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Practical traffic grooming scheme for single-hub SONET/WDM rings

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Abstract. In SONET/WDM networks, one fiber supports multiple wavelengths and each wavelength supports several low rate tributary streams. ‘Traffic grooming’ then is defined as properly using SONET Add/Drop Multiplexer to electronically multiplex and demultiplex required tributary traffic patterns with minimal resource cost (wavelengths and ADMs).

This paper studies traffic grooming problem in single hub SONET/WDM networks and extends existing results. We analyze the real deployments, generalize their results, and study the practical special cases. We prove that BLSR/2 would never be more expensive than UPSR under any traffic pattern. We also present the exact minimum costs of uniform traffic in both UPSR and BLSR/2. We give approximation algorithms for optimal grooming of non-uniform traffic. Finally, we consider how to select the line speeds if there are two different line speeds available.

Keywords: Traffic grooming, SONET/WDM ring, UPSR, BLSR/2, single-hub, bin packing

1. Introduction

Recently as the Internet is booming, and B-ISDN services such as eConference, multimedia communication, VoIP, HDTV, VOD, etc. start to be incentive for customers, bandwidth requirement grows rapidly. At the low end, homes are connected to digital world by existing wires. One trend is to use fast connections such as Cable Modem and DSL technique to access B-ISDN services. Another coming trend is to use fast wireless connections to connect to the digital world, rather than only one telephony channel.

Naturally, following the trend is the usage of optical communication technique, which has originally been used to support the high-end wide area network (WAN) for setting up high-speed MAN feeder networks and LAN access networks. In recent years, as the commercial optical communication standard, SONET/SDH coupling with WDM (Wavelength Division Multiplexing) technique then has been deployed to provide B-ISDN services for customers. SONET rings are embedded in WDM rings, and one wavelength supports one SONET ring if without considering self-healing mechanism. Figures 1, 2 and 3 give some examples of its deployments.

Along with such an immigration are new engineering problems of network designing and planning of SONET/WDM MANs and LANs. Among those new engineering problems, we focus on the traffic grooming problem. Unlike the way we consider the long-haul fiber networks in which each fiber needs many repeaters (say, EDFA-Erbium-doped fiber amplifier) and carries larger data volumes and thus the number of wavelengths is a rare resource, for MAN and LAN deployments we often can assume we have enough fibers to lighten. Thus we assume that wavelengths are sufficient and the terminating devices dominate cost. Indeed, till now the bottleneck of optical communication applications lies on the O/E and E/O boundaries and. Though SONET/SDH ADM provides proper and cost-efficient multiplexing/demultiplexing and O-E/E-O conversions for SONET networks, it is still expensive. So when planning to set up SONET/SDH networks in metropolitan and local areas, we will focus on how to minimize the number of SONET/SDH Add/Drop Multiplexers (SONET ADMs).

Two types of SONET self-healing rings have been widely used: A unidirectional path-switched ring (UPSR) consists of two unidirectional counter-propagating fiber rings, referred to as *basic rings*; A two-fiber bidirectional line-switched ring (BLSR/2) comprises of two unidirectional counter-propagating basic rings as well.

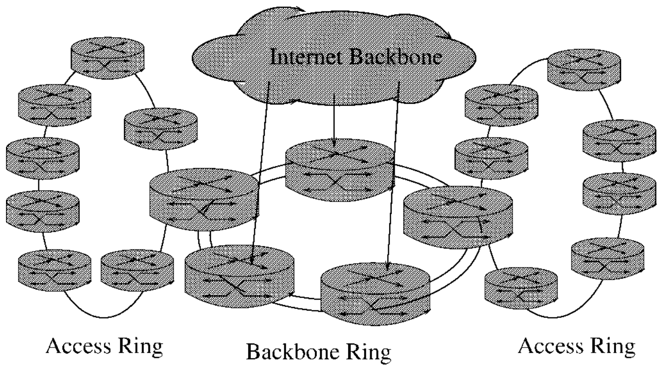

Fig. 1. SONET/SDH over WDM, a likely Metropolitan deployment.

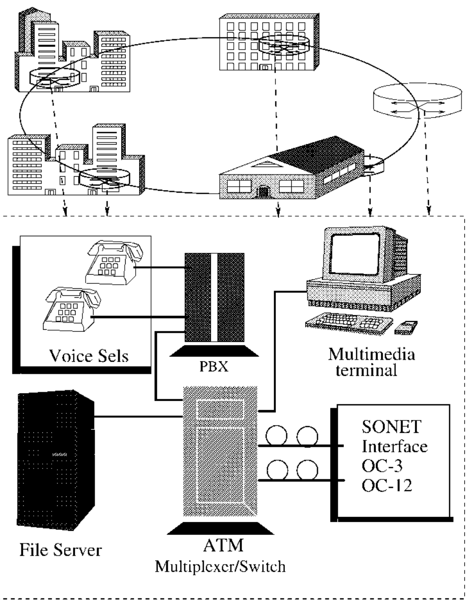

Fig. 2. SONET/SDH access network (LAN) deployments. Here each ring node will support the telephony and digital services for one building. Ring nodes are connected to a hub at center office. Ring nodes will add/drop OC-3/OC-12 tributary streams contained in wavelength channels to ATM switches. Here OC-3 (155 Mbps) and OC-12 (622 Mbps) are two basic ATM bit rates.

In addition to the network architectures, the minimum ADM cost also varies upon the traffic pattern and traffic demands. The traffic could have some regular patterns such as one-to-all and all-to-all, or any irregular pattern. The traffic demands may be uniform (i.e., all traffic have the same amount of demands) or non-uniform. Each traffic demand itself is given as an integer number of low speed (tributary) streams. Alternatively, it can also be represented by its *traffic granularity*, defined as the ratio of its demand to the transmission capacity of a single wavelength. A traffic is said to be a *full-wavelength traffic*, a *sub-wavelength traffic* or a *super-wavelength traffic* if its traffic granularity is equal to one, less than one, or greater than one respectively.

The minimum ADM problem has been discussed in a number of recent works [4,6–8,11,13,14]. [6] and [8] studied optimal grooming of arbitrary full-wavelength lightpaths. [4,13] and [14] provided grooming of uniform $1/2$, $1/4$, and $1/8$-wavelength traffic. [7] and [11] gave some preliminary results on the traffic groomings in single-hub rings. In [11], an optimal grooming of uniform one-to-all traffic in single-hub UPSR rings was presented. Then it can be generalized to any uniform traffic. It also gave lower and upper bounds on the ADM cost of uniform

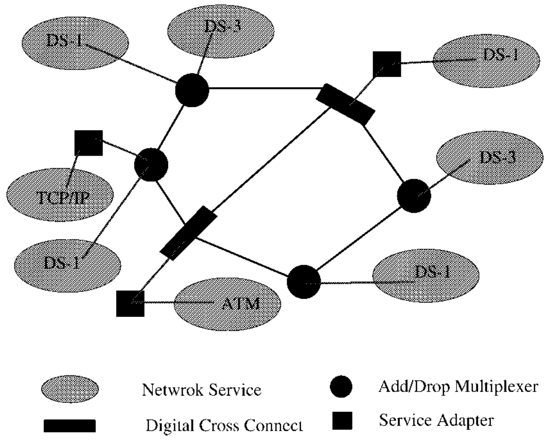

Fig. 3. Typical SONET Networks. Here DS-1 is a connection with capacity of 24 linking to PBX of small companies and DS-3 is with capacity of 672 for bigger companies. TCP/IP is for Intranet such as campus networks and ATM provides statistical multiplexing for B-ISDN connections with different QoS (Quality of Service) requirements.

all-to-all traffic without a hub. In [7], lower and upper bounds on the ADM cost of uniform all-to-all traffic in both single-hub UPSR and single-hub BLSR/2 were obtained. The economics of these two types of rings were then justified by the two lower bounds. A remark on this justification is that it makes logical sense only if the lower bounds are sufficiently close the optimum. In addition, [7] also briefly discussed the criteria for using UPSR vs. BLSR rings and to mix two types of line speeds on a single SONET/WDM ring. In this paper we will further the works in [7] and [11] and provide stronger results.

