stringent quality of service restrictions. One challenge for link scheduling lies in interferences among concurrent transmissions. Unlike the wired networks, the signal interference casts significant effect on the fundamental limit on the data throughput that any scheduling protocols (centralized or distributed) can achieve. It is well-known that a number of scheduling problems (*e.g.*, maximum throughput scheduling) become NP-hard to solve when considering wireless interference, while their counter-parts are solvable in polynomial time for wired networks. Thus, the scheduling protocols for wireless networks (even the benchmark performances obtained by centralized scheduling approaches) often rely on heuristics that approximately optimize the throughput.

We address a fundamental problem for scheduling in wireless communications: Maximum Weighted Independent Set of Links (MWISL): given an input set of links, a subset of links is an *independent set of links* iff they can transmit concurrently. Assume each link is associated with a positive weight (representing the benefit of transmitting along the link), the objective is to seek an independent set of links with maximum weighted sum under given interference model. We focus on MWISL under *physical interference model*, where a signal is received successfully if the Signal to Interference-plus-Noise Ratio is above a threshold depending on hardware and coding method. In this practical model, the definition of a successful transmission accounts also interference generated by transmitters located far away. Thus, traditional methods in graph-based interference models (*e.g.*, protocol interference model, RTS/CTS model *et al.*)

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cannot be directly applied here. The MWISL problem has several variations: such as different power assignments and different weight distributions. Even for the special case where every link has unit weight, MWISL is proved to be NP-hard [6, 7].

Our main contribution is the algorithm design for MWISL problem under the oblivious power assignment setting where each link l (or its corresponding sender) is assigned a transmission power $c \cdot \|l\|^{\beta}$, here c is a constant and $0 < \beta \leq \kappa$ is a constant. We can prove that our solution satisfies the interference constraints and achieves constant approximation ratio of the optimum when $\beta = \kappa$. Our main idea is like this: by using partition and shifting strategies, we find multiple sets of well-separated links and then select the one with the largest weight.

The rest of the paper is organized as follows. Section 2 formulates our problems. Section 3 presents our algorithm design for the MWISL problem. Section 4 outlines the related work. Finally, Section 5 concludes the paper.

2 Network Model

All the networking nodes V lie in plane and have a maximum transmission power P. The Euclidean distance between any pair of nodes is denoted by $\|uv\|$. Let r be the distance between a closest pair of nodes in V. The path-loss over a distance l is $\eta l^{-\kappa}$, where κ is *path-loss exponent* (a constant greater than 2 and 5 depending on the wireless environment), and η is the *reference loss factor*. Since the path-loss factor over the distance r is less than one, we have $\eta < r^{\kappa}$. In an *oblivious power assignment*, a node u transmits to another node v always at the power $c\,\|uv\|^{\beta}$ for some constants $c > 0$ and $0 < \beta \leq \kappa$. This assumption implicitly imposes an upper bound on the distance between a pair of nodes which directly communicate with each other: For u to be able to directly communicate with v, we must have $c\,\|uv\|^{\beta} \leq P$ and hence $\|uv\| \leq (P/c)^{1/\beta}$. Let ξ be the noise power, and σ be the *signal to interference and noise ratio* (*SINR*) threshold for successful reception. Then, in the absence of interference, the transmission by a node u can be successfully received by another node v if and only if $\frac{c\|uv\|^{\beta} \cdot \eta \|uv\|^{-\kappa}}{\xi} \geq \sigma$ which is equivalent to $\|uv\|^{\kappa-\beta} \leq \frac{c\eta}{\sigma\xi}$. Note that when $\|uv\|^{\kappa-\beta} = \frac{c\eta}{\sigma\xi}$, link uv can only transmit alone since any other link will conflict with uv. Thus we can disregard these links in A and assume that

$$\|uv\|^{\kappa-\beta} < \frac{c\eta}{\sigma\xi}$$

Therefore, the set A of communication links consists of all pairs (u, v) of distinct nodes satisfying that $\|uv\|^{\beta} \leq P/c$ and $\|uv\|^{\kappa-\beta} < \frac{c\eta}{\sigma\xi}$. Let R be the maximum length of the links in A. Then,

$$R^{\kappa} = R^{\beta} \cdot R^{\kappa-\beta} < \frac{P}{c} \cdot \frac{c\eta}{\sigma\xi} = \frac{P\eta}{\sigma\xi} < \frac{P}{\sigma\xi} r^{\kappa}.$$