The paper is structured as follows. In Section 2, we formulate the traffic grooming problem after showing that we need only concentrate on one-to-all simplex traffic. We prove that BLSR/2 always costs no more than UPSR under any traffic, and show that the search for optimal grooming can be confined to a narrow subset of valid groomings, referred to as *canonical groomings*. In Section 3, we construct optimal canonical groomings of uniform one-to-all traffic in both UPSR and BLSR/2 rings and derive the analytic expression of the minimum ADMs. An approximation scheme for nonuniform traffic request is presented. We also discuss how to select the line speeds if there are two line speeds available.

2. Problem formulation and canonical grooming

We consider a single-hub SONET/WDM ring comprising of $N+1$ nodes numbered $0, 1, \ldots, N$ clockwise, with the hub placed at node 0. The traffic demand and the transmission capacity of each wavelength are in terms of the basic low-rate (e.g., OC-3) traffic streams. Let g be the transmission capacity of a single wavelength.

We establish a reduction from grooming of any duplex traffic to grooming of one-to-all duplex traffic, and from grooming of one-to-all duplex traffic to grooming of one-to-all simplex traffic. Thus any optimal grooming of one-to-all simplex leads to an optimal grooming of one-to-all duplex and an optimal grooming of all-to-all duplex. Therefore, from then on we concentrate on only one-to-all simplex traffic.

2.1. Reduction to one-to-all simplex traffic

Assume that the traffic between any pair of nodes is full-duplex and the traffic demand between node i and j is r_{ij}. As the traffic stream between any pair of nodes must be routed through the hub, any traffic pattern can be treated as a number of duplex requests between the hub and all other nodes. To be more specific, in the equivalent one-to-all duplex traffic, the traffic demand between node i and hub is

$$r_i = \sum_{j \neq i} r_{ij},$$

for all $1 \leqslant i \leqslant N$. Thus it is sufficient for us to consider only one-to-all duplex traffic.

In the following we take a further step of reduction. Let ADM_d $(r_1, \ldots, r_N)$ be the minimum ADM cost of a one-to-all duplex traffic, in which the demand between node i and hub is r_i for $1 \leqslant i \leqslant N$. Let ADM_s $(r_1, \ldots, r_N)$ be the minimum ADM cost of a one-to-all simplex traffic, in which the demand from hub to node i is r_i for $1 \leqslant i \leqslant N$. Obviously, in either UPSR or BLSR,

$$ADM_d(r_1, \ldots, r_N) \geqslant ADM_s(r_1, \ldots, r_N),$$

as the one-to-all duplex traffic is a superset of one-to-all simplex traffic. On the other hand, any grooming of the simplex traffic naturally gives rise to a grooming of the corresponding duplex traffic with the same cost in the following way: Let w be any wavelength used in the grooming of the simplex traffic, and let r_i^w be the portion of the demand from hub to node i carried in wavelength w. Now consider the following grooming of the duplex traffic: we use the same set of wavelengths used in the grooming of the simplex traffic, each wavelength w carries r_i^w units of demand from hub to node i and r_i^w units of demand from node i to hub for all $1 \leqslant i \leqslant N$. It's easy to see that such grooming is a valid solution and it uses the same number of ADMs. Thus

$$ADM_d(r_1, \ldots, r_N) \leqslant ADM_s(r_1, \ldots, r_N),$$

which implies that

$$ADM_d(r_1, \ldots, r_N) = ADM_s(r_1, \ldots, r_N).$$

The following lemma summarizes this reduction.

Lemma 2.1. *The minimum ADM cost of any one-to-all duplex traffic is same as that of the corresponding one-to-all simplex traffic.*

2.2. Problem formulation

So from now on, we will only concentrate on the grooming of one-to-all simplex pattern. Thus for single hub SONET/SDH (over WDM) networks, the *traffic grooming problem* has the following formulation.
Instance: a traffic set $\{r_i | 1 \leqslant i \leqslant N\}$, assuming one wavelength supports g tributary streams.
Solution: A valid assignment of tributary streams in wavelengths to the traffic set.
Objective: Minimize the number of needed ADMs.

For example, assume in Fig. 2, four buildings a, b, c, d need capacity 30 OC-3's, 20 OC-3's, 9 OC-3's, 17 OC-3's to connect to the center office node e. Then the optimal solution is shown in Fig. 4 and uses 12 SONET/SDH ADMs (if considering self-healing we have to double this number).

2.3. UPSR vs. BLSR/2

In [7], the economics of single-hub UPSR and single-hub BLSR/2 are justified by comparing the lower bounds on the minimum ADM cost of uniform all-to-all duplex traffic, which is essentially the minimum ADM cost of corresponding uniform one-to-all simplex traffic according to the reductions made in the previous section.

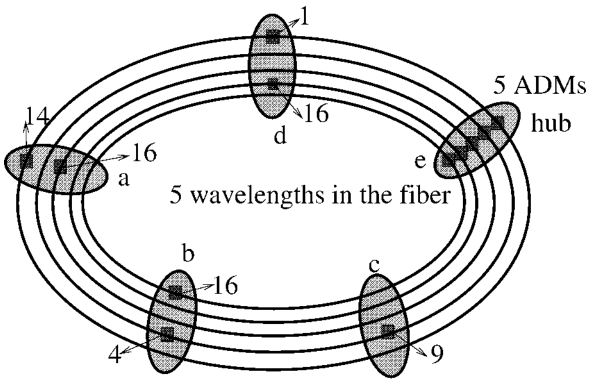

Fig. 4. The optimal grooming scheme for the given instance. Here for this instance, the communication capacity requests are: $a – e$ 30 OC-3's, $b – e$ 20 OC-3's, $c – e$ 9 OC-3's, $d – e$ 17 OC-3's. Each wavelength channel supports $g = 16$ OC-3's. Assume e is a node at some center office.

Logically, the conclusion drawn from such comparison is reasonable only if the lower bounds are sufficiently close to the optimum. Furthermore, the conclusion may still not be persuasive by just considering uniform requests. In this section, we prove that under any type of traffic, the single-hub BLSR/2 costs no more than the single-hub UPSR. The argument applies to any traffic pattern.

Theorem 2.2. *Given any set of traffic demands, the single-hub BLSR/2 costs no more than the single-hub UPSR.*

Proof. Consider any grooming of the given set of demands in UPSR. Let w be any wavelength used in working ring in the UPSR, and let r_i^w be the portion of the demand from hub to node i carried in wavelength w. Now consider the following grooming in the BLSR/2: each wavelength w is used in both rings of the BLSR/2, and in each ring the wavelength w carries $r_i^w/2$ units of demand from hub to node i for all $1 \leqslant i \leqslant N$. It's easy to see that such grooming is a valid solution and it uses the same number of ADMs as in UPSR. Thus the theorem is true. □

In Section 3.1, we will quantitate the exact cost difference if the given traffic is uniform.

2.4. Canonical grooming

In [11], it claimed that the search of optimal grooming of uniform traffic in UPSR can be confined to those canonical groomings defined by us later. We give a formal proof of the claim and generalize this property to arbitrary traffic pattern with arbitrary traffic demands in both UPSR and BLSR/2.

Given a set of demands $\{r_1, \ldots, r_N\}$ in a UPSR and the wavelength capacity g, a grooming is said to be a *canonical grooming* if at each node $1 \leqslant i \leqslant N$, its demand is carried in $\lceil r_i/g \rceil$ wavelengths, among which $\lfloor r_i/g \rfloor$ wavelengths each carries g units of demands to node i, and the remaining one, if there is any, carries $r_i \bmod g$ units of demands to node i.

Given a set of demands $\{r_1, \ldots, r_N\}$ in a BLSR/2 and the wavelength capacity g, a grooming is said to be a *canonical grooming* if at each node $1 \leqslant i \leqslant N$, its demand is carried in $\lceil r_i/(g/2) \rceil = \lceil 2r_i/g \rceil$ wavelengths (counting each wavelength used in both directions as two), among which $\lfloor r_i/(g/2) \rfloor = \lfloor 2r_i/g \rfloor$ wavelengths each carries $g/2$ units of demands to node i, and the remaining one, if there is any, carries $r_i \bmod g/2$ units of demands to node i.

The next lemma states that when looking for optimal traffic grooming, we can pay attention to only canonical groomings.

Lemma 2.3. *Given any set of demands in UPSR or BLSR/2, there is a canonical grooming with minimum ADM cost.*

Proof. We prove the lemma by transforming any given optimal grooming into a canonical grooming with the same cost in a number of steps. The procedure at each step is as follows. Suppose that the current optimal grooming is not canonical. Then at some node i, two portion of its demands, $0 < f_2 \leqslant f_1 < g$, are carried in two wavelengths w_1 and w_2 respectively. We consider two cases.

Case 1: $f_1 + f_2 \leqslant g$. We use an unused wavelength to carry the two portion of demands f_1 and f_2. Then in the new wavelength two ADMs are used. But the two ADMs used in the wavelengths w_1 and w_2 are removed. So the ADM cost does not increase.

Case 2: $f_1 + f_2 > g$. We swap all traffic except f_1 carried in wavelength w_1 with the $g - f_1$ portion within f_2 in wavelength w_2. In the resulting grooming, wavelength w_1 carries the full g units of demands to node i, and wavelength w_2 carries $f_1 + f_2 - g$ units of demands to node i. The total ADM cost remains the same.

It's easy to see that one canonical grooming can be reached after a finite number of such procedures. The resulting canonical grooming has the minimum ADM cost and thus is optimal. □

In the next section, we will apply Lemma 2.3 to find minimum ADM cost of both uniform traffic and non-uniform traffic by designing (sub)optimal canonical groomings.

3. Practical solutions

3.1. Uniform traffic grooming

In this section, we present optimal canonical grooming of uniform traffic in both single-hub UPSR and single-hub BLSR/2. We assume that the traffic demand from the hub to each other node is r.

We first consider the optimal grooming of uniform traffic in single-hub UPSR.

If $r \bmod g = 0$, then the optimal canonical grooming is unique in the sense that each wavelength carries g units of demands exclusively to some node. Thus each node contributes $2 \cdot (r/g) = 2r/g$ ADMs, half at the node itself and half at the hub. So the total ADM cost in the working fiber is $N \cdot (2r/g) = 2Nr/g$. The total ADM cost is then $4Nr/g$.

Now we assume that $r \bmod g > 0$. In any canonical grooming, at each node there are $r - r \bmod g$ portion of demands carried in $\lfloor r/g \rfloor$ wavelengths exclusively. These demands use $2N\lfloor r/g \rfloor$ ADMs in the working fiber. In any optimal grooming, the remaining demands at each node, referred to as *residue demands*, must use a minimum ADM cost. This can be achieved in the same way as in [11]. We partition the N nodes into $\lceil N/\lfloor g/r \bmod g \rfloor \rceil$ groups of at most $\lfloor g/r \bmod g \rfloor$ nodes. The residue demands of nodes in each group are carried in a single wavelength. These residue demands totally require $N + \lceil N/\lfloor g/r \bmod g \rfloor \rceil$ ADMs in the working fiber. Thus the total ADMs used in the working fiber is

$$N\left\lceil \frac{r}{g} \right\rceil + N\left\lfloor \frac{r}{g} \right\rfloor + \left\lceil \frac{N}{\left\lfloor \frac{g}{r \bmod g} \right\rfloor} \right\rceil .$$

Let

$$F(g,r,N) = \begin{cases} \dfrac{2Nr}{g}, & \text{if } r \bmod g = 0, \\ N\left\lceil \dfrac{r}{g} \right\rceil + N\left\lfloor \dfrac{r}{g} \right\rfloor + \left\lceil \dfrac{N}{\left\lfloor \frac{g}{r \bmod g} \right\rfloor} \right\rceil, & \text{otherwise.} \end{cases}$$

Then the minimum ADM cost in the working fiber is $F(g,r,N)$, and the total ADM cost is $2F(g,r,N)$.

Similarly, the minimum ADM cost in BLSR/2 is $F(g/2, r, N)$. The optimum canonical grooming can be constructed in the similar way.

The next theorem summarizes the above discussions.

Theorem 3.1. *The minimum ADM costs of uniform traffic demand with rate r in UPSR and BLSR/2 are $2F(g, r, N)$ and $F(g/2, r, N)$ respectively.*

In Section 2.3, we have proved the BLSR/2 always costs no more than UPSR under any traffic patter. When the traffic is uniform, this can be verified by the inequality

$$F\left(\frac{g}{2}, r, N\right) \leqslant 2F(g, r, N).$$

Notice that the cost difference of UPSR and BLSR/2 is $2F(g, r, N) - F(g/2, r, N)$.

3.2. Non-uniform traffic grooming

3.2.1. General approach

It was proved in [11] that the optimal grooming of non-uniform *sub-wavelength* traffic grooming for UPSR is NP-complete. By applying Lemma 2.3, we can prove that the optimal grooming of arbitrary non-uniform traffic is NP-complete in both UPSR and BLSR/2. The reduction is also made from the well-known bin packing problem.

Lemma 3.2. *Bin packing problem reduces to single hub traffic grooming problem.*

Proof. Due to the canonical lemma, assume each request r_i requires $r_i \bmod g$ tributary streams lying in one fractional wavelength and all other tributary streams lying in $\lfloor r_i/g \rfloor$ wavelengths exclusively. Each wavelength supports g tributary streams. So we have to solve a bin packing problem: to pack N objects into as few as possible bins, where the i-th object requires capacity of $r_i \bmod g$ and the bin size is g. The inverse reduction also holds. □

In the next, we present approximation algorithms. Given a traffic demands $r_1, \ldots, r_N$, a canonical grooming is constructed as follows. At each node i, we carry $r_i - r_i \bmod g$ portion of demands in $\lfloor r_i/g \rfloor$ wavelengths exclusively. Let

$$C_{base} = \sum_{i=1}^{N} \left\lceil \frac{r_i}{g} \right\rceil + \sum_{i=1}^{N} \left\lfloor \frac{r_i}{g} \right\rfloor,$$

and let Opt be the minimum ADM cost. Then the minimum ADM cost at the hub required by the residue demands is $Opt - C_{base}$.

Let $\mathcal{A}$ be any approximation algorithm for the bin-packing problem with approximation ratio of α. We apply $\mathcal{A}$ to groom the residue demands of all non-hub nodes. Then the cost at the hub required by the resulting grooming of the residue demands is at most $\alpha(Opt - C_{base})$. So the total ADM cost of the grooming constructed in this way is

$$C_{base} + \alpha(Opt - C_{base}) = \alpha \cdot Opt - (\alpha - 1)C_{base}.$$

Then the approximation ratio of this scheme is $(\alpha \cdot Opt - (\alpha - 1)C_{base})/Opt = \alpha - (\alpha - 1)C_{base}/Opt$. Notice that the number of ADMs used at the hub is at most the total ADMs used at all non-hub nodes. Hence, we have $C_{base} \geqslant Opt - C_{base}$. So the number of ADMs used by the above scheme is within $(\alpha + 1)/2$ factor of the optimum.