Hence,

$$\frac{R}{r} < \left(\frac{P}{\sigma\xi}\right)^{1/\kappa}.$$

A set I of links in A is said to be *independent* if and only if all links in I can transmit successfully at the same time under the oblivious power assignment, i.e., the SINR of each link in I is above σ. We denote by $\mathcal{I}$ the collection of independent sets of links in A. Given a link weight function $d \in \mathbb{R}_+^A$, the problem **Maximum Weighted Independent Set of Links** (**MWISL**) seeks a set $I \in \mathcal{I}$ with maximum total weight $d(I) = \sum_{a \in I} d(a)$.

3 Algorithm Design

In this section, we present our algorithm design for MWISL in the oblivious power assignment setting under physical interference model.

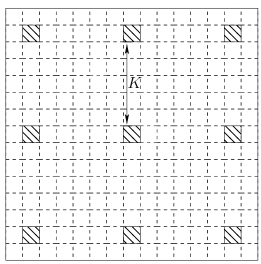

Fig. 1. Grid partition of the plane

We employ a grid partition of the plane (Fig. 1). Let $\ell = R/\sqrt{2}$. The vertical lines $x = i \cdot \ell$ for $i \in \mathbb{Z}$ and horizontal lines $y = j \cdot \ell$ for $j \in \mathbb{Z}$ partition the planes into half-open and half-closed grides of side ℓ (here $\mathbb{Z}$ represents the integer set):

$$\{[i\ell, (i+1)\ell) \times [j\ell, (j+1)\ell) : i, j \in \mathbb{Z}\}.$$

For all $i, j \in \mathbb{Z}$, we denote A_{ij} to be the set of links in A whose senders lie in the grid

$$[i\ell, (i+1)\ell) \times [j\ell, (j+1)\ell).$$

We first give a sufficient condition for a set I of links to be independent. Let

$$K = \lceil \sqrt{2}\left((4\tau)^{-1}\left(\sigma^{-1} - \xi(c\eta)^{-1}R^{\kappa-\beta}\right)\right)^{-1/\kappa} + \sqrt{2}\rceil$$

Generally, K's value depends on R, we can see that when $\beta = \kappa$,

$$K = \lceil \sqrt{2}\left((4\tau)^{-1}\left(\sigma^{-1} - \xi(c\eta)^{-1}\right)\right)^{-1/\kappa} + \sqrt{2}\rceil$$

which is a constant independent of R.

Lemma 1. *Consider any two nonnegative integers k_1 and k_2 which are at most K. Suppose I is a set of links satisfying that for each $i, j \in \mathbb{Z}$, $|I \cap A_{ij}| \leq 1$ if $i \bmod (K+1) = k_1$ and $j \bmod (K+1) = k_2$ and $|I \cap A_{ij}| = 0$ otherwise. Then, I is independent.*

Proof. Consider any link $a = (u, v)$. The wanted signal strength is

$$c \|a\|^{\beta} \cdot \eta \|a\|^{-\kappa} = c\eta \|a\|^{\beta-\kappa} \geq c\eta R^{\beta-\kappa}.$$

Consider any link $a' = (u', v')$ in I other than a. We have $\|u'u\| \geq K\ell$. Therefore,

$$\|u'v\| \geq \|u'u\| - \|uv\| \geq K\ell - R = \left(K/\sqrt{2} - 1\right) R.$$