There are a number of bin packing approximation algorithms developed [3]. The off-line First-Fit-Decreasing (FFD) bin packing method first sorts the input objects in the decreasing order, and assigns the bins sequentially for objects. The assigned bin is the first bin that still can fit the current object. It gives a $11/9$ approximation for the minimal number of bins used [3]. In turn it gives a $10/9$ approximation non-uniform traffic grooming algorithm as following. Assume we have one queue to store all residual demands and one queue for used ADMs at the hub. First we sort the residual demands $r_i \bmod g$ at all non-hub nodes decreasingly, and put them into a queue. Then for each non-assigned residual demand in the queue, we use the first ADM with sufficient spare capacity in the used ADMs queue to carry it; if all used ADMs can not carry it, we use a new ADM to carry it and append this ADM to the used ADMs queue.

3.2.2. Special cases

For real world application, the number of tributary streams one wavelength can support is limited by SONET protocol. For example, besides historically SONET supports T1, E1, T3 and other streams, now in ISDN networks it is generally used to support OC-12 (ATM base rate 622 Mbps) and OC-3 (ATM base rate 155 Mbps) by wavelength channels with speed OC-48 and OC-192. Thus one OC-48 can support $g = 4$ OC-12's and $g = 16$ OC-3's. One OC-192 supports $g = 16$ OC-12's and $g = 64$ OC-3's.

Let $d_i = r_i \bmod g$ be the residual demand at node i, where r_i is integer traffic demand at node i. So for several specific g's, we consider how to solve the integer bin packing problem exactly. At following paragraphs we give the optimal solutions for $g = 2, 4, 8$ and the proof is omitted. We also find that some solutions are very similar (but not totally the same) with the FFD, which suggests that FFD is really a good heuristic for SONET traffic grooming problem.

The Case $g = 2$. We are considering a bin packing problem where each bin has capacity $g = 2$ and each object has volume 1. Assume we have k nodes with residual 1, then we exactly need $\lceil k/2 \rceil$ ADMs.

The Case $g = 4$. Now consider the case where $d_i \in \{1, 2, 3\}$. Assume we have n_1 1's, n_2 2's, n_3 3's among all residuals. The following steps give an optimal solution:

1. First we need n_3 ADMs for those 3's. We also can fill $\min(n_1, n_3)$ 1's to these ADMs.
2. Now we need $\lceil n_2/2 \rceil$ ADMs for those 2's. We may also fill at most 2 1's if n_2 is odd and there is any 1's remaining unfilled.
3. Now if $n_1 > n_3 + 2(n_2 \bmod 2)$, we need $\lceil (n_1 - n_3 - 2(n_2 \bmod 2))/4 \rceil$ ADM's for remaining 1's.

So exactly we need

$$n = \begin{cases} n_3 + \left\lceil \dfrac{n_2}{2} \right\rceil, & \text{if } n_1 \leqslant n_3 + 2(n_2 \bmod 2), \\ n_3 + \left\lceil \dfrac{n_2}{2} \right\rceil + \left\lceil \dfrac{n_1 - n_3 - 2(n_2 \bmod 2)}{4} \right\rceil, & \text{otherwise,} \end{cases}$$

ADMs to groom the residual traffic.

The Case $g = 8$. Now $d_i \in \{1, 2, 3, 4, 5, 6, 7\}$ and assume we have n_1 1's, n_2 2's, $\ldots$, n_7 7's. The following steps give an optimal solution:

1. First we need n_7 ADMs for those 7's, n_6 ADMs for those 6's, n_5 ADMs for those 5's. $\lceil n_4/2 \rceil$ ADMs for those 4's. Assume the set of ADMs are A_7, A_6, A_5, A_4.
2. For ADMs from A_7, we may fill at most $\min(n_1, n_7)$ 1's. Update $n_1 \leftarrow \max(0, n_1 - n_7)$.
3. For ADMs from A_6, if we still have $n_1 > 0$ 1's, we have to select 1's and 2's. We prefer to select 2's first since the remaining 1's give the most freedom for future filling. So we select $\min(n_6, n_2)$ 2's and fill into ADMs from A_6. If $n_6 > n_2$, we select another $\min(2(n_6 - n_2), n_1)$ 1's and fill into ADMs from A_6 if there is any. Update n_2 and n_1 accordingly.
4. Consider ADMs from A_5. We may select 1's, 2's or 3's or the mixture of 1's and 2's to fill. We select them in the following order (1) 3's (2) pairs of 1 and 2 (3) 2's (4) 1's. Update n_1, n_2, n_3 accordingly.

5. Select all 4's into pairs and assign each pair an ADM. We may need an extra ADM if n_4 is odd. So now we have at most 1 ADM not full from A_4. So we only need solve the remainder instance with n_1 1's, n_2 2's, n_3 3's, and an half-full ADM.
6. Merge all 3's into $\lfloor n_3/2 \rfloor$ pairs. And pack each pair into an ADM. Assume the set of ADM used at this step is A_3.
7. Fill as many as possible 2's into A_3, and maybe an non-full ADM from A_4. Update n_2.
8. Use $\lfloor n_2/4 \rfloor$ ADMs, each contains 4 2's. We maybe use an extra ADM if $4 \nmid n_2$.
9. Fill as many as possible 1's into ADMs not full used. Update n_1.
10. Use $\lceil n_1/8 \rceil$ ADMs to contain n_1 1's.

3.3. *Select speeds with two line speeds available*

In the previous section, we assume that all SONET rings have the same line speed. In this case, the higher the line speed, the smaller the number of ADMs. On the other hand, the higher the line speed, the higher the cost of the ADM. However, the cost of ADM does not increase linearly with the line speed. The cost model adopted in [7] assumes that the cost ratio between an *OC*-4n ADM and an *OC*-n ADM is 2.5. If the traffic demand is uniform, then the best line speed can be selected by comparing the total ADM cost for each line speed.

However, if we allow the SONET rings to have different line speeds, we have to partition the traffic from each node into the SONET rings of different line speeds. After the partition, the traffic grooming algorithms developed in the previous sections can be applied to the rings of any particular line speed. Thus a solution has two components, the partition of the traffic, and the groomings of the traffic in rings of each speed. Both components affect the overall cost. Because there are a very large number of possible traffic partitions, it's impossible to find the best solution by enumeration. This is true even if all traffic demands are uniform. So efficient algorithms or criteria should be developed to find traffic partitions which may lead to the minimum ADM cost. This section is intended to address this problem.

To simplify the problem, we assume that there are only two line speeds $g_1$and g_2 with $g_2 = 4g_1$ as did in [7]. We also adopt the same cost model used in [7]. We assume that the cost of an ADM of speed g_1 is one, and the cost of an ADM of speed g_2 is 2.5. A simple approach presented in [7] is that for each traffic demand with value r, assign $r \bmod g_2$ traffic to the SONET rings with speed g_1 and $r - r \bmod g_2$ traffic to the SONET rings with speed g_2. The performance of this approach comparing to the optimal assignment was not discussed in [7]. In this section, more general solutions will be developed and their optimality will also be proven. In particular, a complete optimal solution for uniform traffic demands is obtained.

3.4. *Basic properties*

As there are only two type of speeds, we call a SONET ring of speed g_1 as a low-speed ring, and a SONET ring of speed g_2 as a high-speed ring without any ambiguity. Similarly, we call a SONET ADM of speed g_1 as a low-speed ADM, and a SONET ADM of speed g_2 as a high-speed ADM. For the simplicity of presentation, g_1 is scaled to one and all demands are scaled accordingly. Thus $g_1 = 1$, $g_2 = 4$ and all demands are fractional numbers or integers.

In this section, we will study the selection of line speed in UPSR in detail. The analysis can be extended to BLSR as well. Because the ADM cost of the working ring is exactly the same as the protection ring, we can only consider the cost of the working ring. Assume the demand between node i and hub is r_i for $1 \leqslant i \leqslant n$. Then any traffic partition can be represented by an n-dimensional vector

$$f = (f_1, \ldots, f_n),$$

where $0 \leqslant f_i \leqslant r_i$ is the amount of the traffic between node i and hub placed to a low-speed ring. For any traffic partition, we can groom the traffic carried in low-speed rings and the traffic carried in high-speed rings separately.

If both the grooming of the traffic carried in low-speed rings and the grooming of the traffic carried in high-speed rings are canonical, we call the overall grooming is canonical too.

In the following, we will present some basic properties of optimal traffic partitions.

Lemma 3.3. *In any optimal traffic partition $f = (f_1, \ldots, f_n)$, $f_i < 3$ for all $1 \leqslant i \leqslant n$, and there is an optimal solution $f = (f_1, \ldots, f_n)$ with $f_i \leqslant 2$ for all $1 \leqslant i \leqslant n$.*

Proof. We prove the first part of lemma by contradiction. Let $f = (f_1, \ldots, f_n)$ be any optimal traffic partition with $f_i \geqslant 3$. Then in a canonical optimal grooming, there at least three low-speed rings devoted exclusively to node i. If we move the traffic carried in any three of these low-speed rings into high-speed ring, we save 6 low-speed ADMs and uses two new high-speed ADMs, and thus decrease the cost by 1. This contradicts to the optimality of $f = (f_1, \ldots, f_n)$. We now prove the second part of lemma by contradiction. Let $f = (f_1, \ldots, f_n)$ be any optimal traffic partition which contains the least number of entries that are more than two. Suppose $f_i > 2$ for some $1 \leqslant i \leqslant n$. Then in a canonical optimal grooming of the traffic demands $\{f_1, \ldots, f_n\}$ into low-speed rings, at least $\lceil f_i \rceil + \lfloor f_i \rfloor$ ADMs are devoted to node i. Now we place such f_i amount of traffic from node i into $\lceil f_i/4 \rceil$ new high-speed rings, i.e., set $f_i = 0$. Then in the new solution, a cost of at least $\lceil f_i \rceil + \lfloor f_i \rfloor$ is saved from the rings of speed g_1 while a cost of $5\lceil f_i/4 \rceil$ is added to the rings of speed g_2. As

$$\lceil f_i \rceil + \lfloor f_i \rfloor \geqslant 5 \left\lceil \frac{f_i}{4} \right\rceil,$$

when $f_i > 2$, the new solution has no more cost than the solution f but contains one less entries which are more than two. This contradicts to the selection of f. Therefore, the lemma is true. □

Intuitively, if a traffic can fill a high-speed ring, it should fill fully as many high-speed rings as possible to take advantage of the lower cost per bandwidth of the higher speed ring. The next lemma verifies such intuition.

Lemma 3.4. *There is an optimal traffic partition $f = (f_1, \ldots, f_n)$ with $f_i \leqslant r_i \bmod 4$ for all $1 \leqslant i \leqslant n$.*

Proof. We prove the lemma by contradiction. Let $f = (f_1, \ldots, f_n)$ be any optimal traffic partition satisfying that $f_i \leqslant 2$ for all $1 \leqslant i \leqslant n$ and the cardinality of the set

$$\{1 \leqslant i \leqslant n \mid f_i > r_i \bmod 4\},$$

is the smallest. Assume that $f_i \leqslant r_i \bmod 4$ for some node i. Then in a canonical optimal grooming of the traffic carried in high-speed rings, in addition to $\lfloor (r_i - f_i)/4 \rfloor$ high-speed rings which are devoted exclusively to node i, one high-speed ring carries the remaining $4 - f_i + r_i \bmod 4$ amount of traffic from node i. This high-speed ring must also carry traffic from other nodes, for otherwise we can fill this ring fully with the traffic from node i without any additional cost but the amount of traffic placed in low-speed rings is $r_i \bmod 4$, which contradicts to the selection of $f = (f_1, \ldots, f_n)$. Let $x_i > 0$ be the amount of the traffic carried in this ring from nodes other than node i. Then $x_i > 1$ for otherwise we can decrease the total ADM cost by 0.5 by moving x_i to a dedicated low-speed ring, which again contradicts to the optimality of $f = (f_1, \ldots, f_n)$. As

$$4 - f_i + r_i \bmod 4 + x_i \leqslant 4,$$

we have

$$1 < x_i \leqslant r_i \bmod 4 + x_i \leqslant f_i \leqslant 2.$$

This implies that x_i is from only one node, say j, for otherwise the portion of the traffic from some node is less than one and again we can decrease the total ADM cost by moving it to a dedicated low-speed ring. Now we look at the f_i amount of traffic from node i carried in low-speed rings. In a canonical optimal grooming, one ring carries the traffic of amount 1 from node i only, another ring carries $f_i - 1$ amount of traffic from node i and may carry additional traffic from other nodes. Finally we relocate all traffic in these three rings as follows. Fill the high-speed ring fully with the traffic from node i. Fill the first low-speed ring fully with the traffic from node j. In the second low-speed ring, keep the original traffic not from node i, and place r_i mod 4 amount of traffic from node i and $x_i - 1$ amount of traffic from node j. With this modification, one high-speed ADM is saved but one additional low-speed ADM is used. So the total cost is decreased by $2.5 - 1 = 1.5$, which again contradicts to the optimality of $f = (f_1, \dots, f_n)$. □

From the above lemma, there is an optimal solution in which $\lfloor r_i/4 \rfloor$ high-speed rings are dedicated $r_i - r_i$ mod 4 amount of traffic from node i for all $1 \leqslant i \leqslant n$. Thus from now on, we assume that $r_i < 4$ for all node i. For any traffic partition $f = (f_1, \dots, f_n)$, let

$$S(f) = \{1 \leqslant i \leqslant n \mid 0 < f_i < r_i\},$$
$$U(f) = \{1 \leqslant i \leqslant n \mid f_i = 0 \text{ or } r_i\}.$$

Thus the traffic from any node in $S(f)$ is carried in both low-speed rings and high-speed rings, and the traffic from any node in $U(f)$ is carried in either low-speed rings or high-speed rings but not both.

The next lemma states that at any node, if the traffic of this node is carried in both types of rings, then the amount of traffic carried in low-speed rings is at most one; and if there is some traffic carried in a high-speed ring, its amount is more than one.

Lemma 3.5. *Let $f = (f_1, \dots, f_n)$ be any optimal traffic partition. Then for any $1 \leqslant i \leqslant n$, neither $1 < f_i < r_i$ nor $0 < r_i - f_i \leqslant 1$ is possible.*

Proof. Assume that $1 < f_i < r_i$. Then in a canonical optimal grooming, the total cost of ADMs used by the traffic r_i is at least

$$2 + 1 + 2.5 = 5.5,$$

as at least 2 low-speed ADMs are needed at node i, at least 1 low-speed ADM is needed at the hub, and at least 1 high-speed ADM is required at the node i. But if the traffic r_i is entirely carried by a high-speed ring, the cost of ADMs is at most $2.5 + 2.5 = 5 < 5.5$, which contradicts to the optimality of $f = (f_1, \dots, f_n)$. Now we assume that $0 < r_i - f_i \leqslant 1$. We remove the $r_i - f_i$ amount of traffic from the high-speed ring and put it in a dedicated low-speed ring. With this modification, at least one high-speed ring is saved and two additional low-speed ADMs are used. So the total cost is decreased by

$$2.5 - 2 = 0.5,$$

which again is impossible as $f = (f_1, \dots, f_n)$ is already optimal. □

As a corollary of Lemma 3.5, in any canonical optimal grooming, any high-speed ring can carry traffic from at most three nodes.