The total interference to a from all other links in I is at most

$$\begin{aligned}
&\sum_{(x,y)\in\mathbb{Z}^2\setminus\{(0,0)\}} cR^{\beta} \cdot \eta \left(\sqrt{x^2+y^2} \cdot \left(K/\sqrt{2}-1\right) R\right)^{-\kappa} \\
&= c\eta R^{\beta-\kappa} \left(K/\sqrt{2}-1\right)^{-\kappa} \sum_{(x,y)\in\mathbb{Z}^2\setminus\{(0,0)\}} \left(\sqrt{x^2+y^2}\right)^{-\kappa} \\
&\leq 4c\eta R^{\beta-\kappa} \left(K/\sqrt{2}-1\right)^{-\kappa} \left(\sum_{i=1}^{\infty} i^{-\kappa} + \sum_{x=1}^{\infty}\sum_{y=1}^{\infty} \left(\sqrt{x^2+y^2}\right)^{-\kappa}\right) \\
&\leq 4c\eta R^{\beta-\kappa} \left(K/\sqrt{2}-1\right)^{-\kappa} \left(\frac{\kappa(1+2^{-\frac{\kappa}{2}})}{\kappa-1} + \frac{\pi 2^{-\kappa/2}}{2(\kappa-2)}\right) \\
&= 4\tau c\eta R^{\beta-\kappa} \left(K/\sqrt{2}-1\right)^{-\kappa},
\end{aligned}$$

where

$$\tau = \frac{\kappa(1+2^{-\frac{\kappa}{2}})}{\kappa-1} + \frac{\pi 2^{-\kappa/2}}{2(\kappa-2)}.$$

Thus the SINR at the receiver of the link is at least

$$\frac{c\eta R^{\beta-\kappa}}{\xi + 4\tau c\eta R^{\beta-\kappa}\left(K/\sqrt{2}-1\right)^{-\kappa}} \geq \sigma$$

since

$$K/\sqrt{2} - 1 \geq \left((4\tau)^{-1}\left(\sigma^{-1} - \xi(c\eta)^{-1} R^{\kappa-\beta}\right)\right)^{-1/\kappa}$$

Next, we give a necessary condition for a set I of links to be independent. Let

$$\omega = \left\lceil \frac{2^{\kappa} P}{\sigma^2 \xi} + 1 \right\rceil.$$

Lemma 2. *For any $I \in \mathcal{I}$ and any $i, j \in \mathbb{Z}$, $|I \cap A_{ij}| \leq \omega$.*

Proof. Let $I_{ij} = I \cap A_{ij}$. Assume $a = (u, v)$ be the shortest link in I_{ij}, consider any link $a' = (u', v')$ in I_{ij} other than a, the distance between the sender u' and v satisfies

$$\left\|u'v\right\| \leq \left\|u'u\right\| + \|uv\| \leq \left\|\sqrt{2}\ell\right\| + \|uv\| \leq 2R,$$

The SINR at a from all other links in I_{ij} is at most

$$\frac{c\left\|a\right\|^{\beta}\cdot\eta\left\|a\right\|^{-\kappa}}{\sum_{a'\in I_{ij}\setminus\{a\}}c\left\|a'\right\|^{\beta}\cdot\eta\left\|u'v\right\|^{-\kappa}}\leq\frac{\left\|a\right\|^{-\kappa}}{\sum_{a'\in I_{ij}\setminus\{a\}}\left\|u'v\right\|^{-\kappa}}\leq\frac{r^{-\kappa}}{\left(|I_{ij}|-1\right)\left(2R\right)^{-\kappa}}$$

Since $\frac{r^{-\kappa}}{(|I_{ij}|-1)(2R)^{-\kappa}}\geq\sigma$, we have $|I_{ij}|\leq\frac{2^{\kappa}}{\sigma}\frac{R^{\kappa}}{r^{\kappa}}+1<\frac{2^{\kappa}}{\sigma}\frac{P}{\sigma\xi}+1=\frac{2^{\kappa}P}{\sigma^{2}\xi}+1$.