The next lemma states that, at any node, when a traffic demand from a node is at most one, it should be always put in a low-speed ring; and when a traffic demand is more than three, it should be always put in a high-speed ring.

Lemma 3.6. *Let $f = (f_1, \dots, f_n)$ be any optimal traffic partition. Then for any $1 \leqslant i \leqslant n$, if $r_i \leqslant 1$, $f_i = r_i$; and if $r_i > 3$, $f_i = 0$.*

Proof. The first part follows directly from Lemma 3.5. Now we assume that $r_i > 3$ and $f_i > 0$. From Lemma 3.3 and Lemma 3.5, $0 < f_i \leqslant 1$, and thus $r_i - f_i > 2$. The $r_i - f_i$ amount of traffic from node i must share some traffic from other nodes, for otherwise we can put all traffic from node i in the high-speed ring and decreases the cost by at least one. From Lemma 3.5 if there is some traffic, from any node, carried in a high-speed ring, its amount is more than one. Thus the $r_i - f_i$ amount of traffic from node i share one high-speed ring with some amount, denoted by x_i, of traffic from exactly one node, say j. Note that

$$1 < x_i \leqslant 4 - r_i + f_i.$$

So we consider the following modification to a canonical optimal solution. We replace the f_i amount of traffic from node i in some low-speed ring by the f_i amount of traffic from node j. This may save one low-speed ADM. We then place the $x_i - f_i$ in a dedicated low-speed ring as

$$x_i - f_i \leqslant 4 - r_i < 1.$$

This adds two low-speed ADMs. Finally, we place all traffic from node i in the high-speed ring originally carrying the $r_i - f_i$ amount of traffic from node i and x_i amount of traffic from node j. This saves one high-speed ADM. Thus after the modification, the total ADM cost is decreased by at least $2.5 - 2 = 0.5$, which contradicts to the optimality of $f = (f_1, \ldots, f_n)$. □

The above lemma implies that if $r_i \leqslant 1$ for any node $1 \leqslant i \leqslant n$, then all traffic must be carried in low-speed rings. In particular, if the traffic is uniform with amount r, the total ADM cost is $F(1, r, n)$. If $r_i > 3$ for any node $1 \leqslant i \leqslant n$, then all traffic must be carried in high-speed rings. As in the canonical grooming, the traffic demand from any node must be carried in a dedicated high-speed ring. Thus $2n$ high-speed ADMs are needed with cost $5n$ in total. A remark is such cost only accounts for the working ring, if we consider the protection as well, the total cost should then be doubled.

4. All traffic demands are at most two

In the next lemma, we show that when the traffic demand from each node is at most two, then there is an optimal traffic partition in which none of them is carried in both low-speed rings and high-speed rings.

Lemma 4.1. *If $r_i \leqslant 2$ for all $1 \leqslant i \leqslant n$, then there is an optimal traffic partition f with $S(f) = \emptyset$.*

Proof. We prove it by contradiction. Let $f = (f_1, \ldots, f_n)$ be any optimal traffic partition with the smallest $|S(f)|$. Let $i \in S(f)$ and consider any canonical optimal grooming. From Lemma 3.5, $0 < f_i \leqslant 1$ and $r_i - f_i > 1$. Thus in any canonical optimal grooming, the traffic from node i is carried in exactly one low-speed ring and exactly one high-speed ring. We concentrate on the high-speed ring carrying the $r_i - f_i$ amount of traffic from node i. It can carry traffic from at most three nodes. First of all, it must also carry some traffic from other nodes, for otherwise we can fill it with all traffic from node i and decreases the cost by at least one. Secondly, it is impossible that this high-speed ring carries the traffic from only two nodes, for otherwise we can put all traffic from these two nodes in this high-speed ring, which can also save at least one low-speed ADM. Thus this high-speed ring must carry traffic from exactly three nodes. We denote the other two nodes other than node i by j and k. We show that $j, k \in U(f)$. Suppose to the contrary. We modify the placement of the traffic from these three nodes as follows. We use the high-speed ring to carry the whole traffic from node i and the whole traffic from node j and nothing else. We add at most two new dedicated low-speed rings to carry the traffic from node k. We save one high-speed ADM and add at most two more low-speed ADMs. Thus the modification decreases the total cost by at least 0.5, which contradicts to the optimality of f. Therefore both j and k are in $U(f)$, that is all traffic from node j and node k are carried in the high-speed ring. As $r_i - f_i > 1$,

$$r_j + r_k \leqslant 4 - (r_i - f_i) < 4 - 1 = 3.$$

So we can modify the placement of the traffic from nodes i, j and k as follows. We place all the traffic from node i and nothing else in two new low-speed rings, and use at most three new low-speed rings to carry all traffic from nodes j and k. Then four high-speed ADMs are saved, and at most ten low-speed ADMs are added. The resulting solution has the same cost as f but it contains one less nodes whose traffic are carried in both low-speed rings and high-speed rings. This contradicts to that $|S(f)|$ is the smallest. Thus the lemma is true. □

4.1. All traffic demands are at most $3/2$

The next lemma states that when the traffic demand from each node is at most $3/2$, then we can put all traffic in the low-speed rings.

Lemma 4.2. *If* $r_i \leqslant 3/2$ *for all* $1 \leqslant i \leqslant n$, *then the traffic partition* $f = (f_1, \dots, f_n)$ *where* $f_i = r_i$ *for all* $1 \leqslant i \leqslant n$ *is optimal.*

Proof. We prove it by contradiction. Let $f = (f_1, \dots, f_n)$ be any optimal traffic partition with $f_i = 0$ or r_i for all $1 \leqslant i \leqslant n$ and the smallest number of zero entries. Consider any canonical optimal grooming. As any high-speed ring carries traffic from at most three nodes. We consider the following three cases. If a high-speed ring carries traffic from only one node, we can use at most two new low-speed rings to carry all traffic from this node. This modification saves two high-speed ADMs and uses at most four low-speed ADMs. Thus the cost is decreased by 0.5, which contradicts to the optimality of $f = (f_1, \dots, f_n)$. If a high-speed ring carries traffic from two nodes, we can use at most three new low-speed rings to carry all traffic from these nodes. This modification saves three high-speed ADMs and uses at most seven low-speed ADMs. Thus the cost is decreased by 0.5, which also contradicts to the optimality of $f = (f_1, \dots, f_n)$. If a high-speed ring carries traffic from three nodes, we use at most four new low-speed rings to carry all traffic in this high-speed ring. This modification saves four high-speed ADMs and uses at most ten low-speed ADMs. The resulting solution has the same cost as f, but the number of zero entries is decreased by three, which contradicts to the selection of f. Therefore, the lemma is true. □

The above lemma implies if the traffic is uniform with demand $r \leqslant 3/2$, the minimum cost of ADMs is $F(1, r, n)$.

4.2. All traffic demands are more than $3/2$

We now consider the traffic with demands more than $3/2$ but at most two.

Lemma 4.3. *Suppose that* $3/2 < r_i \leqslant 2$ *for all* $1 \leqslant i \leqslant n$. *If* n *is even, then the traffic partition* $f = (f_1, \dots, f_n)$ *where* $f_i = 0$ *for all* $1 \leqslant i \leqslant n$ *is optimal. If* n *is odd, then for any* $1 \leqslant j \leqslant n$ *the traffic partition* $f = (f_1, \dots, f_n)$ *where* $f_i = 0$ *for* $i \neq j$ *and* $f_j = r_j$ *is optimal.*

Proof. We also prove it by contradiction that there is an optimal traffic partition $f = (f_1, \dots, f_n)$ with $f_i = 0$ or r_i for all $1 \leqslant i \leqslant n$ and at most one non-zero entries. Let $f = (f_1, \dots, f_n)$ be any optimal traffic partition with $f_i = 0$ or r_i for all $1 \leqslant i \leqslant n$ and the smallest number of non-zero entries. Assume that $f_i = r_i$ and $f_j = r_j$. Consider any canonical optimal grooming. There are two low-speed rings devoted to node i and two low-speed rings devoted to node j. We relocate the traffic from node i and node j to one new high-speed ring. This modification saves 8 low-speed ADMs and uses 3 high-speed ADMs. The total cost is decreased by 0.5. This contradicts to the optimality of $f = (f_1, \ldots, f_n)$. Now let $f = (f_1, \ldots, f_n)$ be any optimal traffic partition with $f_i = 0$ or r_i for all $1 \leqslant i \leqslant n$ and at most one non-zero entries. Note that in any canonical optimal grooming, each high-speed ring must carry traffic from two nodes, for otherwise we can move it to two low-speed rings and the cost would be decreased by 1. Thus if n is even, $f_i = 0$ for all $1 \leqslant i \leqslant n$, and if n is odd, there is exactly one $1 \leqslant i \leqslant n$ with $f_i = r_i$. □

From the above lemma, if $3/2 < r_i \leqslant 2$ for all $1 \leqslant i \leqslant n$ we can provide optimal grooming as follows. If n is even, then all traffic is carried in high-speed rings, and each high-speed ring carries the whole traffic from two nodes. It requires totally $1.5n$ high-speed ADMs (in the working ring only) with total cost $3.75n$. If n is odd, then the traffic from one node is carried in two low-speed rings to carry the whole traffic from a node, and the traffic from all other nodes are carried in the high-speed rings, with each ring dedicated to a pair of nodes. Thus 4 low-speed ADMs and $1.5(n-1)$ high-speed ADMs are used. So the total ADM cost is

$$4 + 1.5(n-1) \cdot 2.5 = 3.75n + 1.5.$$

5. All traffic demands are more than two

In general, each high-speed ring can carry traffic from at most three nodes. The next lemma states that if all traffic demands are more than two, then in any canonical optimal grooming no high-speed ring can carry traffic from three nodes.

Lemma 5.1. *If $r_i > 2$ for all node i, then in any canonical optimal grooming each high-speed ring carries traffic from at most two nodes.*

Proof. We prove it by contradiction. Consider a canonical optimal grooming with traffic partition $f = (f_1, \ldots, f_n)$. Assume that three nodes i, j and k appear in a high-speed ring. Then $i, j, k \in S(f)$ for otherwise

$$(r_i - f_i) + (r_j - f_j) + (r_k - f_k) > 2 + 1 + 1 = 4.$$

As

$$(r_i - f_i) + (r_j - f_j) + (r_k - f_k) \leqslant 4,$$

we have

$$f_i + f_j + f_k \geqslant r_i + r_j + r_k - 4 > 2.$$

As $f_k \leqslant 1$, $f_i + f_j > 1$, so are $f_i + f_k$ and $f_j + f_k$. This means that all the three nodes must appear in three distinct low-speed rings. Assume these three rings carry x_i, x_j and x_k amount of the traffic from other nodes respectively. Then we have

$$x_i + x_j + x_k \leqslant 3 - (f_i + f_j + f_k) < 1.$$

Note that

$$r_i + r_j + r_k \leqslant f_i + f_j + f_k + 4 \leqslant 7.$$

As $r_k > 2$, $r_i + r_j < 5$, so are $r_i + r_k$ and $r_j + r_k$. Now we relocate the traffic carried in these three low-speed rings and the high-speed ring as follows. We place the whole traffic from node i in the high-speed ring, place the whole traffic from node j and $4 - r_j$ amount of traffic from node k in a new high-speed ring, and place $r_j + r_k - 4$ amount of traffic from node k in a low-speed ring as

$$0 < r_j + r_k - 4 < 1.$$

The x_i, x_j and x_k amount of the traffic from other nodes are carried exclusively in another low-speed ring. After the relocation, we save three low-speed ADMs and add one high-speed ADM. So the total cost is decreased by 0.5, which is a contradiction. □

The following lemma states that if all traffic demands are greater than two, we can concentrate on those canonical grooming in which exactly one node in each high-speed ring has its whole traffic carried in this high-speed ring.

Lemma 5.2. *If $r_i > 2$ for all $1 \leqslant i \leqslant n$, then there is a canonical optimal grooming in which exactly one node in each high-speed ring has its whole traffic carried in this high-speed ring.*

Proof. We prove it by contradiction. Consider a canonical optimal grooming with traffic partition $f = (f_1, \dots, f_n)$ with $f_i \leqslant 2$ for all $1 \leqslant i \leqslant n$. From Lemma 3.5, $f_i \leqslant 1$ for all $1 \leqslant i \leqslant n$. Thus for all $1 \leqslant i \leqslant n$,

$$r_i - f_i > 2 - 1 = 1.$$

If a high-speed carries traffic from only one node, then it must carry the whole traffic from that node. Now we consider a high-speed ring which carries traffic from two nodes $i, j \in S(f)$. We relocate the traffic from node i and node j as follows. The high-speed ring carries r_i amount of traffic from node i, and $4 - r_i$ amount of traffic from node j. We replace the original f_i amount of traffic from node i in a low-speed ring by f_i amount of traffic from node j. The cost of the result grooming is not increased. We repeat such procedure for all high-speed rings which each carries traffic from two nodes that are both in $S(f)$. In the end, we come up with a grooming in which each high-speed ring carries the whole traffic from at least one node. Finally we use a canonical grooming to place all traffic carried in low-speed rings. Then the resulting grooming satisfies the requirement given in the lemma. □

5.1. All traffic demands are more than $5/2$

When all traffic demands are greater than $5/2$, the following lemma gives an optimal traffic partition.

Lemma 5.3. *If $r_i > 5/2$ for all $1 \leqslant i \leqslant n$, then the traffic partition $f = (f_1, \dots, f_n)$ where $f_i = 0$ for all $1 \leqslant i \leqslant n$ is optimal.*

Proof. We consider a canonical optimal grooming with the traffic partition $f = (f_1, \dots, f_n)$ in which each high-speed ring carries the whole traffic from at least one node. Assume that $f_i > 0$ for some $1 \leqslant i \leqslant n$. From Lemma 3.5, $f_i \leqslant 1$. Furthermore, the high-speed ring where node i appears must carry the whole traffic from another node, say j, and no other traffic. As

$$(r_i - f_i) + r_j \leqslant 4,$$

we have

$$f_i \geqslant r_i + r_j - 4 > 1.$$

This contradicts to $f_i \leqslant 1$. □

The above lemma suggests that if all traffic demands are more than $5/2$, we should carry all traffic in high-speed rings. In this optimal traffic partition, the canonical grooming is unique and each high-speed ring carries exclusively the whole traffic from only one node. Thus the minimal total ADM cost (in the working ring) is $5n$.

5.2. *All traffic demands are at most* $5/2$

Finally we consider the traffic with demands at most $5/2$ but more than two. The next lemma states that if all traffic demands are at most $5/2$, then in any optimal grooming there is at most one high-speed ring which carries exclusively the whole traffic from exactly one node.