Input: A set of links $\mathcal{L}=\{l_1,l_2,\cdots,l_n\}$

$d_{\max}\leftarrow 0$;
for $k_1=0,\cdots,K$ *and* $k_2=0,\cdots,K$ **do**
  for $i,j\in\mathbb{Z}$ *and the grid* $g_{i,j}$ *contains links from* $\mathcal{L}$ **do**
    if $i\equiv k_1 \mod (K+1)$ *and* $j\equiv k_2 \mod (K+1)$ **then**
      select one link with the maximal weight whose sender lies within $g_{i,j}$
  All the selected links form a set $I_{k_1k_2}$;
  if $d(I_{k_1k_2})>d_{\max}$ **then**
    $\mathcal{S}\leftarrow I_{k_1k_2}$; $d_{\max}\leftarrow d(I_{k_1k_2})$;
return *A schedule* $\mathcal{S}$ *containing a subset of links in* $\mathcal{L}$*.*

Algorithm 1. Scheduling for MWISL

Our partition-based scheduling method for input $\mathcal{L}$ is shown in Algorithm 1.

The correctness of the algorithm follows from Lemma 1. Next, we derive its approximation bound.

Theorem 1. *The approximation ratio of our algorithm for MWISL is at most* $(K+1)^2\omega$*.*

Proof. Let I^* be a maximum-weighted independent set of links. For any pair of nonnegative integers k_1 and k_2, let $I^*_{k_1k_2}$ denote the set of links in I^* which lie in the grids

$$[i\ell,(i+1)\,\ell)\times[j\ell,(j+1)\,\ell)$$

for all $i,j\in\mathbb{Z}$ satisfying that $i \bmod (K+1)=k_1$ and $j \bmod (K+1)=k_2$. By Lemma 2, the set $I^*_{k_1k_2}$ contains at most ω links from each grid

$$[i\ell,(i+1)\,\ell)\times[j\ell,(j+1)\,\ell)$$

with $i \bmod (K+1)=k_1$ and $j \bmod (K+1)=k_2$. Hence,

$$d\left(I^*_{k_1k_2}\right)\leq\omega d\left(I_{k_1k_2}\right)\leq\omega\max_{0\leq k_1,k_2\leq K}d\left(I_{k_1k_2}\right).$$

Therefore,

$$d\left(I^*\right)=\sum\nolimits_{0\leq k_1,k_2\leq K}d\left(I^*_{k_1k_2}\right)\leq\left(K+1\right)^2\omega\max_{0\leq k_1,k_2\leq K}d\left(I_{k_1k_2}\right).$$

So, the theorem holds.

4 Literature Review

For general graph-based interference model, the maximum throughput link scheduling problem is NP-complete [5]. Both probabilistic scheduling protocols [5, 10, 11] and distributed link scheduling protocols [13, 14] are proposed to maximize the throughput.

For physical interference model, the problem of joint scheduling and power control has been well studied in [3, 4]. In [12], a power-assignment algorithm which schedules a strongly connected set of links in poly-logarithmic time is presented. [6] shows that the scheduling problem without power control under physical interference model, where nodes are arbitrarily distributed is NP-complete. A greedy scheduling algorithm with approximation ratio of $O(n^{1-2/(\Psi(\alpha)+\epsilon)}(\log n)^2)$, where $\Psi(\alpha)$ is a constant that depends on the path-loss exponent α, is proposed in [1]. Notice that this result can only hold when the nodes are distributed uniformly at random in a square of unit area. In [6], an algorithm with a factor $O(g(L))$ approximation guarantee in arbitrary topologies, where $g(L) = \log \vartheta(L)$ is the diversity of the network, is proposed. In [2], an algorithm with approximation guarantee of $O(\log \Delta)$ was proposed, where Δ is the ratio between the maximum and the minimum distances between nodes. Obviously, it can be arbitrarily larger than $\vartheta(L)$.

Recently, Goussevskaia *et al.* [7] proposed a method for MWISL in the unweighted case which was claimed to have constant approximation bound. Unfortunately, as observed in Xu and Tang [16], their method [7] works correctly in absence of the background noise. Wan *et al.* [15] resolved this issue by developing the first correct constant-approximation algorithm. Most Recently, Halldorsson *et al.* [9] presented a robustness result, showing that constant parameter and model changes will modify the min-length link scheduling result only by a constant. They [8] also studied the scheduling problem under power control.

5 Conclusions

We studied MWISL problem under the physical interference model with oblivious power assignment. Some interesting questions are left for future research. The first is to extend our algorithm to deal with a more general path loss model. The second is to develop constant approximation algorithms for MWISL under uniform power and adjustable power assignment settings.

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