Lemma 5.4. *If* $r_i \leqslant 5/2$ *for all* $1 \leqslant i \leqslant n$*, then in any optimal grooming at most one high-speed ring carries exclusively the whole traffic from exactly one node.*

Proof. We prove it by contradiction. Consider an optimal grooming with traffic partition $f = (f_1, \dots, f_n)$ in which there are two high-speed ring dedicated to node i and node j repulsively. We relocate the traffic from node i and node j as follows. We place r_i amount of traffic from node i, and $\min\{4 - r_i, r_j\}$ amount of traffic from node j on one high-speed ring, and if $r_i + r_j > 4$ we place $r_i + r_j - 4$ amount of traffic from node j on one low-speed ring. This modification saves one high-speed ADM and adds at most two low-speed ADMs. The cost is decreased by at least 0.5, which is a contradiction. □

From Lemma 5.1, 5.2 and 5.4, if n is even and $2 < r_i \leqslant 5/2$ for all $1 \leqslant i \leqslant n$, then there is a canonical optimal grooming in which half nodes have their traffic carried in high-speed rings and half node have their traffic carried in both high-speed rings and low-speed ring, and each high-speed is *fully* filled with the whole traffic from one node in the first half and a portion of traffic from a node in the second half. If n is odd and $2 < r_i \leqslant 5/2$ for all $1 \leqslant i \leqslant n$, then there is a canonical optimal grooming in which the traffic from one node is carried exclusively in a high-speed ring and the traffic from other nodes are carried in the same way as the number of nodes is even. However, how to select the set of nodes to be carried wholly in high-speed rings and how to form node pairs to appear in high-speed rings remains open. But if the traffic is uniform, these two questions can be easily solved. We can select any $\lceil n/2 \rceil$ nodes to be carried wholly in high-speed rings, and the pairing between those nodes and the remaining nodes can be selected arbitrarily. Thus, for uniform traffic with demand $2 < r \leqslant 5/2$, the total ADM cost in the working ring is

$$3.25n + F\left(1, 2r - 4, \frac{n}{2}\right),$$

if n is even, and is

$$5 + 3.25(n-1) + F\left(1, 2r - 4, \frac{n-1}{2}\right) = 1.75 + 3.25n + F\left(1, 2r - 4, \frac{n-1}{2}\right),$$

if n is odd.

6. Summary

For uniform traffic demands, we have provided optimal traffic partition and grooming, which is summarized in Table 1. For non-uniform traffic demands, optimal or suboptimal solutions have been developed depending on the range of all demands. If all demands are at most 1.5, then all of them are carried in low-speed rings. If all traffic demands are greater than 1.5 but less than two, then with even n, all of them are carried in high-speed rings and the total cost of ADMs in the working ring only is $3.75n$; with odd n, all of them except an arbitrary one are carried in high-speed rings and the total cost of ADMs in the working ring only is $3.75n + 1.5$. Such costs remain the same as long as all demands are greater than 1.5 but less than two. If all traffic demands are greater than 2.5, all of them are carried in high-speed rings and the total cost of ADMs in the working ring only is $5n$. Such cost also remain

Table 1
Select Line Speeds For UPSR

Range of all r's	$(f_1, f_2, \dots, f_n)$
$(0, 1\frac{1}{2}]$	$f_i = r, \forall i$
$(1\frac{1}{2}, 2]$, $n = 2k$	$f_i = 0, \forall i$
$(1\frac{1}{2}, 2]$, $n = 2k+1$	$f_i = 0, \forall i \neq j; f_j = r$
$(2, 2\frac{1}{2}]$	$f_{2i-1} = 0$, $f_{2i} = 2r - 4$
$(2\frac{1}{2}, 4]$	$f_i = 0, \forall i$

Table 2
Select Line Speeds For BLSR/2

Range of all r's	$(f_1, f_2, \dots, f_n)$
$(0, \frac{3}{4}]$	$f_i = r, \forall i$
$(\frac{3}{4}, 1]$, $n = 2k$	$f_i = 0, \forall i$
$(\frac{3}{4}, 1]$, $n = 2k+1$	$f_i = 0, \forall i \neq j; f_j = r$
$(1, 1\frac{1}{4}]$	$f_{2i-1} = 0$, $f_{2i} = 2r - 2$
$(1\frac{1}{4}, 2]$	$f_i = 0, \forall i$

the same as long as all demands are greater than 2.5. When all traffic demands are greater than two but less than 2.5, the solution is a little complicated. We first pair up the n nodes. If n is odd, some node is stand-alone and its whole traffic is carried in a high-speed ring. For each pair of nodes i and j, we use a high-speed ring to carry the whole traffic from node i and the remaining capacity is used to carry the traffic from node j.

The above argument is restricted to UPSR. However, it can be extended to BLSR as well. Table 2 lists the optimal traffic partition of uniform traffic demands.

References

[1] G. Călinescu and P.-J. Wan, On ring grooming in optical networks, submit for publication, 1999.

[2] G. Călinescu and P.-J. Wan, Approximation algorithms for optimal generation of primitive rings, submit for publication, 1999.

[3] E.G. Coffman, Jr., M.R. Garey and D.S. Johnson, Approximation algorithms for bin-packing – a survey, in: *Approximation Algorithms for NP-Hard Problems*, PWS Publishing Company, Boston, 1997, pp. 46–93.

[4] C. Colbourn and P.-J. Wan, Minimizing drop cost for SONET/WDM networks with $1/8$ wavelength requirements, submit for publication, 1999.

[5] O. Gerstel, P. Lin and G. Sasaki, Cost effective traffic grooming in WDM rings, in: *Proc. IEEE INFOCOM '98*, Vol. 1, 1998, pp. 69–77.

[6] O. Gerstel, P. Lin and G. Sasaki, Wavelength assignment in a WDM ring to minimize cost of embedded SONET rings, in: *Proc. IEEE INFOCOM '98*, Vol. 1, 1998, pp. 94–101.

[7] O. Gerstel, P. Lin and G. Sasaki, Combined WDM and SONET network design, in: *Proc. IEEE INFOCOM '99*, 1999, pp. 734–743.

[8] L.W. Liu, X.-Y. Li, P.-J. Wan and O. Frieder, Wavelength assignment in WDM rings to minimize SONET ADMs, in: *IEEE INFOCOM '2000*, 2000, pp. 1020–1025.

[9] X.-Y. Li, P.-J. Wan and L.W. Liu, Select line speeds for traffic grooming, in: *Proc. IEEE ICC 2000*, Vol. 1, 2000, pp. 495–499.

[10] X.-Y. Li, P.-J. Wan, L.W. Liu and O. Frieder, Practical traffic grooming scheme for single-hub SONET/WDM rings, in: *Proc. LCN 2000*, 2000, pp. 556–564.

[11] E. Modiano and A. Chiu, Traffic grooming algorithms for minimizing electronic multiplexing costs in unidirectional SONET/WDM networks, in: *CISS '98*, Princeton, NJ, 1998.

[12] B. Mukherjee, *Optical Communication Networks*, McGraw-Hill, New York, 1997.

[13] J. Simmons, E. Golstein and A. Saleh, Quantifying the benefit of wavelength Add-Drop in WDM rings with distance-independent and dependent traffic, *Journal of Lightwave Technology* **17**(1) J (1999), 48–57.

[14] P.-J. Wan and C. Colbourn, Combinatorial design of bidirectional SONET/WDM rings with $1/2$ and $1/4$ wavelength uniform traffic, submit for publication, 1999.

[15] X. Zhang and C. Qiao, An effective and comprehensive solution to traffic grooming and wavelength assignment in WDM rings, in: *SPIE Proc. of Conf. All-optical Networking*, Vol. 3531, 1998, pp. 221–232.